Model Perspective: Digital or Analog?

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Abstract

What the reality is and what it is described in theory or model is not one thing. What we use and understand is just a model. For answering this question in a model, we have two ways: to include them in basic hypotheses or deduce them from basic hypotheses. I’ll try to check the two approaches. Further more, what we see in macroscopic scale is absolutely analog. If the answer in the model we use is digital, I want to know how could digital units form an analog representation.

1. Introduction & Primarily thoughts on this topic

Is reality digital or analog? The point here is that the reality is continuous or discrete. From the view of a person without much understanding of physics, the answer is probably analog. The length of everyday object seems to be completely analog. Some three thousand years age Chinese sage Chuang Tzu made his famous remark "A foot of hammer, day take the half, various world-periods inexhaustible" [1]. Perhaps analog view is a natural one, and because of this it was shared by a great quantity of people soon after the dawn of human civilization.

The digital perspective, although seems not so natural, also existed long ago. In ancient Greece, where there was a dense philosophical atmosphere, Leucippus and his famous student Democritus developed the ancient atomist [2]. They believed that the reality is made of numerous inseparable atoms, which were digital units in their theory. This philosophy inspired scientists to develop modern atomism.

While philosophy can inspire thoughts, a merely philosophical approach is of no use for answering the question "digital or analog" here. In my opinion, a philosophy or a theory is just a model. We can make any assertion as long as it is self-consistent. However, experiments and proofs are needed to make sure the validity of a particular model. Without checking with experiments or proofs, it is just a tricky brain game that we can play for endless time, but it has no practical use in this way. Besides, even if a model corresponds with any experiments we have done, we still cannot say that the reality is just what the model is. We can never know the result of the next experiment. Besides, what we understand in the model and what it really is is not one thing. The reality and a model could produce the same results but in fact the internal mechanism is quite different. That's possible. Anyway, the point here is that discussing which model is the best and reflects the reality is somewhat nonsense [3]. What we need is a model that works well in both fitting the past and predicting the future.

2. On basic assumptions.

For a model or a theory, a set of basic assumptions are needed for reasoning. Therefore, there are two possible approaches to answer the question "digital or analog".

The first way is to directly assume the reality is digital or analog. Let that be included in basic assumptions and use them to build up the whole system. Actually this is what most theory or model did in the past. For example, particles are basic units when we analyze problems in the field of classical mechanics, and physical quantities of particles are assumed to be analog. Actually this way was adopted since Leucippus and Democritus, and it had worked well until the beginning of the 20th century. "Clouds" shadowed the sky of the physics [4]. Building a model in this way seems not effective. A more in depth understanding is needed. That is, excluding the reality is digital or analog from basic assumptions, let that be a logical deduction of the basic assumptions.

Another way is the digital approach. Though digital and analog seems to be compatible in a philosophical way [5][6], it's beyond my knowledge now and it's not the subject here. What's more, a philosophical approach is somewhat useless, as I discussed in the third paragraph. I'll try to develop my opinion in a purely model perspective.

For looking into a typical digital model, I'll check the quantum mechanics I have just learnt last semester in the course of atomic physics [7][8]. Basic assumptions of the quantum mechanics are as follows [9]:

1. For every quantum state, there is a relevant wave function represent this state. And the square of this wave function's modulus represent the probability of this quantum state.
2. The Uncertainty Principle: The result of multiplying the uncertainty of location Δq and the uncertainty of momentum Δp is no less than a constant. $\Delta q \Delta p \leq \frac{\hbar}{2}$.
3. If $|\psi_1\rangle$ is a quantum state of a particle, $|\psi_2\rangle$ is another quantum state of the same particle, then $|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle$ is also a quantum state of the same particle (c_1 and c_2 are two constants). In other words, quantum states can be added, and the result is also a quantum state.
4. The wave function of the quantum state must be consistent with Schrodinger Equation: $i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = (\frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r}))\psi(\mathbf{r},t)$.

Although the concept of the particles are still used here, physical quantities are not assumed to be analog or digital. In fact, there's nothing about the topic of "analog or digital" here in the basic assumptions. It's more like basic assumptions in a mathematical theory as the Euclidean Geometry, especially the last three. And the answer to the question "analog or digital" becomes the result of mathematical logic based on the four basic assumptions above. The potential well of infinite depth serves as a good example. Now let's look into the problem of potential well to follow the mathematical logic that how basic assumptions produce the result of digital.

The condition of the potential well problem is stated as this: the potential field of a particle is not time varied, and in the area of $0 \leq x \leq a$, $0 \leq y \leq b$, $0 \leq z \leq c$, its potential $V(\mathbf{r}) = 0$. Outside this area its potential is infinite ($V(\mathbf{r}) \to \infty$).

Since in this problem the potential field is not time varied, the Schrodinger Equation became equivalent to the stationary Schrodinger

Equation $-\frac{\hbar^2\nabla^2\psi(\mathbf{r})}{2m}=E\psi(\mathbf{r})$, which is easier to solve than the standard Schrodinger Equation. The stationary Schrodinger Equation is a typical second partial differential equation. The process of its solution is as follows [10][11]:

$\psi(\mathbf{r})$ can be expressed as a form of multiplication: $\psi(\mathbf{r})=\psi(x)\psi(y)\psi(z)$.

Substitute this form into the stationary Schrodinger Equation: $-\frac{\hbar^2}{2m}(\psi(y)\psi(z)\frac{\partial^2\psi(x)}{\partial x^2}+\psi(x)\psi(z)\frac{\partial^2\psi(y)}{\partial y^2}+\psi(x)\psi(y)\frac{\partial^2\psi(z)}{\partial z^2})=E\psi(\mathbf{r})$.

Divide the equation by $\psi(\mathbf{r})$ in both sides: $-\frac{\hbar^2}{2m}(\frac{\partial^2\psi(x)}{\psi(x)\partial x^2}+\frac{\partial^2\psi(y)}{\psi(y)\partial y^2}+\frac{\partial^2\psi(z)}{\psi(z)\partial z^2})=E$.

The result seems quite symmetrical. The right side of the equation has nothing to do with variables x, y, and z. To make this equation valid, the left side should also have nothing to do with the variables. So it can only be a constant. Therefore, we can assume: $\frac{\partial^2\psi_1(x)}{\psi_1(x)\partial x^2}=-c_1^2$, $\frac{\partial^2\psi_2(y)}{\psi_2(y)\partial y^2}=-c_2^2$, $\frac{\partial^2\psi_3(z)}{\psi_3(z)\partial z^2}=-c_3^2$. c_1 , c_2 , c_3 are three constants.

$$\frac{\partial^2\psi_1(x)}{\psi_1(x)\partial x^2}=-c_1^2\rightarrow\psi_1(x)=A_1\sin(k_1x+\delta_1)$$

Similarly, $\psi_2(y)=A_2\sin(k_2y+\delta_2)$, $\psi_3(z)=A_3\sin(k_3z+\delta_3)$.

Considering the boundary conditions:

$$\psi_1(0)=0,\psi_1(a)=0,\psi_1(x)\rightarrow\infty(x<0\text{ or }x>a)$$

$$\psi_2(0)=0,\psi_2(b)=0,\psi_2(y)\rightarrow\infty(y<0\text{ or y}>b)$$

$$\psi_3(0)=0,\psi_3(c)=0,\psi_3(z)\rightarrow\infty(z<0\text{ or z}>c)$$

So, $k_1=\frac{n_1\pi}{a},k_2=\frac{n_2\pi}{b},k_3=\frac{n_3\pi}{c}$, n_1,n_2,n_3 are natural numbers. And $\delta_i=0(i=1,2,3)$, $c_i=k_i(i=1,2,3)$. A_1,A_2,A_3 can be figured out by normalization $\int_{-\infty}^{\infty}|\psi(x_i)|^2\,dx_i=1(i=1,2,3)$, but it's not needed here for my discussion so I won't calculate it here.

Therefore, the result is $\psi(\mathbf{r}) = A\sin(\frac{n_1\pi x}{a})\sin(\frac{n_2\pi y}{b})\sin(\frac{n_3\pi z}{c})$. And the energy of this particle (energy eigenvalue): $E = \frac{n_1^2\pi^2}{a^2} + \frac{n_2^2\pi^2}{b^2} + \frac{n_3^2\pi^2}{c^2}$ (n_1, n_2, n_3 are natural numbers as I mentioned above). The digital result is here. It's discrete. It's based on nothing but purely mathematical logic, and the basic assumptions for the reasoning mentioned nothing about digital or analog.

In the model we use today like quantum mechanics, we can deduce digital result from more in-depth hypotheses. For a model perspective, that could be the answer to the question "digital or analog". However, everyday objects are completely analog. Now I'll try to find out how digital units form an analog representation.

3. How may digital Units form an analog representation

Everyday objects are almost completely analog. However, if we look at matter in a microcosmic scale, things are quite the contrary. How do two contradictory aspects be consistent in one universe? Perhaps infinite and statistical regularity is a useful approach here. If we can solve the problem how microcosmic digital units form a macroscopic analog representation, then the contradiction here is solved. I'll try to find out how digital units form an analog representation through two important experiment made in history.

First, if digital units are small enough, summing up numerous digital units can form an analog representation. Actually it's the reason what we use to explain natural phenomenon in most cases as far as I can see. Miligen Petroleum Dripping Experiment [12] showed that very clearly. Electric quantity seemed quite analog both in everyday life and in macroscopic experiments before. But Miligen's experiment proved that electric quantity is digital. It is because its digital units are so small that makes its sum appears to be analog.

Another possible approach is by interaction between digital units. Perhaps some interaction may form an analog representation. Besides, these digital units may come out from an analog field. We can compare it with the double-slit interference. I consider light as analog here, when it pass through the slits it become digital in space distribution. What we see at last is analog (although varied in light intensity). There are possibilities that this is what it is in reality, like fields->particles->macroscopic scale matter. This is just a sort of philosophical thinking or even imagination and in need of proof since I lack knowledge here.

4 .Conclusion & Further discussion

Is reality digital or analog? We don't know the answer yet, personally I doubt if there is such an answer within human's reach. What's useful for us is building up a model that works well in both explaining the past and predicting the future. Perhaps the question could be changed to "are physical quantities analog or digital in the model that works well".

By simply assuming the reality is digital or analog seems not working. Setting more in-depth hypotheses may work as we did in quantum mechanics. Although it seems work well till now, we don't know what it will develop to be in the future or even be abolished. No matter what model we use, the model must be self-consistent, and more importantly it should be consistency with natural phenomenons and experiments.

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