The Cosmic Code

Gene T. Yerger

Abstract

The classical description of nature is characterized by the notion of continuity. In the search for a digital reality underlying this representation of the physical world it would be advantageous to identify a property of elementary particles such as electrons that would support such a discrete quantum perspective. In my theory, I propose that the inherent angular momentum known as spin that is possessed by all fundamental particles is just such a characteristic. Spin is an intrinsic property of electrons much like mass and charge. Most importantly it is quantized, meaning that when measured, the spin along any direction can only take a number of discrete values. In the case of the electron, the spin can take on only two orientations: either parallel to the direction in which it is being measured (spin up) or anti-parallel to the direction it is being measured (spin down.) I will argue that this two valued degree of freedom constitutes a part of a *cosmic code* that serves to support an electron's internal periodic mechanism – the electron clock - that is responsible for the actualization of the reality of the particle itself.

Introduction: The Story of Spin

At the dawn of the twentieth century the physicist Max Planck, who is widely regarded as the father of the *old quantum theory,* proposed that electromagnetic energy could be absorbed and radiated only in *discrete packets* that he called *quanta* [1].This result was the first appearance of a new idea, that natural law on small scales is characterized not by continuity but by discreteness.

In his model of the hydrogen atom, Niels Bohr, the father of the *new quantum theory*, borrowed Planck's idea of an energy quantum and postulated that electrons could only occupy certain stable orbits around the nucleus [2]. Each stable orbit would have a fixed amount of energy, which had to be a multiple of Planck's basic quantum. In this scheme, there were simply no orbits in between those that an electron could occupy; electrons could indeed radiate away their orbital energy, but only in discrete packets equal to the energy difference between two orbits.

Bohr's preliminary work on the hydrogen atom assigned *quantum numbers* to the possible orbital states. Each quantum number was associated with a physical property of the orbit that could affect the electron's energy. In the hydrogen atom, for instance, there are quantum numbers identified by the letters **n**, **m** and **l** which represent respectively the radius, speed, and spatial orientation of an electron's orbit. In Bohr's scheme, changes in an electron's energy could only occur in *integer* increments of the fundamental energy quantum determined by Planck. The success of this model further reinforced the notion that, at least on the microscopic level, energy was something that existed in discrete packets rather than over a continuous range.

Atomic spectroscopy had also revealed the splitting of spectral lines into triplets when atoms are placed in an external magnetic field. This is called the Zeeman effect. For some atoms, however, certain spectral lines would split into *two* rather than three – an anomaly, and difficult to explain. Arnold Sommerfeld explained the *anomalous* Zeeman effect by proposing an additional degree of freedom in the electrons [3]. This required a *fourth* quantum number, in addition to the three that Bohr used to define the electron's orbit. This quantum number had to describe a heretofore unnoticed angular momentum associated with the electron itself.

In November 1925, two young Dutchmen from Leiden, George Uhlenbeck and Samuel Goudsmit, published a paper proposing that if the electron were treated as an extended object rather than a point, it could possess an additional degree of freedom in the form of a rotation or *spin* [4].In other words, they imagined the electron as a small sphere rotating on its axis as it orbited the nucleus of the atom, just as the Earth rotates on its axis as it revolves around the Sun. The rotating electrical charge of the electron should therefore create a small intrinsic magnetic field (also called a magnetic moment). It turned out that the magnetic moment associated with this spin *could take on only two orientations* in an external magnetic field: *parallel or anti-parallel to the field lines* and provided the two degrees of freedom required. The word *spin* was forever attached to this intrinsic feature of an electron, and provided a nice two-valued quantum number: one for *spin up* (parallel to the magnetic field) and the other for *spin down* (anti-parallel to the magnetic field). The very concept of spin, however, remains something of a mystery to this day for two reasons.

The first involves the measurement of spin. If you set up a coordinate system to determine the orientation of the electron, the spin vector will point either parallel or anti-parallel to the chosen coordinate axis with equal probability (50 % up and 50% down). Unmeasured, the electron's spin vector orientation is unknown. Physicists say that the electron has no *definite* spin. At the instant of measurement, however, the spin becomes definite and the vector points in one of the two directions along the chosen coordinate.

The second uniquely quantum characteristic of spin is that when an electron is rotated through a full 360 degrees, its spin does not return to its original position. Only an *additional* 360 degree rotation will return the spin to its original state. The term *spin* is therefore somewhat of a misnomer, a happenstance of early theoretical developments. But spin will play a major role in all that is to follow, so for now I will merely suggest trying to suppress the mental picture of a *spinning* point charge. It should simply be treated as one more fundamental property of an electron (or any other particle), like charge and mass, that physicists hope one day to explain.

The Schrödinger Equation

The time-dependent Schrödinger equation is one of the most fundamental equations of quantum physics and describes how the wave function of a physical system evolves. The wave function represents the probability that the electron is in a particular region, but is not related to the *substance* of the electron itself. The wave function was said to specify the *state* of a quantum system, a radical departure from classical physics. In classical mechanics, when the state of a system is fully specified, one can determine any given outcome; in quantum systems even a full specification cannot assign any definite certainty to a result. Rather, the results of a potential measurement remain *indefinite*. If an experimental procedure is performed on the system to elicit a definite result (i.e., a measurement), then according to quantum mechanics the outcome is a matter of *objective chance* (i.e. a roll of the dice).

The probabilities of quantum mechanics can be represented by vectors in a complex (that is, using complex numbers as opposed to real numbers) *Hilbert space* of possible states. The Hilbert space is simply an infinitely long list of the idealized wave functions that would give exact values of the quantity to be measured. The *state vector* is just a series of (complex) numbers called *amplitudes* identifying how much of each state is in the wave function, and thus the probability that the corresponding measurement outcome will occur.

The Schrödinger equation quantifies the evolution of the state vector over time, and the state vector characterizes the *limit* of our physical knowledge regarding a quantum mechanical system. It only represents our statistical knowledge of probable experimental outcomes. But even the information in the state vector is at once removed from our knowledge, because the probabilities themselves are not represented by the wave function *or* the state vector but by the *product* of the wave function and its complex conjugate. This means that *the wave function itself has no direct physical reality*. Rather, the real probability for the particle to be found at each point in space is given by the *absolute value squared* of the wave function. This process of *complex conjugation* yields the probability density that can describe an uncertain or unknown quantity or give the probability of finding a particle within a certain volume of space.

The Schrödinger equation, however, failed to address several key issues. The first weakness is that although the evolution of the wave function is described by the equation, there is no explicit (mathematical) mechanism for *collapsing the wave function* and actualizing a particular physical reality. Schrödinger's equation only describes the behavior of quantum systems in the *absence* of measurement, which is not in itself very useful. In other words, at one instant the electron could be almost anywhere within a space defined by the wave function; an instant later the process of measurement has changed the wave function in such a way that the electron has a nearly 100% chance of being located at the observation point. This implies that the act of observation somehow *instantly* changes the wave function into a new form that is consistent with the measurement. Whatever its cause, this process appears to actualize the reality that we experience. This is known as the *measurement problem*. A second weakness is that the equation does not conform to the demands of special relativity. This failing leads it to exclude an entire class of particles known collectively as antimatter. Its third and most important limitation, in terms of my theory, is that it does not account for or explain the *spin* degree of freedom. Accounting for its effects requires that spin be introduced *ad hoc*, not as a natural consequence of the fundamental theory. Because of its shortcomings, the Schrödinger equation cannot fully explain the behavior of fundamental particles such as electrons – too much is left out.

The Dirac Equation

In January of 1928, the physicist Paul Dirac submitted a paper entitled simply "The Quantum Theory of the Electron, I" to the *Proceedings of the Royal Society* [5]. This one paper was destined to radically change and extend our understanding of the electron while at the same time introducing a new physical model which is at once difficult to interpret and paradoxical. The immediate impact of Dirac's work, however, was that he correctly derived the spin and magnetic moment of the electron starting from only the basic demands of quantum mechanics and the special theory of relativity.

One of the greatest achievements of Dirac's theory was that it *demanded* a spin term for the electron, in order for the total angular momentum to be conserved. In other words, unless the electron had a spin the theory did not work. This is exactly the kind of constraint that a good physical theory should possess – spin could now be interpreted as a consequence of previously understood principles, rather than an *ad hoc* addition to the model. Dirac's equation was not the whole story, but it would eventually lead to the correct relativistic generalization of Schrödinger's equation for particles with **½ ħ** spin. A mathematical treatment of spin was not the only byproduct of the Dirac equation.

The solution to the Dirac equation is a wave function with four separate components. The first two correspond to the spin up and spin down states of an electron with *positive energy*. The other two components correspond to the spin up and spin down states of an electron with *negative energy*. These negative energy solutions were a direct result of the equation's compliance with special relativity. Dirac suspected that the negative energy components of his wave function *implied* the existence of a different form of matter, one that was unknown at the time. But postulating a new form of matter would have created a great stir in the physics community, and at first Dirac lacked the courage to propose his idea. Since the particles associated with the negative energy states were positively charged, in 1929 he suggested that they might represent the proton. At about the same time, however, he communicated to Bohr his suspicion that the new particle should not only have a positive charge but also *the same mass* as the electron[6]. In 1931 Dirac finally took the plunge and proposed the existence of an *anti-electron* (they are now called positrons)[7].

In the early days following its publication there were some who further analyzed Dirac's equation and explored explicit solutions for the electron's behavior. One of those physicists was Erwin Schrödinger. In analyzing wave packet solutions of the equation, Schrödinger noticed the existence of *interference* between the positive and negative energy electron states. This led to circular oscillations at the speed of light of the localized electron wave function with a frequency of:

$$\mathbf{2m_{electron}c^2/\hbar = 1.55268816x10^{21}\ s^{-1}} \qquad \mathbf{(1)}$$

The period of this oscillation is:

$$\mathbf{\hbar/2m_{electron}c^2 = 6.44044328x10^{-22}\ s} \qquad \mathbf{(2)}$$

Schrödinger dubbed this rapid oscillatory motion *Zitterbewegung* (ZBW), which is German for *trembling motion* [8]. In continued analysis of the ZBW, however, additional peculiar features were revealed. One apparent consequence of the Dirac equation, based on the work of G. Breit, is the unlikely prediction that any measurement of an electron's instantaneous velocity must be equal to the speed of light (**+/-c**) – *even in its rest frame*. What's more, while the **x**, **y**, and **z** components of velocity commute in classical and non-relativistic quantum theory, in the relativistic theory they do not (i.e. they are not simultaneously measurable) [9,10].

These seemingly intractable issues - that electrons should move at the speed of light even in their rest system and that the spatial components of velocity do not commute with each other – might indicate that there is something fundamentally wrong with the Dirac theory. I will contend, however, that the fact that the electron has the velocity of the speed of light even in its rest system is consistent with the notion that the electron's mass is not fundamental but is rather an emergent property associated with its *motion in time*. Furthermore, the fact that the three spatial velocity components do not commute in Dirac's theory suggests, to this author at least, that space itself is quantized on sufficiently small scales. And finally, I will associate the ZBW phenomenon with the collapse of the wave function and with the actualization of the electron's reality.

The *Zitterbewegung*

Louis de Broglie first proposed the idea that electrons possessed internal clocks in 1924 in his Nobel Prize winning thesis on wave/particle duality [11]. He thought it was the most important aspect of his theory. It was, however, largely forgotten or ignored by the scientific community. More recent preliminary experimental evidence was provided by Marcel Gouanère and his associates [12]. They attempted to verify de Broglie's conjecture via an electron channeling experiment that yielded a resonant energy compatible with the electron's internal clock frequency proposed by de Broglie. In what is to follow, I will demonstrate that this internal clock is related to the paradoxical phenomena in the Dirac single electron theory introduced in the last section.

In this model, I take a new direction in considering this most enigmatic fundamental particle – the electron. My novel approach reinterprets both the role of spin and the interaction of the electron's positive and negative energy states. The free electron is no longer represented by a pure positive energy or pure negative energy state but rather exists in an *intermediate* state. The electron is actually an extended entity consisting of a single positive energy state (actually a probability wave) and six (three pairs of) negative energy states (waves) – all occupying the same quantum state. I will argue that all four components of the Dirac wave function are intimately related via the phenomenon of the ZBW. This will allow me to contend that spin may have an appropriate classical analogue after all.

The ZBW phenomenon arises from the wave-like structure of the electron. The following equation [8] represents the fundamental relationship between the interaction of the positive and negative energy states and consists of the sum on an initial position vector, a displacement which is proportional to time (the positive energy state/wave moving forward in time at **c**) and finally an unexpected term which represents a violent oscillation of the particle with an amplitude equal to its Compton wave length – the ZBW.

$$\mathbf{x_k(t) = x_k(0) + c^2 p_k \mathit{H}^{-1} t + \tfrac{1}{2}\, i c \hbar \mathit{H}^{-1} (\acute{\alpha}_k(0) - c p_k \mathit{H}^{-1})(e^{-2i\mathit{H}t/\hbar})} \qquad \mathbf{(3)}$$

↓ initial position ↓ displacement proportional to time ↓ *Zitterbewegung*

The position operator is $\mathbf{x_k(t)}$ at time $\mathbf{t}$.

The nominal extent of the electron is taken to be the classical electron radius:

$\mathbf{2.8179402894 \times 10^{-15}\ m}$

The electron's quantum of time t is: 9.39963702 X 10^{-24} s which is the time it takes for the positive energy state (wave) moving forward in time at c to cross the electron radius.

The velocity operator is $v_k = ic\acute{\alpha}_k$.

$\acute{\alpha}_k \equiv \tilde{\gamma}_0\, \tilde{\gamma}_k$ which are the Dirac gamma matrices

$\tilde{\gamma}_k \equiv \tilde{\beta}\tilde{\alpha}_x, \tilde{\beta}\tilde{\alpha}_y, \tilde{\beta}\tilde{\alpha}_z$

k=1 to 3

The problem that arises in the Dirac theory is with the velocity operator: The eigenvalues of $\acute{\alpha}_x$ are **+/-1**. This indicates that the eigenvalues of any component of the velocity operator are **+/-c** and leads to the conclusion that the particle moves with the speed of light even in its rest frame (despite having mass**).** It is my contention, however that the velocity operator in the Dirac theory represents a fundamentally different observable than in the non-relativistic theory. In this view, the velocity operator represents the motion forward in time (**c**) of the positive energy state and the motion backwards in time (**-c**) of the negative energy states.

Moreover, while the velocity components in the non-relativistic theory commute, in the Dirac theory the **x**, **y** and **z** components of velocity do not commute; i.e. do not possess a simultaneous reality. However, the following is also true:

$v_x = ic\acute{\alpha}_x$ commutes with **x.**

$v_y = ic\acute{\alpha}_y$ commutes with **y**

$v_z = ic\acute{\alpha}_z$ commutes with **z**

This suggests that the components of the electron's *position* (like spin and velocity) do not have a simultaneous reality as well. This seems to indicate that the electron's proposed motion (in time) is along a single axis of an **x**, **y** and **z** coordinate system at any given time.

The structure of the electron then results from the interaction between its positive energy state (probability wave) *moving forward in time from the past* at the speed of light (**c**) and its negative energy states (waves) *moving backward in time from the future* at the speed of light (**-c**). The positive energy state is moving forward in time from the origin of an **x**, **y** and **z** coordinate system. Each negative energy pair is moving backward in time along its respective axis of this coordinate system. There is one pair moving backward in time from the **x**, **-x** direction, one pair from the **y**, **-y** direction and one pair from the **z**, **-z** direction. This leads to a form of space quantization, suggested by the non-commutation of individual velocity components in the Dirac equation. This feature of the velocity measurement is thought to be related to the ZBW phenomenon.

The single positive energy state is a superposition of a spin up and spin down state. The six negative energy states are all *spin definite*; each pair consists of a spin up state and a spin down state. The sequential alignment of the positive energy state spin vector with an appropriate negative energy future state spin vector indicates the axis along which the motion in time of the positive energy state is directed. The point of interaction between the positive and negative energy states is at what I call the *temporal event horizon*. This delineates the nominal extent of the electron's structure and represents the electron's notion of the present moment.

The positive energy state cycles though six discrete interactions with each of the negative energy states. The interaction of the positive and negative energy states collapses the wave function. The pairs of negative energy states are quantum mechanically entangled and this collapse mechanism actualizes one dimension of the electron's reality. The collapsed wave function, however, does not represent a localized point but an energetic string within a volume on the order of the cube of the Compton wavelength of the electron divided by $\mathbf{4\pi}$ (slightly larger than the temporal event horizon). This interaction sequence is repeated *ad infinitum* and represents the electron's internal clock.

Quantum spin can now take on a rather more straightforward role. Because the electron now has an extended structure, we can ignore those in the physics community who implore us to avoid classical analogies. The spin allows the positive energy state of an electron to *communicate* sequentially with its six future negative energy counterparts. The spin degree of freedom is required in order for electrons to *face* (recall that each pair of negative energy states is spin definite –one state spin up and one state spin down) each of these six negative energy states in turn as described earlier (see Figure 1.).

As it accomplishes this task, the electron executes six rotations of 120 degrees before returning to its original orientation. In effect the electron *sees* the universe differently after its initial 360 degree rotation, which reverses the direction of its spin. Technically, the spin vector is 60 degrees from parallel to this axis and precesses around the magnetic field like a spinning top that is leaning away from the vertical as it spins. In this fashion, the orientation of the positive energy state is constantly being *measured* in spacetime and each interaction constitutes a *self-measurement* of the quantum system that takes place in the present moment.

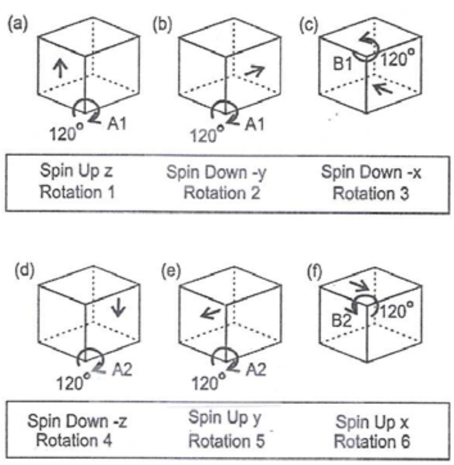

Figure 1 Spin representation of electron: The spin of the electron is brought back to its initial orientation by a sequence of six 120 degree rotations. The rotation axis passes from the center of the cube through either point A1 or A2 or point B1 or B2 as indicated. If all these rotations are added together, we see that a total of 720 degrees or two full revolutions return the cube to its point of origin [13].

The positive energy probability wave represents an infinity of phases all with the same probability amplitude for actualization. At the point of interaction, the negative energy state selects a single phase of the positive energy probability wave. The purpose of this interaction between a determined negative energy state and an indeterminate positive energy state is to choose the phase that serves to maintain the positive energy state's motion forward in time at the speed of light. After each self-measurement, the electron's probability wave function collapses and one of the electron's three dimensions is actualized. In reality the wave function does not disappear but rather changes to a sharp spike representing our near-certainty of the electron's location at that moment. The wave function of the positive energy state then reforms until the next interaction.

The mass of the electron is a derivative of this interaction between the positive and negative energy states. As the spin of the positive energy state lines up with that of each negative energy state the energies of the two states almost (but not quite) cancel each other out. There is a small energy gap between the infinite positive self energy of the positive energy state and the infinite negative self energy of the negative energy state. All but the massive particle's total positive energy is cancelled out, leaving behind an energetic but massless string.

The mass is the amount of the string's total energy that is required to maintain the positive energy state's motion forward in time at the speed of light. In other words, the electron's resistance to a change in its state of motion through space (inertia) is its constant motion in time at the speed of light. The remaining energy consists of the particle's rotational energy that drives its internal clock and its constant angular momentum. As the electron accelerates, more of the total energy is required to maintain the positive energy state's motion forward in time at the speed of light. That energy comes from the electron's rotational energy and as a consequence the electron's mass increases and its internal clock slows (time dilation).

This new understanding of the electron now accounts for the (expanded) role of spin, the collapse of the wave function and the electron's mass. The spin orientation of the unmeasured electron is no longer indefinite but is rather a function of the underlying temporal structure of the particle. Furthermore, the ZBW is a physical process that operates beyond the normal time evolution of the wave function and is not associated with any particular particle behavior but is responsible for the actualization of the particle's reality. And finally, the mass of the electron can now be attributed to a physical process inherent to the particle itself rather than to an interaction with a theorized but yet to be verified Higgs boson. The new paradigm that has been introduced is the notion of motion in time which, in my view, resolves the paradoxical behavior inherent in the Dirac single electron theory.

Conclusion: Messages from the Future

The classical realm – our reality – is based not only on continuity but also on determinism. Determinism is the notion that every physical event is caused by an unbroken series of prior events; this idea is also at the core of classical mechanics. The quantum realm is based on discreteness. It is also decidedly probabilistic. In the murky quantum realm, indeterminism reigns. The position (and momentum, energy, etc.) and spin of an electron are indeterminate until a measurement (observation) is performed. It is generally assumed that no further underlying causal structure can explain these phenomena. It is as if nature wants to surround its most intimate features with a veil of secrecy.

Paul Dirac, who not only developed his relativistic wave equation but also was one of those who contributed a great deal to quantum mechanics in the early days, posed some fundamental questions about the subsequent evolution of quantum theory. In his later years, he consistently suggested that quantum theory would eventually require a radical revision. He concluded that, in the end, determinism will be returned to our most basic physical theory– *but not without cost.* To bring determinism back into our physical theories would require that some cherished assumption that we now accept without question be abandoned [14].

The issue at stake is whether or not quantum probabilities can be interpreted as the occurrence of events which are entirely uncaused by preceding events – do quantum events really happen by pure chance, or as an extension of some logic based on underlying physical principles that we don't yet understand [15]? In the present theory I adopt the second view: that quantum probability is an extension of logic, and that humans and nature lack complete information on this logic in their observations and decision-making. From this perspective quantum uncertainty is real, in the sense that the movements of particles at the quantum level cannot be precisely predicted, but they are not truly random in the sense that they have no prior cause or meaning.

And what might the price that Dirac mentioned be, for this return to determinism?

It may be the sacrifice of an even more cherished idea that underlies all of physics. Regardless of whether we live in a world that is deterministic (in the classical sense) or indeterministic (in the quantum mechanical sense) our fundamental understanding of the universe is based on the concept of cause and effect. By definition, a cause must happen before an effect. If this relationship were to change, all of physics would be stood on its head.

Or would it?

We typically associate cause and effect with temporal evolution. The cause of an event lies in its past, and any other effects of that event lie somewhere in the future. According to this analysis, it is indeed correct to say that the information contained in the probability wave function limits the precision of our knowledge regarding any particular quantum entity such as an electron. In this author's view it is also correct to say, however, that there is a physical process at work beneath the probability wave –the interaction of the positive and negative energy states via the ZBW mechanism I have proposed. And even though this process may be beyond our current capability to detect or calculate, it is nevertheless deterministic in nature. It is, in effect, a process of *retrocausation* whereby the cause is the future (the negative energy states) interacting with the past (the positive energy state) and the effect is in the present moment - the effect being the electron's reality.

References

[1] M. Planck, On the Law of Distribution of Energy in the Normal Spectrum, *Annalen der Physik*, Vol. 4, p. 553 ff (1901)

[2] A. Pais, *Niels Bohr's Times: In Physics, Philosophy and Polity*, Clarendon Press, Oxford, 1991, pp. 133-155

[3] Sin-itiro Tomonoga, *The Story of Spin*, The University of Chicago Press, Chicago, 1997, pp. 1-23.

[4] G.E. Uhlenbeck and S. Goudsmit, *Naturwissenschaften* 47 (1925) 953.

[5] P.A.M. Dirac, The Quantum Theory of the Electron, I.*Proceedings of the Royal Society* (London) A , vol 117, 1 February 1928, pp.610-624.

[6] Helge S. Kragh, *Dirac: A Scientific Biography*, Cambridge University Press, New York, 1990 pp. 90-98.

[7] P.A.M Dirac, Quantised singularities in the electromagnetic field, *Proceedings of the Royal Society* (London) A vol.133, 1 September 1931 pp. 60-72.

[8]E. Schrödinger, Über die kräftefreie Bewegung in der relativistischen Quantenmechanik. B*erliner Ber.* pp. 418-428 (1930).; Zur Quantendynamik des Elektrons. *Berliner Ber.* pp. 63-72 (1931).

[9] G. Breit, An interpretation of Dirac's theory of the electron. *Proc. Nat. Acad. Sci. U.S.A.* 14, 553-559 (1928);

[10] G. Breit, On the interpretation of Dirac's α-matrices. *Proc. Nat. Acad. Sci. U.S.A.* 17, 70-73 (1931).

[11] L. de Broglie, *Recherches sur la théorie des quanta* (Researches on the quantum theory), Thesis (Paris), 1924;

[12] P. Catillon, N. Cue, M.J. Galliard, R.Genre, M. Gouanère, R.G. Kirsch, J.C. Poizat, J. Remillieux, L. Roussel, and M. Spighel, A Search for the de Broglie Particle Internal Clock by Means of Electron Channeling. *Foundations of Physics,* Volume 38, Number pp. 659-664 (2008)

[13] C. W. Misner, K. S. Thorne and J.A. Wheeler, *Gravitation*, W.H. Freeman and Company, New York (1973) pp. 1135-1140.

[14] P.A.M. Dirac, Directions in Physics, Lecture on The Development of Quantum Mechanics , H. Hora and J.R. Shepanski, Editors, John Wiley & Sons Inc, New York P 10 (May 1978)

[15] E.T. Jaynes, Probability in Quantum Theory, Complexity, Entropy and the Physics of Information, W.H. Zurek, Editor, Addison-Wesley Publishing Co., Reading MA (1990)