It From Bit and The Unsmooth Reality

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Abstract

In this short essay it is argued that the "It from Bit" idea is plausible when assuming "generalized bits", resulting from the Kolmogorov superposition theorem, as universal building blocks.

It from Bit or Bit from It ? Is Reality Digital or Analog ? These two questions, theming the present and one of the past FQXI contests are deeply interrelated and still controversial.

It is usually assumed to be an "element of reality", something we can isolate or prepare and investigate, at least in principle. On the other hand, *Bits* are "elements of information", something we can associate with a sequence of *Yes/No* or *True/False* statements, the simplest answers we may get from an investigation. Therefore, as put sometime ago by Wheeler [1]

> [...] it is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer.

Wheeler himself was deeply convinced that it is information what really matters [2]:

> [...] every "it" every particle, every field of force, even the space-time continuum itself derives its function, its meaning, its very existence entirely even if in some contexts indirectly from the apparatus-elicited answers to yes-or-no questions, binary choices, bits. "It from bit" symbolizes the idea that every item of the physical world has at bottom a very deep bottom, in most instances an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes no questions and the registering of equipment-evoked responses; in short, that all things physical are information-theoretic in origin and that this is a participatory universe.

Recent experiments with entangled photons suggests that indeed, an act of observation or even a choice of measurement performed on one photon influence the results obtained for the other one. In an very recent experiment with photon heralding, the act of observation of one photon has brought effectively the entangled twin photon into existence [3]. Another recent experiment demonstrated that a causally separated choice of measurement can influence the behaviour of a quantum object, i.e. whether it behaves like a wave or like a particle [4]. These results are therefore strong *experimental* indications supporting also the Wheeler idea of a participatory universe, at least in the quantum realm and in consequence the "It from Bit" idea.

Another even stronger conclusion is that if we want to live in peace with the special theory of relativity, an individual photon cannot be treated sometimes as behaving definitely as a wave and sometimes behaving definitely as a particle. This leads directly to the thought, that a proper formal description of reality cannot be continuous (for waves) or discrete (for particles), but have to be in some sense *simultaneusly* continuous and discrete.

According to the Nyquist-Shannon theorem [5, 6], a fundamental result in information theory, continuous information may be converted into an equivalent discrete information by sampling, provided that the sampling rate exceeds the doubled bandlimit frequency. Such an approach to physical fields and also to the spacetime itself has been suggested awhile ago by Kempf [7, 8].

Bits, trits or other commonly used units of information are not very convenient to represent complicated and structured information, not to mention an "It from Bit" programme. What is really needed is rather a set of universal "building blocks" with an appropriate capacity for encoding continuous information, "generalized bits" in a sense.

An interesting but not widely known theorem by Kolmogorov on superpositions of continuous functions [9] could pave the way in this direction and has also very interesting interpretative implications. Namely, without going into formal details, this theorem states that *every* multivariate continuous function could be represented in the following simple form:

$$f(x_1, \ldots, x_n) = \sum_{i=0}^{2n} g\left(\sum_{j=1}^{n} \lambda_j \phi_i(x_j)\right) \tag{1}$$

where all constants λ_j and also all (continuous) functions $\phi_i(x_j)$ of one variable do *not* depend on the function f. The function g is in turn a continuous function of one variable, remaining in a one-to-one relationship to f. Therefore $(\lambda_j, \phi_i(x_j)$ are perfect candidates for our "generalized bits".

For almost 30 years this theorem was treated mainly as a fine toy for pure mathematicians, without practical use, until the observation was made by Hecht-Nielsen [10] that Kolmogorov theorem could describe neural networks. Moreover, it turned out at the same time, that due to the Kolmogorov theorem it is also possible to encode an arbitrary continuous multivariate function in a three-layer neural network with continuous activation functions (cf. also the book [11]). Therefore, an universal neural network representation is also possible, at least in principle.

A strong indication that this is a step in the right direction when searching for "generalized bits" for an "It from Bit" programme is provided by the explanation facilitated by the Kolmogorov theorem, why the fundamental physical equations are of second order [12]. Especially interesting in this context is the nonsmoothness of fundamental phenomena resulting from the fact that smooth multivariate functions cannot be generally represented as superpositions of smooth functions of one variable (cf. [13] for details). With this "inherent unsmoothess" it is therefore enough room left for all the weird behavior observed in Nature.

Are we able to determine the "generalized bits" $(\lambda_j, \phi_i(x_j))$ somehow explicitly or calculate them out ? At present, we know that they do exist and are computable (cf. [14, 15] for recent rigorous results). Hence, a (pure) mathematician could probably sleep well with that, but a (genuine) physicist most probably not at all. From the proofs presented in the cited papers it is obvious that "generalized bits" a'la Kolmogorov are generally *very* weird objects.

In conclusion, the "Bit from It" idea, although strange at the first sight, seems plausible, provided we use "generalized bits". Another attractive idea, only mentioned in this essay but closely related to the "generalized bits" approach is the universal neural network representation, something like the "Calculating Space" of Konrad Zuse [16], arguably the first attempt to get the *It* from *Bit*.

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