# UNPREDICTABLE UNPREDICTABLES

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Abstract. Predictability or otherwise of an event is a property not so much of the event, as of the theory that is used for prediction. Unpredictable events can be considered to be 'predictable unpredictables' if there is a theory that predicts the probabilities of their occurrence. Everything else is an unpredictable unpredictable, and these are the most challenging for physics. Guided by an analogy with undecidability in mathematical theories, I consider what kind of physical theory might be required in order to predict those fundamental parameters that in current theories are unpredictable unpredictables.

## 1. Theories of theories

**1.1. Predictability and decidability.** As Donald Rumsfeld might have said:

> *There are predictables. There are things we can predict. There are predictable unpredictables. There are things we now predict that we can't predict. But there are also unpredictable unpredictables. There are things that we can't predict whether we will ever be able to predict them.*

Much of physics, including classical mechanics, special and general relativity, deals in predictables. Other parts of physics, such as thermodynamics and quantum mechanics, deal in predictable unpredictables. These theories are characterised by probabilities and averages, which can be calculated because the model contains appropriate probability spaces in which to do the calculations. If a model contains no such probability space, then such calculations cannot be done, and we are left with unpredictable unpredictables.

The first remark to make is that there is no model-independent definition of predictability. Something that is predictable in one model can be unpredictable in another. The same is true for decidability in mathematics—what is decidable in one model of mathematics can be undecidable in another. Many questions of decidability refer to a particular standard model of mathematics (usually ZFC, that is, Zermelo–Fraenkel set theory with the axiom of choice), but there are plenty of non-standard models that are also used. For example, many questions that can be decided with the axiom of choice cannot be decided without.

The corresponding question for physics is whether a non-standard model can predict (correctly!) some things that are unpredictable in the standard model. The search for hidden variable theories of quantum mechanics is a case in point, as they aim to move certain properties from the category of predictable unpredictables into the category of predictables. Other theories aim to move properties from the category of unpredictable unpredictables into one of the other two categories.

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1.2. **Predicting unpredictable unpredictables.** One can legitimately ask the question, whether consideration of unpredictable unpredictables is a part of physics or not. For example, the fundamental constants of the standard model of particle physics can be treated as unpredictable unpredictables. Their values seem to be 'random', and yet there is no clear probability distribution to define what 'random' means. Worse, the values are not 'random', they are fixed (except for those that 'run' with the energy scale). The values, then, can only be explained by one supreme act of randomness at the origin of the universe in the Big Bang. Is this really an adequate physical explanation, or is it just a fairy story?

To take one particular example, consider the mass ratio of proton to electron, at a little over 1836. Why this number rather than any other? In current theory, this ratio is an unpredictable unpredictable. We would like a theory in which it is predictable. Much speculation has been devoted to trying to convert it into a predictable unpredictable, and applying the principles of quantum mechanics to the Big Bang. This leads to concepts such as the multiverse, that are necessary in order to define an appropriate probability space, but for which there cannot ever be any experimental evidence, even in principle. Would it not be better to speculate instead on ways of converting this mass ratio directly into a predictable?

1.3. **A dubious assumption.** Predictability is the domain of classical physics and relativity. One of the most fundamental concepts of classical mechanics, that remains of the utmost importance in both special and general relativity, is that of an inertial frame (of reference). While this concept cannot be consistently defined, we think we know one when we see it. But do we really?

In practice, the concept of inertial frame changes as the scale of the experiment changes. Experiments in particle accelerators are always analysed on the assumption that the particle accelerator itself is at rest in an inertial frame. But on a gravitational scale it is clearly not reasonable to consider this frame to be inertial. Is it really any surprise that the standard model of particle physics is inconsistent with general relativity, if the two theories are based on two incompatible definitions of inertial frame?

Results concerning decidability in mathematical theories only apply to consistent theories. In inconsistent theories, everything is simultaneously true and false. Much the same happens in inconsistent physical theories—in principle they can be used to predict anything you like. Therefore we must fix this inconsistency before going any further.

General relativity has been very well tested on a Solar System scale, so it is reasonable to take a definition of inertial frame that works on at least that scale. Then we must build a (non-standard) model of particle physics with respect to that definition of inertial frame. In other words, we must adjust the (standard) model for the complicated motion of the particle accelerator, or other experimental apparatus, relative to the Solar System inertial frame.

We cannot just assume that these adjustments are negligible—we must first have a model that predicts what these adjustments are. Only then can we calculate the adjustments, and make an informed decision as to whether or not they are in practice negligible. If they turn out not to be negligible, then perhaps we can devise experiments to measure them, and thereby test a putative non-standard model against the standard model.

## 2. Axioms for a nonstandard model of physics

2.1. **Fundamental parameters.** The motion of a particle accelerator relative to the Solar System inertial frame is quite complicated, so let us begin with a simplified model in which this motion consists of two superimposed circular motions, one of period one year around the Sun, and the other of period one day about the Earth's axis. It is likely that only dimensionless parameters of the motion can be transferred between such vastly differing scales as the Solar System and an atom. There are only two such parameters available, namely the frequency ratio of approximately 365.25 and the angle 23.44° between the axes.

With two parameters one can only reasonably hope to predict two of the most fundamental of the unpredictable unpredictables. Most of these can be interpreted as mass ratios of fundamental particles, and what could be more fundamental than the mass ratios of electron, proton and neutron? To avoid notational clutter, I use the name of the particle to stand for the mass. Then the values retrieved from CODATA 2014, with unnecessary accuracy removed, are

$$\begin{array}{rcl} n/p & \approx & 1.00137842, \\ e/p & \approx & 5.44617021 \times 10^{-4}. \end{array}$$

It is hard to avoid noticing the similarity to the numbers

$$\begin{array}{rcl} 1 + 1/(2 \times 365.25) & \approx & 1.0013689, \\ \sin(23.44^\circ)/(2 \times 365.25) & \approx & 5.44543 \times 10^{-4}. \end{array}$$

The factor of 2, incidentally, arises from the fact that these particles are fermions, which means they have to go through two complete revolutions before returning to their initial state.

2.2. **Thinking the unthinkable.** What do you think? Have I merely produced a pair of meaningless numerical coincidences? Or have I found a clue to a non-standard model that could predict these mass ratios? Let us listen to Donald Rumsfeld again:

> *There are thinkable thinkables. There are things we think we think. There are thinkable unthinkables. There are things we think we don't think, but we think we might think in the light of new research. But there are also unthinkable unthinkables. There are things we can't even think of thinking, that if we ever did think could shatter our entire world-view.*

Predicting the unpredictable requires thinking the unthinkable. Restricting ourselves to thinkable unthinkables has not worked. To make progress, we may have to think unthinkable unthinkables. This is not a safe option. Thinking unthinkable unthinkables can lead to a complete paradigm shift. If we are not prepared for a paradigm shift, then we should not go down this road.

What is interesting for physics is whether in our current state of knowledge we can devise a better model than we currently have, in order to be able to predict a wider range of phenomena than we currently can. The lesson from the mathematics of decidability is that this can only be done by enriching the assumptions on which the theory is based. The lesson from history is much the same—the development of special relativity, for example, required adopting the new and counter-intuitive assumption that the speed of light in a vacuum is the same for all observers.

If the examples I have given are anything to go by, then the new, counter-intuitive, assumption that we must surely adopt is the assumption that the measured masses of the elementary particles depend on the non-inertial motion of the experiment. In the context of the standard model, such an assumption is an unthinkable unthinkable. But if we want to go beyond the standard model, we must be prepared to think unthinkable unthinkables.

2.3. **Meta-predictions.** The development of a complete non-standard model based on this new assumption will be a huge amount of work. Before doing this work, it would be nice to convince ourselves that it is likely to be worthwhile. Rather than making predictions, we would first like to be able to make some meta-predictions about what predictions the non-standard model is likely to be able to make.

The motion of a typical experiment is significantly affected by the gravitational attraction of the Moon on the Earth. This provides two further dimensionless parameters, namely the number of days in a month and the inclination of the Moon's orbit to the ecliptic. The former has many variants, the most significant difference being that between the synodic month of 29.53 solar days and the sidereal month of 27.397 sidereal days. The angle of inclination averages around 5.14°, but varies between 4.99° and 5.30° on a 347-day cycle.

What measured mass ratios are likely to be described by these parameters? Different people may give different answers to this question, but it is hard to dismiss the claims of the pions, that are responsible for binding the nucleus of an atom together. After that, perhaps the kaons, or the $Z$ and $W$ bosons? In all three of these cases, these particles come in charged and neutral variants, and the mass ratio between these two variants is something we might expect a non-standard model to predict. Experimental values are

$$\begin{aligned} \pi^{\pm}/\pi^0 &\approx 1.03403, \\ K^{\pm}/K^0 &\approx .99202, \\ W^{\pm}/Z^0 &\approx .88146. \end{aligned}$$

Possible predictions might involve the following expressions:

$$\begin{aligned} 1 + 1/29.53 &\approx 1.03386, \\ \cos^2(5.14^\circ) &\approx .99197, \\ \cos(23.44 + 5.14)^\circ &\approx .87815. \end{aligned}$$

Is this a large enough collection of meta-predictions to convince us to undertake the necessary work on building a non-standard model? Perhaps not, so maybe we would like some more. I now introduce the last member of Gell-Mann's meson octet, the eta meson, to add to the three pions and four kaons, plus the two most important of the remaining mixing angles in the standard model, the Cabibbo angle and the CP-violating phase. I invite you to compare the experimental values with the meta-predictions:

$$\begin{aligned} K^0/\eta &\approx .90893, & \cos(23.44^\circ)\cos(5.14^\circ) &\approx .9137, \\ \theta_C &\approx 13.02^\circ, & 360^\circ/27.397 &\approx 13.14^\circ, \\ \theta_{CP} &= (68.8 \pm 4.6)^\circ, & 90^\circ - 23.44^\circ &= 66.56^\circ. \end{aligned}$$

It is of course unlikely that any non-standard model would be able to convert all eight of these meta-predictions into actual predictions. Some of them are surely just coincidences. But is it plausible that *all eight* are pure coincidence? I suggest that it is not.

2.4. **Hidden relations.** Even if we are lucky, eight equations gets us nowhere near to predicting the approximately 20 unpredictable unpredictables in the standard model. And we only have four basic parameters of non-inertial motion to play with. So if there is a nonstandard model of this type that predicts all 20, there must be quite a lot of equations between them that do not depend on the non-inertial motion of the experiment. These 20 include the masses of the muon $\mu$ and tau particle $\tau$, and the six quark masses $u, d, s, c, b, t$.

Including also the Higgs boson $H^0$ and the eta-prime meson $\eta'$, the following eight equations, interpreted as equality of mass and charge, are consistent with experiment (if perhaps some are 'more consistent' than others):

$$\begin{aligned} e+\mu+\tau+3p &= 5n \\ e+u &= d \\ \mu &= s+2d \\ \tau &= c+5s \\ 2H^0+2n &= Z^0+W^++W^- \\ c &= s+K^++\eta+\pi^0 \\ b+s+d+\pi^++2\pi^0 &= 5n \\ t+c+d+\eta' &= b+s+u+Z^0+W^+. \end{aligned}$$

Numerical values for both sides of these equations are give in the Appendix. Clearly it is hard to judge the significance of such numerology, and many people will simply dismiss this evidence as meaningless in any case. But one should try to make a reasonable assessment of the size of the pool of potential equations these are chosen from, and the likelihood of obtaining such results by chance.

Consider for example the experimental value of the ratio of the two sides of the first suggested equation:

$$(e+\mu+\tau+3p)/5n \quad = \quad .999996 \pm .000034$$

Unless it can be reasonably argued, and I suggest that it cannot, that this equation is a typical random equation out of at least a million similar equations, then it must at least make one pause for thought.

## 3. More on some oxymorons

3.1. **Variable constants.** It does not very much matter whether or not you are convinced by these meta-predictions. What this analysis forces us to do is to examine the notions of constant and variable. Some approaches to non-standard models do indeed treat some of the fundamental constants as variable. But they have not listened to Donald Rumsfeld:

> *There are constant constants. There are constants that are constant for all time. There are constant unconstants. There are constants here and now that have changed over time, or change with energy. But there are also unconstant unconstants. There are unconstants that vary in a manner we have no notion of.*

If our aim is to calculate the values of the fundamental constants, then we must work in a model that is sufficiently general to allow the constants to vary, and moreover to vary in a way that we do not specify in advance. Within this model, we must then develop some equations that these unconstant unconstants satisfy.

Only when we have developed enough of these equations will we be able to solve them, and deduce the values of the fundamental constants. Only then will we be in a position to make an informed judgement as to whether or not they are really constant. If a particular putative non-standard model says they are not constant, then perhaps we can devise some experiments to measure the variability, and test the model. Hopefully there will be some good experiments that are less expensive and more practical than building a particle accelerator on Mars.

We might even try to test some meta-predictions. Such a test cannot be definitive in the way that tests of genuine predictions can be definitive. But it can still put restrictions on the possible forms a non-standard model could take, so might be worth doing anyway. Take the kaon mass ratio, for example. The meta-prediction suggests (but does not predict!) that this measured mass ratio might vary on a 347-day cycle by as much as $\pm.05\%$. This difference is big enough to detect experimentally if one is specifically looking for it, but small enough to miss if not.

3.2. **Consistent inconsistency.** As we have seen, there is a close analogy between undecidability in mathematics and unpredictability in physics. Undecidability tells us about the limits of any given model of mathematics, while unpredictability similarly tells us about the limits of any given model of physics. It is only an analogy, however—unpredictability has little, if any, relevance in mathematics, and undecidability has little, if any, relevance in physics.

In both subjects, models are chosen primarily for their usefulness. Consistency is a secondary issue. In mathematics, a theory is useful if it can decide lots of questions. In physics, a theory is useful if it can predict lots of phenomena. In both cases, the more useful a theory is, the more assumptions it has to make, and the less likely it is to be consistent.

Hilbert's hopes of a rigorous self-consistent theory of mathematics that could be proved to be self-consistent were dashed by Gödel's work. Hence consistency of a mathematical theory cannot be proved, but can only be tested by experiment. As long as a theory continues to work without producing obvious falsehoods, it is consistent to assume it is consistent.

Physical theories need to be not only mathematically self-consistent, but also consistent with experiment. The current standard models of physics are well-known to be neither, although they are *almost* consistent with experiment. At the risk of over-indulging Donald Rumsfeld, we could say that the standard models of mathematics exhibit a consistent inconsistency, while the standard models of physics exhibit an inconsistent inconsistency. It is probably asking too much to aim for consistent consistency in physical theory, given that it is not possible even in mathematics. But it is surely not unreasonable to aim for consistent inconsistency.

3.3. **The triumph of hope over experience.** Einstein's hopes of a rigorous consistent theory of all of physics have not been realised. But Gödel's theorems do not prove that such a theory is impossible. For physics has one great advantage over mathematics—namely, the existence of the universe. If something exists in the universe, it is, *ipso facto*, consistent. So maybe, just maybe, if we can upgrade the theory from inconsistent inconsistency to consistent inconsistency, then the universe can do the rest, and give us a consistent consistent theory of everything.

## Appendix

Particle masses used in this essay are listed here in units of MeV$/c^2$.

| | | | | | |
|---|---|---|---|---|---|
| $e$ | .5109989461(31) | | | | |
| $\mu$ | 105.6583745(24) | $u$ | $2.3 \pm 0.7 \pm 0.5$ | $\pi^\pm$ | 139.570 |
| $\tau$ | 1776.82(16) | $d$ | $4.8 \pm 0.5 \pm 0.3$ | $\pi^0$ | 134.977 |
| $p$ | 938.2720813(58) | $s$ | $95 \pm 5$ | $K^\pm$ | 493.68 |
| $n$ | 939.5654133(58) | $c$ | $1275 \pm 25$ | $K^0$ | 497.65 |
| $Z^0$ | 91187.6(2.1) | $b$ | $4180 \pm 30$ | $\eta$ | 547.51 |
| $W^\pm$ | 80379(12) | $t$ | $173210 \pm 510 \pm 710$ | $\eta'$ | 957.78 |
| $H^0$ | 125180(160) | | | | |

From these values one easily calculates the following mass values for the two sides of the eight equations in the display in Section 2.4.

| | |
|---|---|
| $4697.805 \pm .16$ | $4697.825$ |
| $2.8 \pm .7 \pm .5$ | $4.8 \pm .5 \pm .3$ |
| $106$ | $105 \pm 5$ |
| $1777$ | $1750 \pm 35$ |
| $252240 \pm 320$ | $251946 \pm 24$ |
| $1275 \pm 25$ | $1271 \pm 5$ |
| $4690 \pm 30$ | $4698$ |
| $175450 \pm 510 \pm 710$ | $175844 \pm 30$ |

With the exception of the second one, all the discrepancies between the two sides are well within one standard deviation.