

To be or not to be strictly deterministic? ¹

Some variations over a well-known theme

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Abstract

The task of this essay is to examine the possible discrepancies between a strictly deterministic description of reality and quantum mechanics with no hidden variables in its interpretational framework. We start this paper by considering some general lines of reasoning about what can be known or not known in principle. Thereafter we analyze certain contradictions which we have obtained from our considerations about what can be known or not known in principle and examine their possible consequences for the framework of a strict determinism as well as for a framework with random events incorporated. We compare these findings with human experience as well as with the limits of logical consistent inferencing. Furthermore some consequences of multiple coexistent finite or even infinite universes are examined. Finally we arrive at the conclusion that for our hitherto most successful scientific theories to be true *and* consistent, it is necessary to assume the existence of consciousness to be at least as fundamentally necessary as these theories seem to be.

1. Introduction

The well-known Austrian philosopher and logician *Ludwig Wittgenstein* once wrote in his famous *Tractatus* the following line of reasoning [1]:

“For the aim of drawing a border between the thinkable and the unthinkable, one would have to think both sides of the border, the thinkable and also the unthinkable.”

For the aim of this essay – namely trying to explore some fundamental boundaries between a strictly deterministic “theory of everything” (in the following simply named as “TOE”) and quantum mechanics with no hidden variables (in the following simply named as quantum mechanics) –, we can use Wittgenstein’s statement as a pretty good starting point. Because if one contemplates his statement a little bit longer, one can easily see that it demonstrates the *impossibility* to finally define a universal set whose elements all have not only the property of being “unthinkable” and of being *countable*, but each element would have to be also individually incorporated into the thinker’s consciousness for the aim of drawing the questionable border at the right place – which in this case would generate a striking contradiction to it’s main axiomatic property of being “unthinkable”. Moreover, Wittgenstein’s line of reasoning seems to imply a much more subtle detail: By applying it to the fundamental question of physical (*im*)possibilities, it seems to prove that it is *impossible*

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to define *all* physical impossibilities as well as all physical possibilities by a general act of deduction.

A similar result was obtained by the well-known Philosopher Karl Popper who combined inductive reasoning with human experience to come to the following result. Popper pointed out clearly the obvious asymmetry between a “proof” and a refutation by showing that *inductive reasoning* doesn’t automatically lead to reliable knowledge for all times. He stated that the consistency of some physical data is not equivalent at all with the possible physical completeness of that data [2]. Hence, three consequences can be deduced at this point from our hitherto findings:

1. Firstly, Wittgenstein’s result, albeit grounded on *deduction*, has to be at least *one* of those impossibilities, which *can* indeed be easily deduced by human reasoning and imagination. Because his result shows, that it is *impossible* to obtain all impossible and/or all possible things by a general deduction, what surely seems to be true.
2. Secondly, we further have to state that the fact of deriving all *impossible* things in general is obviously impossible by deduction, this does *not* necessarily imply that these “undeducable” issues are physically impossible in general in the sense of nonexistent - what simply reflects our experience that the questionable border between both sides can be adjusted by some future increase of knowledge.
3. Thirdly, we also have to state that the fact of deriving all *possible* things in general is also obviously impossible by deduction in the sense of “hidden”, does not necessarily imply that these “undeducable” issues can in general be discovered by some future increase of knowledge.

By summarizing point 1. – 3. one can assert that obviously as an issue of principle we can neither obtain the complete set of physical possibilities by just one step of induction nor can we obtain the complete set of physical impossibilities by just one step of induction, nor both by deduction. Moreover, due to these findings it could be possible that we have a forever persisting asymmetry between at least the accessibility of all these members of the mentioned sets and the easy accessibility by deduction of at least one member of the set of all physical impossible things, namely the one mentioned in 1.

2. The quest for hidden variables

The just mentioned asymmetry between the knowable and the unknowable in respect to its deducability by only just one step seems to be important, because it would indicate that it is not decidable ad hoc, whether some questionable impossibilities have their reasons in the fundamental *absence* (non-existence) of certain physical laws or in some general inability for an intelligent observer to ever gain knowledge about these physical laws. The fundamental question is therefore, whether one can decide between these two alternatives or whether there is a third, yet unexplored *possibility* to bring together both perspectives coherently (for example by some exception of the *tertium non datur*).

As a first result of this paper, with our lines of reasoning we have obtained the almost commonly known phrase that

“The absence of significance for something is not the same as the significance for the absence of something.”

Now, this phrase is a crucial point for our further discussion of the relation between quantum mechanics and reality and also for the possibility/impossibility of the existence of some “hidden”, “yet to be discovered” physical laws. The question about these hidden physical laws in my opinion is the most important one, because if there *wouldn't* exist any hidden and/or strictly deterministic structure beyond quantum mechanics, what would be the alternative? And how would this alternative relate to our additional aim of this paper, namely the determination of some fundamental borders between possibilities/impossibilities in physics? This is not a trivial question, because if there exists a deterministic description that goes beyond quantum mechanics *and* we are able to find such a description, then it is presumable that such a description compresses a lot of potential information in its axioms, whereas, according to the findings of Gregory Chaitin [3], its “program code” should be expected as not too large to discover “itself” at all. That would mean that such a deterministic theory would impose much stronger restrictions on what could in principle be possible in physics than the probable non-existence of such a description would impose. The latter is so, because it is obvious that in a totally random universe everything what “can” happen not only happens (with probability one), but would even happen *infinitely many times*. If this “program-code” would be larger than the complexity of the physical human mind, it would not be possible for the human mind to grasp it (with complexity we mean here the amount of in-built a-priori axioms that make this program code fully consistent and singularly). But if human mind could be *more* complex as this program code, then nature couldn't be anymore solely deterministic in a mathematical sense, because Gödel's theorems wouldn't be valid anymore. Therefore the human mind must for the case of a discoverable TOE be in some way approximately as complex as this program code is. The question now is how complex the human mind is and if this level of complexity demands a calculation time of the discoverable TOE that could be much longer than the existence of our solar system and/or human race.

To be able to answer the two mind-boggling questions at the beginning of the last break at all, one has at first to differentiate between 3 realms: The somewhat fuzzy realm of imagination where Wittgenstein's set of all thinkable things seems to reside, the realm of reasonable thinking where maths and logics reside and the realm of physical reality, where precise laws and logics govern the tide of events - and therefore the set of possibilities in the latter realm seems to be somewhat more restricted to our imagination than the realm of our imagination itself. The latter seems to be evident but should not be generalized unproven, because what can be imagined depends also on personal believes and unconscious patterns of reasoning. For example, like in physics, in our consciousness/soul there operate some very effective and precise laws too, namely some - more or less - unconscious psychodynamics with projection, suppression and so on. Let's take a short look what this could mean for human reasoning: Each time a human being has learned a new pattern of behaviour/thinking by drawing an internal cognitive distinction between former beliefs/knowledge and new gained beliefs/knowledge and executes this new pattern frequently, this pattern goes into the unconscious realms and therefore the former pattern is inhibited of being executed furthermore. This is the case for patterns which are mutually exclusive in such a way, that the former pattern in the unconscious realm is at least temporarily inaccessible for modifications and is, surely not deleted, so surely somewhat “isolated” from the newly learned pattern. So this human being gains some more “freedom” by learning mutually exclusive patterns to the expense of some loss of former “freedom”. Such a pattern could be for example gained by learning an alternative behaviour that “frees” one (due to personal taste) from self-restricting and generalizing negative thinking (about

whatsoever). In this context, assumptions clearly arise more out of *emotions* instead out of facts.

What is not at all as evident as the above mentioned set of consciousness-related (“cognitive”) rules and psychodynamics, is the irrefutable fact that every human act of learning patterns – and therefore also every elementary act of knowledge *and measurement* (the latter obviously in the most cases not through an observer) – happens through an *act of distinction* and accordingly *is* a distinction. Whether this distinction comes out of a pre-existing set of all possible distinguishable things has to be examined in the next step of considering the border between the TOE and quantum mechanics. Probably the most elementary pattern right from the start of all physically existing things would be to draw a line into the void and this would be the prototype, the “mother” of all distinctions and therefore the mother of all information (and informational processes). If this would indeed be the origin of all distinctions, like Spencer-Brown considered it to be [4], an a-priori set of fixed distinctions is hard to imagine, because the only property gained by such a distinction is the distinction between a “oneness” and a “distinction”, and therefore “only” a law of form is obtained that is coinstantaneous its own single property. We want to label this prototype of a distinction here provisionally with “bit”, according to the information theoretic interpretation of quantum mechanical randomness in Zeilinger *et al.* [5].

3. Perceptions from math or math from perceptions?

Trying to answer the question whether the set of all possible distinguishable things exists a-priori in a logical or rather mathematical realm of entities where all lines of distinctions are already drawn is our next step.

There are some proposals to incorporate the whole landscape of mathematics into a concept of physical reality, themed by the reasoning, that thinking of all “thinkable” maths would be easier to comprehend than a single substructure of maths [6, 7, 8, 9]. Furthermore the question for the possible a-priori existence of this complete set of distinctions that would be independent of any perception is analogous to our question for the existence of a hidden deterministic structure beyond quantum mechanics. For the discussion of the lucidity of all mathematical describable worlds being existent, it is important to distinct between two alternatives: The first alternative is that the whole landscape of mathematics exists independent of what is going on in any universe. This landscape would be like a picture, that can be overlooked fully if one has the right distance. This is the platonic view, sometimes also called “the bird’s perspective”. The other view is that mathematics evolves as a consequence of a proceeding distinguishing-process of some yet unknown underlying essence. If one assumes the first alternative to be true, one has to explain why mathematics should be the only essence that can exist at all. Furthermore one has to expose how consciousness can emerge out of a mere symbolic structure. Though it is correct that mathematics and at least our (own) universe fit well together in an astonishing manner, there is a second question left unexplained by the attempt to explain reality by a platonic realm of maths. This second questions is “Why math and not consciousness?”. The second alternative would be to consider math as a creative distinction-process guided by evolving axioms which cannot be proven to be true but only to be consistent within one’s own *belief system*.

For the author it seems that it would be much more natural to assume the essence of all reality to be of various subtle degrees of perception. These degrees would depend on the various present local states of complexity of the fundamental dynamical distinction-process mentioned above. This process then would have been possible due to an initially conscious act of quasi-dividing itself (itself in the meaning of consciousness) into a world that would

seem to be totally independent of the other side of this division, namely the internal world of conscious observers.

The proposed answer to the question "Why maths and not consciousness?" has the advantage to locate "all" maths as evolving structures inside a broader essence of reality without having to explain consciousness through maths, but the other way round.

One can now ask which alternative is more lucid. In respect to this question a completely fixed infinite landscape of mathematical universes would be hard to distinguish from fundamental randomness of all reality. Because in an infinite random process, everything can and will occur, even infinitely many times. The absence of a bird's view for us poses the problem to never being able to distinct between fundamental randomness and fundamental order. Therefore nobody can know how the concrete structure of the complete landscape of maths looks like and if it is truly self-consistent. Hence it is at least questionable to assume this landscape to be fully existent a-priori without ever finally being able for us to carry it out (by some unknown number of calculation steps and therefore some unknown number of distinction processes). Assuming this landscape to be the mother of all reality and of all that can ever exist would mean one has simply shifted Wittgenstein's border to the utmost limit of what one assumes to be existent in general – to the expense of being now an other-directed observer by a universal wavefunction, who cannot detect in principle his own fundamental lack of freedom. This observer couldn't have anymore the ability to partially understand the present in a way that is not determined by the past, but would instead of it being rigorously *thought by pure maths*.

Would the latter result be shocking? Not so much, because also for our proposal here, an observer would be also thought (or in a certain manner "dreamt") by the source of all consciousness. Though, the difference between both approaches lies in the circumstance that for the "consciousness"-approach the observer could really partially act as a co-creator in certain limits.

At this point of our discussion it is important to note, that every TOE is in a certain disaccord to one of our best elaborated scientific theories, namely modern evolutionary theory. In respect to this theory it is widely accepted that consciousness and intelligence evolved due to dynamical macroscopic selection effects which enabled for an organism to extend its lifetime and reproduction chances with the help of internal "precognition". This "extension" of the organism's lifetime with the help of internal "precognition" is well-thought-out by the intuitive insight that the more intelligence an organism has, the more it can predict certain circumstances in the future and avoid them or otherwise handle them.

According to Popper this organism can "simulate" certain circumstances in its consciousness, especially *its own* behaviour regarding certain future events, similar to a scientific experiment. If the organism would act out this visualized behaviour in reality instead of simulating it first, it would risk to fail and to die.

Nonetheless, the sharp contrast between evolutionary theory and a strictly deterministically evolving world is that the explanation of consciousness via evolutionary theory could be only true, if consciousness is a creative and generic issue of reality. If it would be only an epiphenomenon of chemical complexity, one has to ask why complex chemical processes bear this epiphenomenon at all when it makes no difference for the tide of strictly deterministic events. Therefore evolutionary theory and a strictly deterministic TOE cannot be both consistent *and* complete at the same time.

4. A first short answer

We can now give a more precise description of our “consciousness”-approach. Therefore – and taking for granted that the nature’s main structure of logical relations is self-consistent under *all* measurable circumstances (but not necessarily under every instant of their conditional statements!) – one can conclude the source of all reality to be a coherent “oneness” that is able to partially camouflage itself by dividing itself into various and seemingly mutual exclusive subsets whereas continuously conserving its oneness by repeating always one and the same dividing-operation (namely the operation of making a distinction). So at that point of this paper it is inevitably necessary to link our coarse-grained results to the subtle hard facts of quantum mechanics. We now have to prove if our intuition is in accordance with the probabilistic structure of quantum mechanics as well as in accordance with other assumptions of this theory. Concretely with the “random” outcomes of experiments, entanglement, decoherence and superpositions.

5. Linking quantum mechanics with consciousness

Trying to start this enormous task mentioned in the previous break, I would like to begin with the undeniable fact of decoherence [10], discovered by the German physicist Dieter Zeh in 1970.

It is today well-known that a physical measurement is not at all an affair of a one-way influence from the measurement apparatus to the object of interest, as one could conclude by contemplating Heisenberg’s uncertainty principle. Today we know that there are various influences also into the opposite direction. The theory of decoherence gave us a very natural solution of for example the ancient mind-boggling Schrödinger’s cat and also Wiener’s friend. Let us briefly remark here that a measurement apparatus which is itself measured by the environment, which also is itself measured by the further environment and so on, has some element of logical iteration in its system that is not unsimilar to the adding of new axioms into a “Gödel-restricted” system to conserve its consistency.

Now we are at a very crucial point of our whole debate and two questions arise. Firstly, one can ask, what have windy logical axioms to do with hand-tight physical processes? Secondly, isn’t it the most intelligent assumption to consider a Gödel-like system which has undecidable outcomes to be rather incomplete than *inconsistent*? So why has a “Gödel-restricted” system to conserve its consistency at all?

The answer to the first question is a little bit more subtle and affords first answering the second question: Gödel’s undecidability result (“this outcome is undecidable”) itself is an iteration and therefore could be iterated as “it is undecidable, that this outcome is undecidable” – which is surely the same as “undecidable”, but the probabilities for the iterated statement to be true now have somewhat changed. Because, if it is really undecidable *that* something is undecidable, then the assumption that something is undecidable gets somewhat weaker. Well, does this mean that iterations of such a kind govern the evolution of the world’s structure deterministically? The answer is no. The reason for this is due to the following answer to the first question: Evolving axiomatization in mathematics affords some subjective belief into one’s own inductions. Once such an induction is made, the own behaviour is somewhat restricted to one’s line of reasoning. We discussed this psychodynamics above. The reason for probabilities in quantum mechanics could be in my opinion analogous to the mentioned psychodynamics. Because one can understand quantum physical entities as equipped with rudimental perception and in most cases following some group-dynamics generated by the degree to what such an entity can

modify its natural environment, its rudimental system of axiomatization and last but not least its natural type of entity.

I think if we truly are convinced of the fact that there exists consciousness and even understanding inside the cosmos about *itself in the form of us*, it would at least not be too hard to contemplate about the mentioned proposal. Also one should consider that, though it is indeed true, that physical causes lead to physical effects, there is no law that states that there cannot exist in principle additional unphysical causes which could – partially - govern the behaviour of some subtle physical dynamics *by manipulating certain probabilities from the top down, hence by intelligent observers like us to govern their own physical behaviour*. This attempt would indeed afford that our consciousness to some degrees is as *independent* of the physical realm as the mentioned rudimentary perceptions of our constituent parts are (“particles” or whatever it is named). So one can say that this partially independence from the physical realm would be an analytical axiom. The latter would govern the world in an evolving manner through the creation of more and more coherently behaving local (subatomic) entities.

Entanglement would be in this framework the constant attempt of reality to preserve its original self-consistent “oneness” without loosing its self-exploring multiplicity enabled by the above mentioned iteration of the mentioned “bit”-distinction. At this point we have to take a look at set theoretic cases in which an element can be contained in more than one set. In this context, according to Bertrand Russell’s famous set theoretic antinomy [11], whenever one doesn’t strictly separate between a “set” and its “elements”, one gains a certain antinomy. Therefore Russell’s famous “*set of all sets which don’t include themselves as elements*” is such an antinomy. Does Russell’s “set of all sets” contain itself or not? This is undecidable without an additional axiom. The analogy to quantum mechanical duality of which-way information and interference is such, that the related experiments reveal an analogical (concerning Russell’s antinomy) contradiction when performed with single particles (“elements”) that can be considered as independent when which-way-information is gained. But when interference is gained it seems that such a “particle” belongs to a broader set of events (“set”). Concerning entanglement and reality’s attempt to preserve its self-consistency, one can draw the following line of reasoning:

"If Bruce is ill, he coughs" (Conditional statement) (1)

"Bruce is ill" (Antecedent)

"Bruce coughs" (consequence which is valued TRUE) (2)

Now what about if we measure not the "antecedent" first, but the "consequence"?

This would look like the following:

"If Bruce is ill, he coughs" (Conditional statement)

"Bruce coughs" (Antecedent)

"Bruce is ill" (consequence which is valued UNDECIDABLE) (3)

So if Bruce coughs, this will not necessarily mean, that he's indeed ill (but it could mean exactly this and leaves room for probabilities). This is so because, if we take the conditional

statement as a fact, “to be ill” is associated with “cough”. But beyond that we simply know neither if “cough” is the only consequence and therefore the only element in the set of consequences of “to be ill” nor do we know if “to be ill” is the only antecedent and therefore the only element in the set of antecedents of “coughs”. Though “cough” is associated with “ill”, in our physical world the state of “cough” is unfortunately *not sufficient* to indicate “ill”. The analogous appears if we swap antecedent and consequence in the conditional statement (1). If we do so, we finally have two true values (2) and two fuzzy values (3). The latter sums up to one value because it is not distinguishable from each other. At the end we have approximately the proportion of 1/3 (one fuzzy value/two truth values) to which the Bell's inequalities are violated in the experiment of Alan Aspect. So entanglement and superpositions could be explainable firstly due to the fact that some antecedents and consequences of logical propositions could be distributed over distinct physical entities - and therefore places - without losing their “oneness” and secondly due to set theoretic considerations of the essential logical distinction between merely necessary and already sufficient causes and effects.

6. Summary

The result of our considerations would be that due to logically undecidable propositions the world gains some degrees of freedom on the one hand, because every undecidable physical and logical relation must appear from an outside point of view as irreducible random in the sense of equally possible. But from an inside point of view this mustn't be effectively the case, if we assume that microcosmical entities can exhibit a tiny bit of self-government. On the other hand this degree of freedom results in a decrease of freedom for a macrocosmic observer due to automatically arising superpositions somewhere in the environment of the whole process. An example for this decrease of freedom would be the inaccessibility of which-way-information and interference at the same time.

Gödel's incompleteness theorems could have demonstrated that whenever a distinction is made, it is made to the expense of a resulting undecidability. Envision the decision between inconsistency and incompleteness of Gödel's 1931. There are two possibilities: If we choose an axiomatic system to be incomplete, we cannot decide certain propositions from inside this system. If we choose this system to be *inconsistent*, we cannot decide if it is indeed inconsistent. For the latter case we would simply run into an infinite regression of inferencing about true and false values. Therefore the main result of this essay is, by assuming consistence as more valid than completeness, Gödel's undecidability theorems demonstrate that truths can evolve out of beliefs. And for this to be possible there must be an empty space of fundamentally undistinguishable possibilities (for which every proposition can play over this cushion) that go far beyond quantum mechanical possibilities.

Finally, it is important to emphasize that the proposed framework, namely the “consciousness”-approach, isn't a theory at all, scientific or not. Instead, it is an inducing-scheme that attempts to gain some analytical and synthetically a-priori to the aim of understanding from what stuff the world is made of. Therefore the essay's considerations should not be understood as an attempt to deconstruct the multiverse-approach. It is merely the attempt to explore what can be possible in general and what cannot.

By contemplating the explanation of consciousness by evolutionary theory, there is no need to note that consciousness is one of the things in the universe, which are indeed possible to exist. But it is hard to imagine that pure mathematics should have fired into this possibility some breath to awaken the relating equations with the consequence of evolving matter that

at some point of its evolution grasps itself by an – infinite – accident. If math feels like my Qualia feel right now, I would suggest replacing the word “math” by the word “Qualia”. My belief is that for any physical knowledge that refers to abstract symbolism that can isomorphically be communicated, there is no path from abstraction to ultimate reality. So even for our considerations in this paper, there is no direct path from our abstract knowledge to ultimate reality. Because abstract knowledge cannot distinguish between truth and assumption. Therefore abstract knowledge can never leave the world of abstraction. But reality can. And reality is at least what actually is real in our Qualia, not what we assume to be real through abstract physical knowledge. So distinguishing between math and Qualia could reveal that there must be a deeper reality than only math and logical dependencies. Because enhanced awareness, be them scientific, philosophical or whatever, cannot evolve out of mere conditional statements if there isn’t something outside that can distinguish between their truth and their possibility, hence between their (in)completeness and their (in)consistence. But even if Gödel’s theorems are not capable at all to show *which* of all the things that are “thinkable” can be consistent out of themselves, one thing can: The well-known *cogito ergo sum* as an unconditional fact. I assume that the latter’s bird’s perspective of awareness can only be achieved by a conscious observer who isn’t in its own thinking completely determined by physical processes. Therefore all “thinkable” things cannot exist a-priori in a deterministically unveiling platonic realm. But if they nonetheless would, there had to be, for the aim of believing in a discoverable TOE, a level of awareness at which all these things are congruent with the source of their perception, hence being *a-priori* equivalent with consciousness. Otherwise the whole existence of all that can be, inclusively the existence of that TOE, would be deeply self-contradictory and forever not able to get complete hold of itself by abstract knowledge.

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