Nature's grammar, mathematics, settles the physics in Bell-v-Einstein

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Ever-truthful (never lying or misleading; which is handy), in accents ranging freely from big bangs to whispers (which can be tricky), Nature counsels us in many ways and sometimes nicely: yet just one grammar, *mathematics*, governs all her languages. So begins a prologue to her lesson here: Nature in truth responding to FQXi's (2014) theme – *Trick or truth: the mysterious connection between physics and mathematic*s – via that tricky Bell-v-Einstein context. First uniting classical and quantum experiments on bosons and fermions under just one language, Nature reveals neglected laws – laws that settle Bell-v-Einstein in Einstein's favor and quietly shape the realistic philosophy of most working scientists and their concept of spacetime. Seeking to keep pace with her we proceed as follows: 1-Truth, 2-Analysis, 3-Conclusions, 4-Appendix A (Language), 5-References, 6-Technical-endnotes. With Nature presenting maths as the best logic, and little more than undergraduate maths required, newcomers best begin with Appendix A – especially the modeling in Table A1 – questions, critical comments, error-corrections, etc., being very welcome here.

Key-terms: 't Hooft, CLR, Einstein's expectation, Gisin, local-causality

1 Truth

1.1. Trick-free, but taking care with superscripts and the shorthand defined in Appendix A, we begin with revealed truth: *Truth revealed by Nature and thus, forever, beyond dispute.* Based on the unification in Table A1 (elements) and Table A2 (experiments), Nature delivers:

$$E(AB\,|\,\Omega) = \int_\Omega d\lambda\,\rho(\lambda)\,|A\rangle_\Omega\,|B\rangle_\Omega \tag{1}$$

$$= P(A^+B^+|\Omega)\big|A^+B^+\big\rangle + P(A^+B^-|\Omega)\big|A^+B^-\big\rangle + P(A^-B^+|\Omega)\big|A^-B^+\big\rangle + P(A^-B^-\,|\Omega)\big|A^-B^-\big\rangle; \tag{2}$$

$$|A\rangle_\Omega = \varepsilon(s, \mathbf{a}^+, \lambda \mid \Omega)\,\big|A^+\big\rangle + \varepsilon(s, \mathbf{a}^-, \lambda \mid \Omega)\,\big|A^-\big\rangle\,; \tag{3}$$

$$|B\rangle_\Omega = \varepsilon(s, \mathbf{b}^+, \lambda' \mid \Omega)\,\big|B^+\big\rangle + \varepsilon(s, \mathbf{b}^-, \lambda' \mid \Omega)\,\big|B^-\big\rangle\,. \tag{4}$$

$$P(A^\pm B^\pm\,|\,\Omega) = \int_\Omega d\lambda\,\rho(\lambda)\,\varepsilon(s, \mathbf{a}^\pm, \lambda\,|\,\Omega)\,\varepsilon(s, \mathbf{b}^\pm, \lambda'\,|\,\Omega) \tag{5}$$

$$= P(A^\pm\,|\,\Omega)P(B^\pm\,|\,\Omega A^\pm) = P(B^\pm\,|\,\Omega)P(A^\pm\,|\,\Omega B^\pm) \neq P(A^\pm\,|\,\Omega)P(B^\pm\,|\,\Omega); \tag{6}$$

$$P(A^+\,|\,\Omega) = P(A^-\,|\,\Omega) = P(B^+\,|\,\Omega) = P(B^-\,|\,\Omega) = \tfrac{1}{2}; \tag{7}$$

$$P(A^+\,|\,\Omega B^+) = \int_\Omega d\lambda\,\rho(\lambda)\,(\sqrt{2}\,\varepsilon(s, \mathbf{a}^+, \lambda\,|\,\Omega))\,(\sqrt{2}\,\varepsilon(s, \mathbf{b}^+, \lambda'\,|\,\Omega)) = \tfrac{1}{2}(1 + E(AB\,|\,\Omega)),\ etc\,; \tag{8}$$

$$P(A^+\,|\,\Omega B^-) = \int_\Omega d\lambda\,\rho(\lambda)\,(\sqrt{2}\,\varepsilon(s, \mathbf{a}^+, \lambda\,|\,\Omega))\,(\sqrt{2}\,\varepsilon(s, \mathbf{b}^-, \lambda'\,|\,\Omega)) = \tfrac{1}{2}(1 - E(AB\,|\,\Omega)),\ etc. \tag{9}$$

$$\text{If } P(A^\pm B^\pm\,|\cdot) \neq P(A^\pm\,|\cdot)P(B^\pm\,|\cdot)$$
$$\text{then } P(A^\pm B^\pm\,|\cdot) = P(A^\pm\,|\cdot)P(B^\pm\,|\cdot A^\pm) = P(B^\pm\,|\cdot)P(A^\pm\,|\cdot B^\pm). \tag{10}$$

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2 Analysis

2.1. Taking care with analysis to ensure that no step here is negated by experiment, our approach – per Appendix A – is based on commonsense local realism (CLR). Taking care with Nature, we hold a consequence of *realism* to be: 'at all times, the set of beables possessed by a system fully determines all relevant probabilities,' after Gisin (2014). So our (1)-(10) are realistic: and already there – see #6.1 also – we refute Gisin's claim (2014:4) "for a realistic theory to predict the violation of some Bell inequalities, the theory must incorporate some form of nonlocality."

2.2. Then, taking care with truth, here's an important one: that (10) – exhausting the binary options in (5)-(6) – is universal under CLR. So with all results here in full accord with special relativity (SR), quantum mechanics (QM) and experiment, a correct reconnection between maths and physics in Bell-v-Einstein follows:

2.3. For the set C of classical experiments, using (1) with the correlation $\lambda + \lambda' = 0$, and [1]

$$|A\rangle_C = \cos^2 s(\mathbf{a},\lambda)\,\left|A^+\right\rangle + \sin^2 s(\mathbf{a},\lambda)\,\left|A^-\right\rangle; \tag{11}$$

$$|B\rangle_C = \cos^2 s(\mathbf{b},\lambda')\,\left|B^+\right\rangle + \sin^2 s(\mathbf{b},\lambda')\,\left|B^-\right\rangle: \tag{12}$$

$$E(AB\,|\,C) = \int_C d\lambda\,\rho(\lambda)|A\rangle_C\,|B\rangle_C = \int_0^{2\pi} d\lambda\,\tfrac{1}{2\pi}\,|A\rangle_C\,|B\rangle_C \tag{13}$$

$$= \tfrac{1}{8}(2+\cos 2s(\pi\pm(\mathbf{a},\mathbf{b})))(\left|A^+B^+\right\rangle + \left|A^-B^-\right\rangle) + \tfrac{1}{8}(2-\cos 2s(\pi\pm(\mathbf{a},\mathbf{b})))(\left|A^+B^-\right\rangle + \left|A^-B^+\right\rangle) \tag{14}$$

$$= \tfrac{1}{2}\cos 2s(\pi\pm(\mathbf{a},\mathbf{b}))\ \text{when}\ \left|A^\pm\right\rangle = \pm 1,\ \left|B^\pm\right\rangle = \pm 1. \tag{15}$$

2.4. Then, from (14), based on (1)-(2):

$$P(A^+B^+\,|C) = P(A^-B^-\,|C) = \tfrac{1}{8}(2+\cos 2s(\pi\pm(\mathbf{a},\mathbf{b}))); \tag{16}$$

$$P(A^+B^-\,|C) = P(A^-B^+\,|C) = \tfrac{1}{8}(2-\cos 2s(\pi\pm(\mathbf{a},\mathbf{b}))). \tag{17}$$

$$\therefore P(A^+\,|C) = P(A^+B^+\,|C) + P(A^+B^-\,|C) = \tfrac{1}{2} \neq P(A^+\mid CB^+)\,\text{nor}\,P(A^+\mid CB^-);\ \textit{etc.} \tag{18}$$

2.5. For the set Q of quantum experiments, using (1) with the correlation $\lambda + \lambda' = 0$, and [2]

$$|A\rangle_Q = (\surd 2\cos^2 s(\mathbf{a},\lambda) \pm \tfrac{1}{2})\,\left|A^+\right\rangle + (\surd 2\sin^2 s(\mathbf{a},\lambda) \pm \tfrac{1}{2})\,\left|A^-\right\rangle; \tag{19}$$

$$|B\rangle_Q = (\surd 2\cos^2 s(\mathbf{b},\lambda') \mp \tfrac{1}{2})\,\left|B^+\right\rangle + (\surd 2\sin^2 s(\mathbf{b},\lambda') \mp \tfrac{1}{2})\,\left|B^-\right\rangle: \tag{20}$$

$$E(AB\,|\,Q) = \int_Q d\lambda\,\rho(\lambda)|A\rangle_Q\,|B\rangle_Q = \int_0^{4\pi} d\lambda\,\tfrac{1}{4\pi}\,|A\rangle_Q\,|B\rangle_Q \tag{21}$$

$$= \tfrac{1}{2}\cos^2 s(\pi\pm(\mathbf{a},\mathbf{b}))(\left|A^+B^+\right\rangle + \left|A^-B^-\right\rangle) + \tfrac{1}{2}\sin^2 s(\pi\pm(\mathbf{a},\mathbf{b}))(\left|A^+B^-\right\rangle + \left|A^-B^+\right\rangle) \tag{22}$$

$$= \cos 2s(\pi\pm(\mathbf{a},\mathbf{b}))\ \text{when}\ \left|A^\pm\right\rangle = \pm 1,\ \left|B^\pm\right\rangle = \pm 1. \tag{23}$$

2.6. Then, from (22), based on (1)-(2):

$$P(A^+B^+\,|Q) = P(A^-B^-\,|Q) = \tfrac{1}{2}\cos^2 s(\pi\pm(\mathbf{a},\mathbf{b})); \tag{24}$$

$$P(A^+B^-\,|Q) = P(A^-B^+\,|Q) = \tfrac{1}{2}\sin^2 s(\pi\pm(\mathbf{a},\mathbf{b})). \tag{25}$$

$$\therefore P(A^+\,|Q) = P(A^+B^+\,|Q) + P(A^+B^-\,|Q) = \tfrac{1}{2} \neq P(A^+\mid QB^+)\,\text{nor}\,P(A^+\mid QB^-);\ \textit{etc.} \tag{26}$$

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[1] See #6.2 for analyzer-states $|A\rangle_.$ and $|B\rangle_.$ that reproduce the $\mathbf{a}^\pm \sim A^\pm$ and $\mathbf{b}^\pm \sim B^\pm$ relations in (3)-(4).

[2] See footnote 3. See #6.3 re the seeming redundancy here of $\pm$ or $\mp$ and our use of the term *conjugate*.

2.7. For $A^{\pm} = \pm 1,\ B^{\pm} = \pm 1$; comparing (15) and (23):

$$E(AB\,|\,Q) = \cos 2s(\pi \pm (\mathbf{a}, \mathbf{b})) = 2E(AB\,|\,C). \tag{27}$$

$$\therefore\ E(AB|Q_{\frac{1}{2}}) = -\cos(\mathbf{a}, \mathbf{b}) = 2E(AB|C_{\frac{1}{2}});\ E(AB|Q_1) = \cos 2(\mathbf{a}, \mathbf{b}) = 2E(AB|C_1); \tag{28}$$

equals more truth. See #6.4 re the physics in (27), and note that, *given their basis in (1),* (28) accords with CLR, QM and experiment while (16)-(18) and (24)-(26) confirm (10).

2.8. Now (1) and (10) – with their mathematics; these ever-present laws in Nature – are often neglected laws when it comes to the study and critique of Bellian mathematics. Yet – in CLR terms – the limited physical significance of Bell's maths is exemplified in the following view, held typically by many of his supporters:

> "Einstein maintained that quantum metaphysics entails spooky actions at a distance; experiments have now shown that what bothered Einstein [ie, spooky actions at a distance] is not a debatable point but [is, thanks to Bell and Aspect] the observed behavior of the real world," see Mermin (1985:38). That is, via *spooky action at a distance*, particle $p(*)$ in Alice's locale acquires a definite value of a property as a result of the interaction of its twin $p'(*)$ with a device in Bob's locale ...

2.9. But we've just studied four experiments (two classical, two quantum; with bosons and fermions) and there – from (1)-(28), thanks to factorizations (1) and (10) proven per (28) – we find neither need nor evidence for spooky actions at a distance. We therefore side with Einstein (from Mermin 1985:38 above): It is not Nature but metaphysics – in our terms, abstract theory or talk with no basis in reality; think Bell (2004), Brunner *et al* (2014), Gisin (2014a), Mermin (1985), etc – that entails spooky action at a distance.

2.10. So let's examine the experiment that many, like Mermin (1985), rely on: it's our Q_1. From (22), taking $A^+ = B^+ = G$ (Green-flash) and $A^- = B^- = R$ (Red-flash), with $s = 1$:

$$E(AB\,|\,\text{Mermin 1985}) = \tfrac{1}{2}\cos^2(\mathbf{a}, \mathbf{b})(|GG\rangle + |RR\rangle) + \tfrac{1}{2}\sin^2(\mathbf{a}, \mathbf{b})(|GR\rangle + |RG\rangle); \tag{29}$$

ie, without mystery under CLR – and hence without spooky actions, given the factorizations in (1) and (21) – the same colors always flash when the polarizer-settings are the same; the color-pattern is completely random when the polarizer-settings are random. So where do Bell, Mermin, and so many others go wrong?

2.11. From Table A1, and termed *separability*: an object in Alice's locale, spacelike separated from an object in Bob's locale, has independent existence; and vice versa. So, based on Einstein (1948), "the following idea characterizes the relative independence of objects far apart in space" (by which we mean *the principle of local-causality*; see #A4.3-4.4): External influence on an object in Alice's locale has no direct influence on an object in Bob's locale; and vice-versa. Thus (1), with its sharp factorization, cleans up Einstein's intuitive idea mathematically.

2.12. But here's Bell with his own idea (in our terms; with our emphasis and footnote):

> 'Now it is precisely in cleaning up intuitive ideas for mathematics that one is likely to throw the baby out with the bathwater. *So the next step should be viewed with the utmost suspicion*: A theory will be said to be locally causal if the probabilities attached to the local beables in Alice's locale are unaltered by specification of values of local beables in Bob's locale when what happens in the backward lightcone of Alice's locale is already sufficiently specified by condition c,' Bell (2004:239,242).[3]

[3]Bell appears to mix marginal and conditional probabilities here; see #2.14 below, *et seq*; and #6.5.

2.13. Interlude: If Bell's condition c is a relevant pre-condition then it defines the experiment and is certainly included in our condition $\mathit{\Omega}$. Either way, when it comes to deriving the related probabilities – and though we ourselves use them – under CLR (and in general) *the following circumstances are of no consequence in a conditioning-space*: the inclusion of irrelevant conditions (for symmetry in presentation, say) or the repetition of relevant conditions (say, for clarity).

2.14. So, from #2.12, here's our view of Bell's intuition again: 'A theory will be said to be locally causal if the probabilities attached to the local beables in Alice's locale are unaltered by specification of values of local beables in Bob's locale when what happens in the backward lightcone of Alice's locale is already sufficiently specified by condition c.' But comparing (8) with (7) we see that specification of B^+ (a local beable in Bob's locale, and never in Alice's backward lightcone) *does alter* the probability attached to the corresponding A^+ (a local beable in Alice's locale). In short: A conditional probability here does not equate to a marginal one.

2.15. Indeed, Bell discovers this fact for himself. For at Bell (2004:243-244) – via his equations (9)-(14) there; after *mathematically sharpening his intuition* – Bell finds (and we agree; again in our terms and using #2.13 for reasons of both symmetry and clarity: (See also #6.1, #6.5.)

$$P(A^+B^+\,|Q,\mathbf{a},\mathbf{b},c,s,\lambda,\lambda') \neq P(A^+\,|Q,\mathbf{a},\mathbf{b},c,s,\lambda,\lambda')P(B^+\,|Q,\mathbf{a},\mathbf{b},c,s,\lambda,\lambda'). \tag{30}$$

2.16. But Bell does not back-track and invoke the universal consequence of our (10). After (30) and under CLR, that consequence for us – a return to his equation (9) – is a valid restarting-point. Alas, the *mathematical sharpening of Bell's intuition* moves him beyond his (9) to our (30), so he emphatically concludes (Bell 2004:244): QM "can*not* be embedded in a locally causal theory." So, given QM's success, Bell here throws Einstein's baby (local-causality) out with his own bathwater (his misleading intuition) – despite his grave suspicion at #2.12 above.

2.17. The remedy follows: Agreeing with Bell re (30), we now invoke an appropriate variant of (10) and proceed to a conclusion under CLR (with its wholly local factorings) – noting that, re #2.12-2.14, (10) entails the product of marginal and conditional probabilities. [4]

$$\therefore P(A^+B^+\,|Q,\mathbf{a},\mathbf{b},c,s,\lambda,\lambda') = P(A^+\,|Q,\mathbf{a},\mathbf{b},c,s,\lambda,\lambda')P(B^+\,|Q,\mathbf{a},\mathbf{b},c,s,\lambda,\lambda',A^+) \tag{31}$$

$$= P(A^+\,|Q,\mathbf{a},c,s,\lambda)P(B^+\,|Q,\mathbf{b},c,s,\lambda',A^+) \quad \textit{under CLR} \tag{32}$$

$$= \tfrac{1}{2}\int_{\mathit{\Omega}} d\lambda\,\rho(\lambda)\,(\sqrt{2}\,\varepsilon(s,\mathbf{b}^+,\lambda'\,|\,\mathit{\Omega}))\,(\sqrt{2}\,\varepsilon(s,\mathbf{a}^+,\lambda\,|\,\mathit{\Omega})) \quad \textit{based on } (7)-(8), (42) \tag{33}$$

$$= \int_0^{4\pi} d\lambda\,\tfrac{1}{4\pi}(\sqrt{2}\cos^2 s(\mathbf{b},\lambda') \mp \tfrac{1}{2})(\sqrt{2}\cos^2 s(\mathbf{a},\lambda) \pm \tfrac{1}{2}) \quad \textit{based on } (19)-(20) \tag{34}$$

$$= \tfrac{1}{2}\cos^2 s(\pi \pm (\mathbf{a},\mathbf{b})):\ \textit{QED, agreeing with } (24);\ \textit{etc.} \tag{35}$$

2.18. Further, per (33): our theory is realistic – eg, Gisin (2014) and #2.1 – for in *being realistic*: 'at all times, the set of beables possessed by a system fully determines all relevant probabilities.' Thus, per (10) and per (31), when A^+ is an element of the set of beables possessed by the *global* system under $\mathit{\Omega}$, that *global* set of beables *still* fully determines all the relevant *global* probabilities: all of which, per (33) and (1)-(10), remain *locally factorable* under CLR!

2.19. For, from (11)-12) and (19)-(20), each spacelike separated analyzer-output $|.\rangle$ is associated with functions based on $s(\mathbf{a},\lambda)$ and $s(\mathbf{b},\lambda')$; $(.,.)$ here denoting 'angles of attack' for λ and λ'. Spin s is thus the driver (Watson 2013:#19.2) for the polarizer-functions $[\lambda \hookrightarrow \mathbf{a}^\pm \mid \mathit{\Omega}\}$ and $[\lambda' \hookrightarrow \mathbf{b}^\pm \mid \mathit{\Omega}\}$: for s is in units of $\hbar = \frac{h}{2\pi}$ where $\hbar$ is Dirac's constant (the quantum of angular momentum) and h is Planck's constant (the quantum of action). Supplementary tests can confirm the binary polarized-outputs under both $[\lambda \hookrightarrow \mathbf{a}^\pm \mid \mathit{\Omega}\}$ and $[\lambda' \hookrightarrow \mathbf{b}^\pm \mid \mathit{\Omega}\}$.

[4]Under Q, integrals over the product of *relevant conjugates* yield all our probabilities; see #6.3.

3 Conclusions

3.1. Wholly consistent with CLR, (1) and (10) are neglected laws that rightly reconnect maths to the physics in Bell-v-Einstein. By simultaneously satisfying two grand theories (QM/SR), they quash nonlocality and refute Gisin's (2014:4) claim re nonlocality at #2.1 above. In our terms, (1) – *Einstein's general expectation under local-causality* – is grounded in the physics of SR; its components in turn based on *Einstein's local expectation* that spacelike separated outcomes $|.\rangle$ will be causally associated with local beables alone. Similarly, (10) with its symmetric and asymmetric strands – our general product rule for the probability of any compound event – is equally grounded in physics, departures therefrom being impossible (per #2.2).

3.2. (1) and (10) thus: (i) resolve Bell-v-Einstein in Einstein's favor, (ii) recover local-causality from Bell's false expectation per #2.14-2.17, (iii) unite bosons and fermions via spin s in a unified driver of particle-polarizer interactions, (iv) reinforce and quietly shape the realistic philosophy of most working scientists and their concept of spacetime.

3.3. Indeed, the disconnect between Bell's maths and the associated physics arises from Bell's classicality. (i) Bell's naive realism at #A4.7 ignores Bohr's insight at #A4.5 and Bell's own remark at #A4.6: to thereby breach CLR's principle of physical-realism (#A4.5). (ii) Bell's primary expectation at #2.14 holds (in part) classically under condition C; OK per (11)-(12), but not under (18). However, Bell's intuition is misleading, and false quantumly, under condition Q; per (19)-(20).

3.4. In this latter regard, we may attribute the odd forms of (in our terms) Einstein's expectation ε – say (19)-(20) – to the symmetric factorization of asymmetric probabilities: noting that CLR-based probability-based factors of such asymmetric probabilities follow thereafter. But in satisfying the boundary-conditions (1)-(4) under Ω via (11)-12) and (19)-(20), we prove that (10) holds universally: to thus refute all quantumly unphysical 'Bellian factorizations'.

> From the simplicity of (1) to the simple universality of (10), 'our experience to-date justifies us feeling sure that in Nature is actualized the ideal of mathematical simplicity,' after Einstein (1933) ex Pais (1979:910).

3.5. In particular, Bell's theorem should no longer be a constraint on 't Hooft's (2014) [5] program, especially not at 't Hooft 2014:(8.22)-(8.23). That is, reviewing #A4.13 in the light of our results, we conclude that Alice and Bob (whether human or robotic) have sufficient free-will to complete any experiment to our CLR satisfaction. For, in concluding that Bell's theorem is irrelevant to any serious physical theory, we eliminate the need for superdeterminism in the mathematics here. We thus agree with Bell's better intuition (Bell 2004:244) here, "In such 'superdeterministic' theories the apparent free will of experimenters, or any other apparent randomness, would be illusory." !beebnI

3.6. We therefore close with a happy snapshot of Wigner's (1960:14) views and our own:

> "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."
>
> Nature speaks in many ways, from big bangs to whispers, but just one grammar, *beautiful mathematics*, governs all her languages: thus all her laws.

[5] See #6.6 re our differing approaches.

4 Appendix A: Language

A4.1. This essay is based on commonsense local realism (CLR), the union of local-causality (after Einstein 1949: *no information, signal or causal influence propagates superluminally*) and physical-realism (after Bohr, see Bell 2004:xi: *some physical properties change interactively*). Paragraphs and equations are numbered to facilitate discussion; many texts are freely available online (see 5-References); *the work of others is cited and accurately expressed in our terms.*

Table A1: Elements of reality, abstract and concrete, under experiment Ω.
(i) Each element is defined by the context $C_{\frac{1}{2}}, C_1, Q_{\frac{1}{2}}$ or Q_1 per Table A2: For $s = \frac{1}{2}$, device $D(\mathbf{a})$ is (in our terms) a spin-half polarizer-analyzer with its principal-axis oriented $\mathbf{a}$; etc. For $s = 1$, device $D(\mathbf{a})$ is photonic polarizer-analyzer with its principal-axis oriented $\mathbf{a}$; etc. (ii) Alice's locale (with the $D(\mathbf{a}) \leftarrow p(*)$ interactions) is spacelike separated from Bob's locale (with the $p'(*) \rightarrow D'(\mathbf{b})$ interactions).

(a)	Abstract	$[A^{\pm} \Leftarrow \mathbf{a}^{\pm}\{\{\mathbf{a}^{\pm} \hookleftarrow \lambda] \leftarrow p(\lambda, s) \quad \langle \lambda + \lambda' = 0 \rangle_{\Omega} \quad p'(\lambda', s) \rightarrow [\lambda' \hookrightarrow \mathbf{b}^{\pm}\}\}\mathbf{b}^{\pm} \Rightarrow B^{\pm}]$
(b)	Bounds	[Alice's locale] $\cdot\cdot \langle$ Source $\rangle_{\Omega} \cdot\cdot$ [Bob's locale]
(c)	Concrete	$\langle A^{\pm}\| \Leftarrow [D(\mathbf{a})] \leftarrow p(*) \quad \cdot\cdot \langle \text{Source} \rangle_{\Omega} \cdot\cdot \quad p'(*) \rightarrow [D'(\mathbf{b})] \Rightarrow \|B^{\pm}\rangle$

Note: (i) All terms are defined in Table A3. (ii) Vector $\mathbf{a}^{+} = +\mathbf{a}$; under $s = \frac{1}{2}$, $\mathbf{a}^{-} = -\mathbf{a}$; under $s = 1$, $\mathbf{a}^{-} = \perp\mathbf{a}$ (ie, $\mathbf{a}^{-}$ is orthogonal to $\mathbf{a}$); etc. Ω thus defines each experiment completely. (iii) Polarizer-analyzer $[D(\mathbf{a})]$ consists of polarizer-function $[\lambda \hookrightarrow \mathbf{a}^{\pm}\}$ followed by analyzer-function $\}\mathbf{a}^{\pm} \Rightarrow A^{\pm}]$. The binary analyzer-state $|A^{\pm}\rangle$ may represent ± 1, G/R, etc.

A4.2. The principal elements of physical reality (the principal beables) in Bell (1964) and Aspect (2002) are shown at **Table A1**(c), a unified model for the four experiments under discussion here. Re Bell (1964), our unit-vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ replace Bell's $\vec{a}, \vec{b}, \vec{c}$. In trigonometric arguments and contexts, $(\mathbf{a}, \mathbf{b})$ denotes the angle between vectors $\mathbf{a}$ and $\mathbf{b}$; etc. When clarity requires, primes ($'$) identify elements in Bob's locale. $A^{\pm}$ and $B^{\pm}$ (representing Bell's A and B) typically denote ± 1; expectation $E(AB \mid Q_{\frac{1}{2}})$ replaces Bell's equivalent term $P(\vec{a}, \vec{b})$; etc.

A4.3. From Table A1 – *nothing relevant missing, nothing irrelevant found* – every relevant element of the subject physical reality has its counterpart in our analysis and there are no irrelevancies. Specifically, there are no superluminal signals nor action-at-a-distance under CLR:

> "The direct causes (and effects) of events are nearby, and even the indirect causes (and effects) are no further away than permitted by the velocity of light," Bell (2004:239).

A4.4. CLR's local-causality – *no information, signal or causal influence propagates superluminally*: ie, consistent with SR, beables (elements of physical reality) in Alice's locale are causally independent of beables in Bob's locale; and vice-versa. Thus, holding fast to the classical principle of local causality and referring to Table A1$(a)-(c)$, CLR agrees with Einstein (1949:85): The real factual situation of any beable or system in Alice's locale is independent of what is done with any beable or system in Bob's locale, which is spatially separated from the former; and vice-versa. This principle is expressed mathematically in (1)-(4).

A4.5. CLR's physical-realism – *some physical properties change interactively*: ie, consistent with **Bohr's insight**, "the result of a 'measurement' does not in general reveal some preexisting property of the 'system', but is a product of both 'system' and 'apparatus'," Bell (2004:xi-xii). Comparing the Table A1 rows (a) and (c), local particle/device interactions modify each lambda so that it accords with the orientation of a relevant output channel. In Bob's locale, that interaction transforms λ' to $\mathbf{b}^{\pm}$ as indicated by the Ω-qualified polarizer-function $[\lambda' \hookrightarrow \mathbf{b}^{\pm} \mid \Omega\}$; ie, each particle/device interaction yields a particle polarized via an available output channel – and such results may be confirmed by supplementary experiments; etc. Since $\mathbf{b}^{\pm}$ is binary under exclusive-or: all analyzer-outputs $\}\mathbf{b}^{\pm} \Rightarrow B^{\pm}]$ here are binary; etc.

> Based on Bell (1964:196): In a complete physical theory of the type envisioned by Einstein, the beables would have dynamical significance and laws of motion. Our λ and λ' can be thought of as initial values of these pair-wise correlated beables as each particle-pair leaves the relevant source – under C or Q – see Table A1*(a)*.

A4.6. CLR thus supports **Bell's remark**: 'It seems to me [ie, to Bell] that full appreciation of this [ie, Bohr's insight in #A4.5 above] would have aborted most of the 'impossibility proofs' . . . ,' based on Bell's 1987 preface at Bell (2004:xii). But CLR rejects Bell's inconsistency; for Bell earlier championed a contradictory naive realism in Bell (1981; at 2004:147): "To explain this dénouement without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)."

A4.7. From d'Espagnat (1979:166),

> "One can infer that in every particle-pair, one particle has the property A^+ and the other has the property A^-, one has property B^+ and one B^-, ... Such conclusions require a subtle but important extension of the meaning assigned to our notation A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself."

A4.8. Compounding Bell's rejection of Bohr's insight (#A4.5 above), and Bell's contradiction (#A4.7 versus #A4.5-4.6), #2.12-2.18 in the main text also show that Bell's final formulation of his theorem relies on a naive classicality, similar to that in #A4.7.

Table A2: Four experiments (two classical, two quantum) and their unification via set Ω.

Context	Description	Reference
Ω	$\{C_{\frac{1}{2}}, C_1, Q_{\frac{1}{2}}, Q_1\} \subseteq \Omega$	Table A1
C	$\{C_{\frac{1}{2}}, C_1\} \subseteq C$	-
$C_{\frac{1}{2}}$	Classical experiment based on $Q_{\frac{1}{2}}$	#A4.11
C_1	Classical experiment based on Q_1	#A4.11
Q	$\{Q_{\frac{1}{2}}, Q_1\} \subseteq Q$	-
$Q_{\frac{1}{2}}$	Quantum experiment EPRB (B = Bohm)	Bell (1964), #A4.11
Q_1	Quantum experiment EPRBA (A = Aspect)	Aspect (2002), Mermin (1985)

A4.9. Table A2 shows the unification of four experiments under the set Ω; two termed *classical* (without scare-quotes) and two termed *quantum* for convenience in analysis. That is, fully accepting that we live in a quantum world, we use the word *classical* here to identify the two experiments that may be understood by an extension of the classical Malus' Law: eg, re C_1, Étienne-Louis Malus (1775-1812) could have conducted an equivalent classical experiment and derived the correct classical results by considering the intensity of light-beams alone. In other words, from our unified comparative analysis, we seek to show that terms widely used and accepted classically (eg, locality, separability, local-causality, relativity; no spooky action at a distance) might now be permanently accepted and used quantumly.

A4.10. Nevertheless, each experiment here is based on a quantum source: Under quantum Q, each particle-pair leaves the pristine source correlated by the conservation of angular momentum; ie, $\lambda + \lambda' = 0$. Under classical C we enclose the pristine source of its quantum counterpart Q in a black-box so that, under C, each particle-pair leaves the black-box correlated by linear-polarization in one random direction, orthogonal to their line of flight; ie, $\lambda + \lambda' = 0$.[6]

> Based on Bell (1964:196) again: In a complete physical theory of the type envisioned by Einstein, the hidden beables would have dynamical significance and laws of motion; our λ and λ' can be thought of as initial values of these pair-wise correlated beables as each particle-pair leaves the source; see Table A1*(a)*. Allowing λ to be a random beable in 3-space under Q, or in 2-space under C, it is probability zero that two particle-pairs are the same.

A4.11. $C_{\frac{1}{2}}$ is derived from $Q_{\frac{1}{2}}$ as follows (with C_1 derived from Q_1 similarly): (i) Each polarizer-analyzer $\check{D}$ is constrained so that its principal-axis rotates in a plane orthogonal to the line-of-flight of the particles. (ii) $Q_{\frac{1}{2}}$'s particle-source is placed in a black-box (which does not disturb a particle's line-of-flight) to form the $C_{\frac{1}{2}}$-source. (iii) The particle-pairs, originally highly-correlated in the singlet state under $Q_{\frac{1}{2}}$, now leave the $C_{\frac{1}{2}}$-source more weakly correlated: Alice's particle is polarized spin-up with respect to orientation $\check{\lambda}$ orthogonal to the line-of-flight; Bob's particle is polarized spin-up with respect to orientation λ'. This outcome is achieved via yoked linear-polarizers stepping in unison. (iv) Upon leaving the black-box each new λ varies randomly from pair to pair with normalized probability distribution $\rho(\lambda) = \frac{1}{2\pi}$ over the (now) 2-space of λ.

A4.12. To be clear: All correlations here arise from the underlying correlations of separated beables, principally via $\lambda + \lambda' = 0$ from the *initial* conservation of angular momentum in the production of each particle-pair under Q. Under our black-box interventions, a similar relation holds adequately for the related correlation of linear-polarizations under C.

A4.13. Alice (the agent for polarizer-setting $\mathbf{a}$) is spatially separated from Bob (the agent for polarizer-setting $\mathbf{b}$). And (for now), whether human or robot, we allow Alice and Bob to be free-willed. Thus, in the absence of signals between them, Bob's free-will cannot possibly be correlated with Alice's free-will, nor with whatever the source of the particles does. And vice-versa. The point being that, in their respective spaces (ie, 2 under C or 3 under Q), $\mathbf{a}$ and $\mathbf{b}$ are free or random beables (Bell 2014:243; Gisin 2012:Slide 4) independent of λ and λ'; and vice-versa.

A4.14. In (3)-(5), *Einstein's expectation* ε – bound by (1)-(4) – is a consequence of the local-causality associated with SR; there being no action-at-a-distance, etc, allowed or required under CLR. Thus, per Table A1(b), outcomes in Alice's locale are causally independent of beables in Bob's locale; and vice-versa. So, for us, (1) is Nature's law for local-causality in physics:

> "The direct causes (and effects) of events are nearby, and even the indirect causes (and effects) are no further away than permitted by the velocity of light," Bell (2004:239).

A4.15. So the possibilities associated with each ε are limited to relevant beables in the same locale. Thus $\varepsilon(s, \mathbf{a}^{\pm}, \lambda \,|\, \varOmega)\,|A^{\pm}\rangle$ expresses the fact that marginal probabilities and effects $A^{\pm}$ in Alice's locale – spacelike separated from Bob's locale per Table A1*(b)* – will be associated with a function of the relevant beables in her locale, here $s, \mathbf{a}^{\pm}$ and λ alone. Similarly, $\varepsilon(s, \mathbf{b}^{\pm}, \lambda' \,|\, \varOmega)\,|B^{\pm}\rangle$ expresses the fact that marginal probabilities and effects $B^{\pm}$ in Bob's locale – spacelike separated from Alice's locale per Table A1*(b)* – will be associated with a function of the relevant beables in his locale, here $s, \mathbf{b}^{\pm}$ and λ' alone.

[6] Under C_1, some might prefer $\lambda = \lambda'$, but no result changes under the convenient commonality used here.

Table A3: Summary of key terms and symbols		
Symbol	Meaning	Refer
$*$	Particle with spin s, identified by unit-vector λ or λ' in **A1**(a)	**A1**(c)
$'$	Identifies an element in Bob's locale when clarity requires	#A4.2
$\mid .)$	The *conditioning space* in a probability function $P(. \mid .)$	(10)
$\oplus$	XOR, *exclusive*-or	-
ε	*Einstein-expectation*; our term, based on special relativity	#A4.13
λ, λ'	Correlated *beables*; in our terms $\lambda + \lambda = 0$.	#A4.5
Ω	The set of four experiments; see Table **A1**	←
$\mathbf{a}$	Unit-vector, principal-axis orientation of Alice's polarizer	#A4.2
$\mathbf{a}^{\pm}$	Vector-pair for symmetry over three vector states; see next	**A1**(a)
$\mathbf{a}^{+}, \mathbf{a}^{-}$	$\mathbf{a}^{+} = -\mathbf{a}$; $\mathbf{a}^{-} = -\mathbf{a}$ under $s = \frac{1}{2}$; $\mathbf{a}^{-} = \perp\mathbf{a}$ under $s = 1$.	**A1**(a)
$A^{\pm}$	Alice's analyzer-output; input $p(s, \mathbf{a}^{\pm}) \Rightarrow A^{\pm} = \pm 1$ typically	**A1**(a)
$\|A^{\pm}B^{\pm}\rangle$	$= \|A^{\pm}\rangle\|B^{\pm}\rangle$; analyzer-states signaling polarizer-outcomes	(2)
Alice	Free agent, orients the principal-axis $\mathbf{a}$ of polarizer $D(\mathbf{a})$	#A4.13
$\mathbf{b}$	Unit-vector, principal-axis orientation of Bob's polarizer	#A4.2
$B^{\pm}$	Bob's analyzer-output; input $p'(s, \mathbf{b}^{\pm}) \Rightarrow B^{\pm} = \pm 1$ typically	**A1**(c)
beable	Element of physical reality (Bell 2004:174)	#A4.2
Bob	Free agent, orients the principal-axis $\mathbf{b}$ of polarizer $D'(\mathbf{b})$	#A4.13
$\mathbf{c}$	Third unit-vector, Bell (1964:198)	#A4.2
C	The set of classical experiments; see Table **A2**	**A2**
CLR	Commonsense local realism	#A4.1
D (D')	Polarizer-analyzer; principal-axis oriented $\mathbf{a}(\mathbf{b})$ by Alice (Bob)	**A1**(c)
Driver	$s(\mathbf{a}, \lambda)$; driver for polarizer input-output function at $D(\mathbf{a})$; etc	#2.19
$E(AB \mid \Omega)$	Expectation over the outcome AB under condition Ω	(1)
EPR	Einstein, Podolsky, Rosen essay (EPR 1935)	←
EPRB	EPR-Bohm experiment; see Bell (1964)	←
EPRBA	Aspect's EPRB-based experiment (Aspect 2002)	←
h	Planck's constant, the quantum of action	-
$\hbar$	Dirac's constant, the quantum of angular momentum $= {}^{h}/_{2\pi}$	-
$p(*)$, $p'(*)$	Paired-particles associated with locale of Alice, Bob respectively	**A1**(c)
$P(X \mid Y)$	*Marginal probability* of X under general condition Y	(7)
$P(X \mid YZ)$	*Conditional probability* of $X \mid Y$ under special condition Z	(8)
Q	The set of quantum experiments; see Table **A2**	**A2**
$s\hbar$	Intrinsic spin; $s = {}^{1}/_{2}$ (1) for spin-half particles (photons)	-

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6 Technical-endnotes

6.1. Re #2.1: From (1)-(10), and in our terms, we certainly agree with Gisin (2012:Slide 2):

$$P(A^{\pm}B^{\pm}\,|Q,\mathbf{a},\mathbf{b},\lambda,\lambda') \neq P(A^{\pm}\,|Q,\mathbf{a},\lambda)P(B^{\pm}\,|Q,\mathbf{b},\lambda'). \tag{36}$$

And under CLR and (10) – with our spin s included in Q – we respond:

$$\therefore P(A^{\pm}B^{\pm}\,|Q,\mathbf{a},\mathbf{b},\lambda,\lambda') = P(A^{\pm}\,|Q,\mathbf{a},\lambda)P(B^{\pm}\,|Q,\mathbf{b},\lambda',A^{\pm});\ etc. \tag{37}$$

Here, however, is Gisin's (2012:Slide 2) response to (36): "The events at Alice and Bob's sides are not independent! It seems that somehow the two sides are coordinated or 'interact' !?! (but without signaling):

> Spatially separated systems are not logically separated
> ⇒ Quantum Physics is nonlocal." (A)

To which we respond, given our (5)-(6): The events at Alice and Bob's sides are *causally* independent, per (5); and they are *logically* dependent, per (6) – which is an immediate consequence of (5). This is as it should be: the *causally* independent Einstein expectations in (5) – see them; the coefficients in (19)-(20) – are correlated by their common spins s and the $\lambda+\lambda'=0$ correlation of their lambdas. So, in our CLR terms – given that there is neither interaction nor signaling between such systems –

> Some spatially separated systems are logically dependent
> ⇒ Such spatially separated systems are correlated by correlated beables
> like $s\lambda + s\lambda' = 0$. (B)

> "The direct causes (and effects) of events are nearby, and even the indirect causes (and effects) are no further away than permitted by the velocity of light," Bell (2004:239).

To be clear about the meaning in (B) we complete the loop with:

> Some spatially separated systems are not logically dependent
> ⇒ Such spatially separated systems are not correlated by correlated beables. (C)

6.2. Re #2.3 and #2.5: Many functions satisfy (1)-(4) but we'll be satisfied with (38)-(41) for the moment. However, in (11)-(12) and (19)-(20) we employ the *principal* polarizer output-channels **a** and **b** only.

$$|A\rangle_{Cs} = \cos^2 s(\mathbf{a}^+,\lambda)\,\left|A^+\right\rangle + \cos^2 s(\mathbf{a}^-,\lambda)\,\left|A^-\right\rangle; \tag{38}$$

$$|B\rangle_{Cs} = \cos^2 s(\mathbf{b}^+,\lambda')\,\left|B^+\right\rangle + \cos^2 s(\mathbf{b}^-,\lambda')\,\left|B^-\right\rangle. \tag{39}$$

$$|A\rangle_{Qs} = (\surd 2\cos^2 s(\mathbf{a}^+,\lambda) \pm \tfrac{1}{2})\,\left|A^+\right\rangle + (\surd 2\cos^2 s(\mathbf{a}^-,\lambda) \pm \tfrac{1}{2})\,\left|A^-\right\rangle; \tag{40}$$

$$|B\rangle_{Qs} = (\surd 2\cos^2 s(\mathbf{b}^+,\lambda') \mp \tfrac{1}{2})\,\left|B^+\right\rangle + (\surd 2\cos^2 s(\mathbf{b}^-,\lambda')\,\left|B^-\right\rangle \mp \tfrac{1}{2})\,\left|B^-\right\rangle. \tag{41}$$

6.3. Re #2.5 and #2.17: We have not yet studied CLR connections with QM notation and terminology. So (with apologies; and until our CLR/QM study begins) our seemingly redundant use of ± and ∓ is associated with ongoing studies of boson-fermion relations. In this regard, all probabilities under Q here may be derived from integrals over the product of (in our terms) *relevant conjugates*. Thus, under the integral in (21), terms in (19) with ± are multiplied by terms in (20) with ∓. In our terms, such multiplications yield the product of relevant *special*-conjugates whereas $(\surd 2\cos^2 s(\mathbf{a}^+,\lambda) \pm \frac{1}{2})$ is a relevant *standard*-conjugate of $(\surd 2\cos^2 s(\mathbf{a}^+,\lambda) \mp \frac{1}{2})$. Thus,

$$P(A^+\,|\,Q) = \int_0^{4\pi} d\lambda\,\tfrac{1}{4\pi}\,(\surd 2\cos^2 s(\mathbf{a}^+,\lambda) \pm \tfrac{1}{2})(\surd 2\cos^2 s(\mathbf{a}^+,\lambda) \mp \tfrac{1}{2}) = \tfrac{1}{2};\ etc. \tag{42}$$

6.4. Re #2.7 and the physics in the relation $E(AB \mid Q) = \cos 2s(\pi \pm (\mathbf{a}, \mathbf{b})) = 2E(AB \mid C)$ in (27). Under Q: $(\mathbf{a}, \mathbf{b})$ is the angle between unit-vectors $\mathbf{a}$ and $\mathbf{b}$ in 3-space, while λ and λ' are unit-vectors in the same 3-space (correlated by $\lambda + \lambda' = 0$). Under C: $(\mathbf{a}, \mathbf{b})$ is the angle between unit-vectors $\mathbf{a}$ and $\mathbf{b}$ in the 2-space orthogonal to the line-of-flight of the particles, while λ and λ' are unit-vectors in the same 2-space (correlated by $\lambda + \lambda' = 0$; footnote 5 refers). The remarkable spherical-symmetry under Q is a consequence of the conservation of angular momentum when each particle-pair is produced. Thus the factor of 2 in (27)-(28) reflects the fact that the polarizer-analyzers operate over the same space as that in which the particle-pairs are correlated: the unfettered 3-space symmetry under Q yielding twice the correlation of the black-box induced (and hence correlation-reduced) 2-space symmetry of λ and λ' under C.

6.5. Re #2.12: Under classical C we can take control of each black-box (the one used in $C_{\frac{1}{2}}$ and the one used in C_1) and totally control the orientation of each λ, and hence of its correlated twin λ'. With respect to Bell (2004:239,242), at #2.12 in our terms: " ... what happens in the backward lightcone of Alice's locale is [then] already sufficiently specified ..." via our direct knowledge of each λ. However the probabilities attached to the local beables in Alice's local are still altered by the specification of local beables in Bob's locale: as in the classical case at (18). In this way, from this alternative perspective, we anticipate the negative findings – re Bell's (2004:239) intuition – at #2.14 *et seq.*

6.6. Re #3.5: Bound by commonsense local realism (CLR), we reject Bell's theorem and eliminate the need for 't Hooft's superdeterminism. Our approach thus differs diametrically from 't Hooft's, who "did not refute Bell's theorem but by-passed it by accepting superdeterminism," after G 't Hooft (2014, pers. comm., 1 July).

> On one supposition we absolutely hold fast; that of local/Einstein causality: "The real *factual* situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former," after Einstein (1949:85).