A Tale of Two Relativities

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Abstract

Some fundamental aspects of gravitation, quantum mechanics and cosmology are identified, as well as departures from modern premises. A synthesis of modern ideas yields a radically different picture of the universe. In gravitation, a relativistic accounting of redshift associated with potential energy is required, leading to a novel gravitational relativity. In quantum mechanics, Planck's hypothesis about quantization of energy levels is essential, while the notion that quantum mechanics belongs in the domain of the small is questionable. In cosmology, recognizing the galactic redshift of light in the context of Planck's hypothesis dispenses with the big bang, inflation and dark energy.

Introduction

My aim is to show where conventional physical theories are not aligned with modern foundations, and furthermore, to present modern alternatives. The reader can then decide to what degree these theories are fundamentally sound or fundamentally questionable.

The discussion actually involves three relativities. A novel gravitational relativity is relegated to the Appendix. Let's start with Galilean *vs* special relativity.

Gravitation

Fundamental: **Inverse square law of gravity, relativity**
Questionable: **Non-relativistic Newtonian potential energy**

The inverse square law of force is at the base of Newtonian gravitation. It follows conceptually from the notion of uniformly distributed lines of force directed toward the center of a sphere. Since the same number of lines pass through any concentric spherical shell, and the area of a shell varies as the square of its radius, the number of lines passing through a patch of fixed area on the spherical shell varies inversely as the square of the radius. By integrating the force over radial distance, the inverse square law yields Newtonian gravitational potential energy. Newtonian potential energy was used in the development of general relativity to verify that weak field approximations agreed with classical predictions, and it was then essentially embedded in Schwarzschild's exact metric solution.

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The deep connection between classical gravitation and general relativity can be demonstrated using Gullstrand-Painlevé coordinates [1], which form a metric knowing only the radial escape velocity as a function of radius.[1] It turns out that the radial escape velocity required for general relativity is the same as Newtonian escape speed. This coincidence indicates that the foundation of general relativity is entrenched in the classical realm.

And that presents a problem for gravitational theory, because the process of integration that resulted in Newtonian potential energy corresponds to a Galilean transformation when potential is associated with gravitational redshift through the Einstein equivalence principle. Whereas an ordinary integral corresponds to continuous addition, a *product integral* corresponds to continuous multiplication. The basic premise here is that a redshift must be treated relativistically. A relativistic calculation of gravitational potential energy can be performed using the product integral to account for having to multiply factors when calculating the potential energy associated with composing a relativistic redshift [2]. This link between the product integral and relativity will be developed further in the context of constant acceleration (see Appendix).

Relativistic gravitational potential energy is exponential in form, always positive, and has a maximum value equal to its rest energy far from other matter. Its value decreases in the presence of a gravitational field. Unlike classical gravitation, where there is no limit to potential energy, a relativistic treatment indicates that gravitational potential energy of an object is limited to its rest energy. The relativistic version will be called *Machian potential energy* to acknowledge Mach's principle, that rest energy is gravitational potential energy of an object due to its separation from other matter.

Newtonian gravitational potential energy is approximately equal to Machian potential energy minus the rest energy. A negative quantity by convention, Newtonian gravitational potential energy is then not really a bottomless well, but instead, gravitational potential energy is taken from the finite tower of an object's positive rest energy.

A metric with exponential terms results from substituting the radial escape velocity associated with Machian potential energy into the Gullstrand-Painlevé formulation. It does not involve a singularity and ought to satisfy first order tests of general relativity. Invoking Gullstrand-Painlevé is a bit of a trick to get the Machian metric using special relativity in a way that is independent of general relativity, thus providing part of what is ultimately needed. Multiplicative concepts like the product integral will likely be required in a reformulation of gravitational theory, which would then presumably replicate the Machian metric.

After starting with the first assumption, the inverse square force law, a relativistic accounting of gravitational potential energy departs from the classical

[1] Gullstrand-Painlevé coordinates and the river model will be found to be highly relevant. "In the river model, space itself flows like a river through a flat background, while objects move through the river according to the rules of special relativity [1]."

derivation. In either case, the inverse square law is foundational. But are there really lines of force directed to the center of an object? The concept of lines of force can only be seen as a descriptive fabrication, and may be far from what might be considered a cause or mechanism of gravity. However, the associated inverse square law of gravitational force encapsulates a fundamental concept of conservation at the base of both classical and modern gravitation.

Quantum Theory

Fundamental:	**Waves, Planck's hypothesis**
Questionable:	**Quantum effects only at small scale**

Given that the wave function [3] may be considered central to quantum theory, it follows that waves are fundamental. What are these waves? They are generally assumed to be complex-valued plane waves, and the use of Fourier analysis in the theory presumes that they are standing waves. The notion of waves is closer to our experience of physical reality than lines of force, although the wave function demands an extremely high number of waves with slightly different frequencies.

Planck's hypothesis, which resolved the ultraviolet crisis of black body radiation, is that the energy of an ocillator, whose natural frequency is ν, is restricted to energy levels which are integral multiples of $h\nu$, where h is Planck's constant. A transition between energy levels requires the gain or loss of a quantum of energy, $h\nu$. Planck's hypothesis was found to be characteristic of a quantum mechanical oscillator in one dimension, having a zero-point energy of $h\nu/2$.

This is supposed to apply even to classical oscillators, such as a bouncing ball or a pendulum, which are not small in scale. In these cases of large scale, the frequency is low, and the interval between energy levels can be undetectable, so it may be natural to tend to associate quantum effects with small scale. However, the association with small scale is due to difficulty with detection at large scale, and is not a theoretically imposed restriction.

Cosmology

Fundamental:	**Quantized decay of light energy**
Questionable:	**Hubble's law as recession**

Planck's hypothesis originated in the context of electromagnetic energy. Redshifted light from distant sources can be viewed, in terms of Planck's hypothesis, as a quantum process. From this viewpoint, a photon changes state between cycles, losing a fixed quantum of energy in the transition. Notably, the energy lost per photon cycle is Hh, where $H \approx 2 \times 10^{-18}$ Hz is Hubble's constant, which implies that the natural frequency of the oscillator is $\nu = H$. This exceptionally low frequency of natural oscillation implies cosmic scale, and presents a problem for detection of the transition quantum, Hh, because of its exceedingly low energy. Fortunately, the advent of supernova cosmology allows the cumulative change in light energy to be assessed reliably over great distances.

While the light from distant supernova is redshifted, a supernova event, which predictably brightens and fades over a period of days, is observed to take longer as the distance is greater. The recession hypothesis requires two equal dimming factors to account for decreased light – one due to recession velocity, the other to time dilation. The quantum hypothesis fits the supernova data if there is only one dimming factor, and this requires complementarity between received photon energy and distant time dilation observed at the source. The quantum hypothesis faces a hard constraint here because there are no parameters. The model with one dimming factor [4] must fit the supernova data without adjustment, which it seems to, or be rejected (see Endnotes).

As a result of this critical difference in accounting for the dimming of light, the quantum approach is incompatible with recession, producing a different distance scale. It would be difficult to incorporate quantum aspects into the paradigm of recession. Furthermore, the Machian approach to gravitation invalidates the recession hypothesis because inflation requires an unlimited well of potential energy.

Incidentally, as McEachern points out [5], effects such as quantum correlation can be expected from any process limited to one bit of information, even classical processes. A transition of one quantum between energy levels is a one bit process, so there is a possibility of distant correlations at astronomical scale.

The redshift is as much about light as it is the cosmos. Following Planck's hypothesis leads to a completely different concept of the redshift, and the universe, but the idea of recession and a big bang must be abandoned. In the quantum picture, the energy of redshifted light undergoes exponential decay by losing a quantum of energy, Hh, per cycle. This energy can be expected to accumulate near the zero-point, likely forming a cold superfluid, and perhaps hosting the Higgs field, or manifesting as dark matter.

Synthesis

Cosmology inferred from Planck's hypothesis, Machian gravitation, and quantum mechanics share a common mathematical characteristic – they are based to a significant extent on exponential functions, which are essentially multiplicative. The wave function operates on waves that are complex exponentials. In cosmology, it is the decay of light energy. In gravitation, it is the Machian scale factor.

So, the foundation of general relativity is not entirely relativistic, and big bang cosmology disregards a basic property of electromagnetic radiation. It would appear that an understanding of redshifts, gravitational or cosmological, is fundamental to our understanding of the universe.

Just like any other redshift, the gravitational redshift of light demands a modern treatment, and that leads to Machian potential energy. Curiously, as discussed in the Appendix, there is an alternate Machian relativity that is related to Machian escape velocity. The difference is due to the way kinetic energy and rest energy are related. In an accelerating rocket far off in space out of gravity's reach, kinetic energy is added to rest energy. In the case of free fall by gravity, kinetic energy is subtracted from rest energy.

While it may not be surprising that there are two relativities arising from this difference, the nature of what is moving in Machian relativity is puzzling. Machian relativity takes place in a rest frame, with no regard for motion aside from the kinetic energy of a moving object. A possible, if sketchy, solution to the puzzle is suggested through a bit of holistic detective work.

Machian relativity is compatible with quantum mechanics regarding the simplicity of their mutual assumptions about space and time. Suppose that the rest frame of Machian relativity is space, and that space is populated by, or perhaps formed by, vast numbers of quanta at, or near, the inferred zero-point energy of electromagnetic radiation, $Hh/2$. Recall that the use of Fourier transforms in the wave function implies that the waves are standing waves. But there is no fixed firmament which constrains these waves, the ether, to any position.

The phase, or relative position, of any wave is not fixed, but only has relevance in relation to other waves, whose coherence presumably sets the background frame of reference for the river model of gravitation [1]. The changing phase of the underlying waves corresponds to motion through space, because space itself (or the ether) would be moving. Thus, on imagining a link between gravitation and changing phase, an escape velocity is required in order to exceed the speed of space moving inward to a gravitating object. Machian relativity would then be an account of the rate of change of phase, or phase velocity, of underlying ether waves moving toward a gravitating object.

And that takes us all the way back to something that looks a lot like lines of force!

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Appendix

Einstein equivalence principle

The principle supposes equivalence between a system of measurement undergoing uniform acceleration and one in a gravitational field. Redshift and a change in gravitational potential are proportional, to first order, which is sufficient for the product integral.

Let us follow the steps of the derivation in the accelerated system which is undergoing uniform acceleration g in the upward direction to mimic gravity. The time required for radiation emitted downward to travel a distance h to meet the rising measurement system is h/c, where c is the speed of light. In that time, the measurement system has acquired a velocity $v = gh/c$ upward. From the special theory of relativity, the radiated energy is then detected by the measurement system and found to be increased approximately by a factor of $1 + gh/c^2$ corresponding to a redshift $z = -gh/c^2$ (a blue shift). For the system in the gravitational field, the increase in the energy of detected radiation is then attributed to the increase in gravitational potential given by gh.

Product integral and relativity

The idea behind the application of the product integral in relativity can be illustrated with an example. The total redshift, when it is influenced by more than one factor, such as Doppler shift, gravitational shift and Hubble shift, is given by

$$1 + z_{\text{total}} = (1 + z_{\text{Doppler}})(1 + z_{\text{Gravity}})(1 + z_{\text{Hubble}}) \tag{1}$$

When the redshifts are small, the total redshift can be approximated by the sum of the constituent redshifts, but the multiplicative form must be maintained for full accuracy. That is the difference between Galilean and relativitistic treatments.

The product integral is a continuous version of the above product. It takes a function, chops it into tiny pieces, adds one to each tiny piece, and multiplies them altogether. Note that it is sufficient to carry first order accuracy in each term which is to be multiplied because the part associated with relativistic redshift will be infinitesimal compared to one, simplifying its application greatly. Redshift can be linked to gravitational potential energy through the Einstein equivalence principle, hence the product integral can be used to evaluate relativistic gravitational potential energy.

The calculation of potential energy is formulated so that each infinitesimal spherical shell of matter contributes to the redshift, or to relativistic potential energy, in same way as in Eq. (1), except using the product integral. Redshifted photon energy and gravitational potential energy can be assumed as one-to-one, to first order, from the Einstein equivalence principle. For an object in a gravitational field with classical potential energy, u, its relativistic potential energy is $U = mc^2 \exp(u/mc^2)$, where the rest energy is presumed constant. In the absence of a field, $u = 0$, and U is the rest energy. In a gravitational potential well, $u = -GMm/r$, and the Machian gravitational potential energy falls to $U = mc^2\sigma$ where $\sigma = \exp(-GM/rc^2)$ is the scale factor. In comparison, the conventional Schwarzschild scale factor for general relativity would be $\sigma_{\mathrm{GR}} = (1 - GM/rc^2)^{-1/2}$. There is a natural link to the differential geometry of general relativity, which is not provided by the Schwarzschild scale factor (see Endnotes).

Note that the Machian form of potential energy is from an exponential map of the classical potential energy. This is a distinction that separates gravitation from the rest of relativistic dynamics, and has been at the root of problematic aspects of general relativity related to the Schwarzschild singularity. An exponential map is available, and might be used in general relativity for gravitation, while keeping intact the non-gravitational relativistic dynamics of the conventional relativititistic rocket, for example.

Relativistic rocket: A tale of two relativities

Aside from an exponential map, there is another underlying difference between gravitational acceleration and acceleration induced through some other means. For example, contrast the power required by a hovering helicopter or rocket, against the apparently effortless opposition by gravity. This peculiar energetic aspect of gravity can be modeled by changing some assumptions about a relativistic rocket [6], where a rocket undergoes constant acceleration at 1g.

To be clear, the point is not to model acceleration of a powered rocket, but its opponent, gravity, acting on any object in free fall. In the gravitational treatment, the most significant difference is that kinetic energy is subtracted from the rest energy of any freely falling object, instead of being added to its rest energy. From a subjective point of view, a person on an accelerating rocket will be able to physically sense a force in the same way as a stationary subject in a gravitational field. One is moving, the other is not. Likewise, a person in a rocket that is stationary (or not accelerating) will experience weightlessness, and so will a person in free fall accelerating due to gravity.

Obviously, acceleration due to gravity varies with distance, so a model of constant acceleration is not a realistic depiction of gravity, although it turns out that a restriction to constant acceleration is not necessary. The intention of this artificial exercise is to show that escape velocity associated with Machian potential energy calculated using the product integral is the same as a relativistic velocity composed in the rest frame. This links the energetic considerations handled by the product integral to specific space-time aspects of relativistic gravitational acceleration detailed in Eqs. (2)-(3) which follow.

First, only the dynamics are important in this model of constant gravity. The acceleration will occur effortlessly, in the sense that no fuel is required and mass is not ejected. Second, acceleration is usually applied in the frame of the moving rocket, but instead constant acceleration will be applied in the rest frame where measurements are made. Third, time dilation or length contraction due to motion of an object in a gravitational field are assumed not to be involved

– only the kinetic energy of a moving object is relevant to the gravitational analogy.

A relativistic velocity can be composed by considering a time, T, divided into N intervals of $\Delta t = T/N$ each. Let $\alpha\Delta t$ be the change in velocity of the rocket that would be brought about in one time interval by a constant acceleration, α, under classical Galilean assumptions. A relativistic velocity for interval n can be composed approximately as

$$v_n = \frac{v_{n-1} + \alpha\Delta t}{1 + v_{n-1}\alpha\Delta t/c^2} \tag{2}$$

and the distance traveled can be approximated similarly as

$$r_n = r_{n-1} + v_{n-1}\Delta t \tag{3}$$

The resulting velocity corresponds to the escape velocity associated with relativistic potential energy calculated using the product integral. Here, the classical potential energy is given by $u = -m\alpha r$, and relativistic "Machian" potential energy is $U = mc^2 \exp(u/mc^2)$, where the rest energy, mc^2, is presumed constant. Using the distance, r, from Eq. (3), the Machian escape velocity would be given by

$$v_{\text{esc}} = c\,[1 - \exp(-2\alpha r/c^2)]^{1/2} \tag{4}$$

which comes from countering the reduction in potential energy in a "gravitational well" due to a decrease in the scale factor, $\sigma = \exp(-\alpha r/c^2)$, with the increase in kinetic energy associated with an object's escape velocity. A balance occurs when $\sigma\gamma = 1$, where $\gamma = (1 - v_{\text{esc}}^2/c^2)^{-1/2}$ is the Lorentz factor.

As numerical confirmation, running the above approximations $N = 10^8$ times, starting with $v_0 = r_0 = 0$, gives seven figure agreement with the Machian escape velocity for periods out to $T = 10^5$ years. Increasing the number of intervals, N, beyond that does not improve the agreement.

To see how Machian escape velocity corresponds to a relativistic velocity, note that the recursion for velocity, Eq. (2), can be rearranged to give the acceleration

$$a = \lim_{\Delta t \to 0} \frac{v_n - v_{n-1}}{\Delta t} = \alpha\left(1 - \frac{v^2}{c^2}\right) \tag{5}$$

which is the same as the time derivative of Eq. (4), the escape velocity. This holds even for classical gravitational potential energy, $u = -GMm/r$, where $\alpha = -GM/r^2$, the classical gravitational acceleration, is not constant.

Machian relativity takes place in a rest frame, and its only application seems to be in relation to gravitational escape velocity associated with Machian potential energy. Consider the condition for escape, $\sigma\gamma = 1$. Symbolically, $\sigma\gamma$ is the product of gravitation and motion, respectively given by $\sigma \leq 1$ and $\gamma \geq 1$, and represents a separation between gravitation (in an exponential map) and the usual relativity of motion. Machian gravitational potential energy produced by the product integral has a clear relativistic interpretation in terms of redshift.

The meaning of the corresponding Machian relativity is less clear, but it must be associated with gravitation.

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Endnotes

The following material is adapted from [2] to show that gravitational potential energy composed relativistically using the product integral has an alternative, but equivalent, expression in terms of a radial distance element which undergoes length contraction in a gravitational field. This seems to be a natural link to the differential geometry of general relativity, which is not provided by the Schwarzschild scale factor.

Dimensional variability in general relativity

Bowler [7] has shown that general relativity is a gauge theory in which the fundamental dimensions of length, time and mass vary radially with the dimensionless scale factor respectively as $L = L_o\sigma$, $T = T_o\sigma^{-1}$ and $M = M_o\sigma^{-3}$. As shown in Table 1, which demonstrates the variablility of some physical quantities, energy becomes

$$E = E_o\,\sigma(r) \tag{6}$$

in the presence of a gravitational field at radius, r, compared to the original energy, E_o, sufficiently far from the field where $\sigma(\infty) = 1$.

Table 1: Dimensional Variability in General Relativity

	Radial	Transverse
Length: L	$L_o\,\sigma$	L_o
Time: T	$T_o\,\sigma^{-1}$	$T_o\,\sigma^{-1}$
Energy: E	$E_o\,\sigma$	$E_o\,\sigma$
Mass: M	$M_o\,\sigma^{-3}$	$M_o\,\sigma^{-1}$
Velocity: v	$v_o\,\sigma^2$	$v_o\,\sigma$
Acceleration: a	$a_o\,\sigma^3$	$a_o\,\sigma^2$
Momentum: p	$p_o\,\sigma^{-1}$	p_o
Force: f	f_o	$f_o\,\sigma$
Newton: G	$G_o\,\sigma^8$	$G_o\,\sigma^3$
Planck: h	h_o	h_o
$-GM/rc^2$: Φ	Φ_o	Φ_o

An equivalent formulation

Encapsulating the calculation of Machian gravitational potential energy by the redshift rule using the product integral for the free fall example gives

$$\frac{U(R)}{mc^2} = \exp\left(\frac{-1}{mc^2}\int_\infty^R F(r)\mathrm{d}r\right) = \sigma(R). \tag{7}$$

But, from the identity $\int \mathrm{e}^u(\mathrm{d}u/\mathrm{d}x)\mathrm{d}x = \mathrm{e}^u$, there is the indefinite integral

$$\int \exp\left(\frac{-GM}{rc^2}\right)\frac{GM}{r^2c^2}\mathrm{d}r = \sigma(r). \tag{8}$$

Equation (8) can be expressed in the form of a definite integral of the gravitational force, $F(r)$, as

$$-\int_{\infty}^{R} F(r)\sigma(r)\mathrm{d}r = U(R) - mc^2 \tag{9}$$

which is approximately equal to the Newtonian potential energy. Equation (8) can also be written as another definite integral,

$$-\int_{0}^{R} F(r)\sigma(r)\mathrm{d}r = U(R). \tag{10}$$

This last equation deserves special note because the limits of integration are different from those used for the free fall discussion in (7). Since a radial force, because of its dimensions, is independent of the scale factor, $\sigma(r)$ can be associated with the element of radius, dr, in (9) and (10). Thus, the potential energy function calculated by the redshift rule can be interpreted as a spatial integration of the Newtonian gravitational force from a point mass in which the element of radial distance traveled is contracted by the exponential scale factor.

Supernova luminosity distance vs redshift

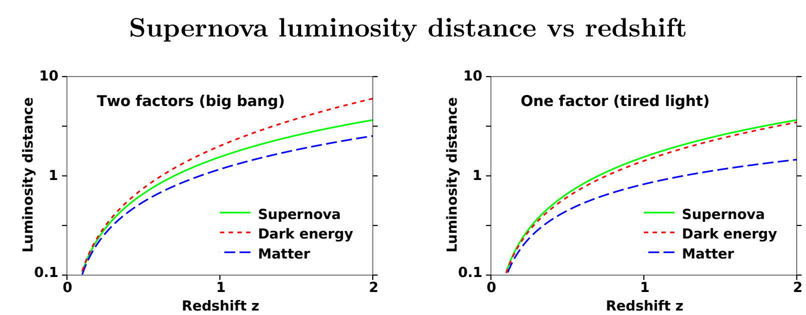

Figure 1: *Left:* Big bang interpretation requires two correction factors to determine luminosity distance (normalized by Hubble length). Supernova data are represented by a combination of matter and dark energy shown as the solid green line. Pure matter model is shown as blue dashes. Pure dark energy model is shown as red dots. *Right:* Quantum tired light requires only one factor resulting in smaller luminosity distances for both models. Supernova data indicate a pure dark energy model consistent with tired light. (Adapted from [4])