Mathematics is Physical

By Sara Imari Walker

Abstract: The advances of theoretical physics over the last several centuries have provided profound insights into the structure of reality at the smallest and largest scales in our universe. But they have fallen short of explaining the scale of our everyday experience, that is we cannot yet explain the existence of you or I or any life for that matter, or even more simply stated that mathematical objects are made real by ingenuity of some physical systems (i.e. us). Our current best explanations in physics tend to appeal to what amounts to fine-tuning: assuming the universe started in a low-entropy state that just so happened to have in its future this essay among other things. It is not difficult to think of examples where properties that are not exactly physical - well not in the sense of mass or charge or energy - can nonetheless be causal. These are ubiquitous phenomena that appears to pervade our perception of reality – basically anytime we perceive that information, or abstractions have causal consequences. It is however difficult to explain this beyond mere anecdote. In this essay I attempt to push thinking in that direction.

On her desk sits a funny sort of object, which looks very much like a disfigured bronze paperweight. It casts an unusual shadow in the low morning sun because of how it seems to fold in on itself every which way. I am studying it, but she is not paying attention. Instead she is going-on about some-or-other kind of rock from Antarctica, which I learn she acquired from the recently retired professor in the neighboring office.

"What's this?" I ask.

"Oh that's my favorite one!" she snaps into attention picking up the object and marveling at it as if for the first time. "I completely overlooked it at first, I was so disappointed not to get another smart watch for conference swag like I had the year before." Now she is animated, hopping from her chair to lean against the side of the desk. "But … then when I brought it back here, I noticed this note in the bottom of the box. Read it and tell me what you think." She thrusted toward me a folded piece of cardboard paper.

"Cool?" I replied not really getting it, but knowing it would be explained in the next breath.

"Yes very! This is nothing less than a mathematical object 3D printed into reality. It did not exist, then existed as an abstraction and through human creativity and ingenuity was made physical."

It is not difficult to think of examples like this where things that are not exactly physical - well not in the sense of mass or charge or energy - can nonetheless be causal. The example of a mathematical object being driven into existence by technology is manifestation of a much more ubiquitous phenomenon that appears to pervade our perception of reality – that is that information, or abstractions have causal consequences. It is however difficult to explain this beyond mere anecdote.

The advances of theoretical physics over the last several centuries have provided profound insights into the structure of reality at the smallest and largest scales in our universe. But they have fallen short of explaining the scale of our everyday experience, that is we cannot yet explain the existence of you or I or any life for that matter, or even more simply stated that mathematical objects are made real by ingenuity of some physical systems (i.e. us). Our current best explanations in physics tend to appeal to what amounts to fine-tuning: assuming the universe started in a low-entropy state that just so happened to have in its future this essay, that paperweight, and you reading this. The difficulty is in deciding whether or not additional explanation is necessary.

It has been the tradition of physics to do *Geddanken* – or thought – experiments when problems are very stubborn: Maxwell had his demon and Einstein had his trains. The experiment now needed is one that can probe the intersection of mathematics and physical reality – that is, the intersection of abstractions and the transformations of matter they can cause.

What follows is an impoverished attempt to construct those experiments, meant to illuminate how mathematics provides a concrete window into the physics of information – that is, how abstractions exist and interact with the material world. It is not complete and is only meant to provoke thinking in the direction of concrete steps to build better intuition, and thereby better theories and experiments for understanding the role of information (viz. math), if it indeed has a role beyond a useful descriptive tool, in the structure of reality.

We can do a thought experiment first to illustrate the point made in the desk ornament more precisely.

Imagine a perfect circle. Now try to draw it (or better yet, if you have clay you could attempt to mold a perfect sphere).

That's it, experiment done. A perfect circle – a truly perfect one – cannot exist as such. It is an idea humanity invented by taking the ratio of the circumference to the radius of a circle and coming up with an irrational number Pi that cannot be computed with finite resources. (How close did you get to perfection?) Humans have in mind the *idea* of a perfect circle, but one could never be produced with infinite precision, no more than we will ever be able to build a computer with enough resources to calculate the infinite digits in the number Pi. However, a perfect circle *can* influence the physical world as can the concept of Pi: for example, as in our experiment, you imagined a mathematical object we formalize as a circle and attempted to create one. Mathematical descriptions of circles or spheres are prominent in formulations of some physical theories also. These enable humans to do new and inventive things (for example launching satellites to space due to knowledge of gravitation formulated in a mathematical compression referred to as a 'law of nature', see [1]) . It is in this sense that we can say circles (or spheres), as mathematical objects, exist. They exist by structuring what transformations can/do happen, based on knowledge of the mathematical structure we call a circle or its absence. If I know about the mathematical abstraction that is a circle, I have more causal power in the physical world than I would otherwise.

Thus, our use of mathematics to formalize knowledge about the world can also demonstrate how information about the world (abstractions) can be causal, whether or not those abstractions are computable with finite resources (e.g. pi).

We can consider another property of mathematics that parallels important properties of information beyond how it is causal. What separates information from other physical attributes such as charge or mass? Take for example the charge of an electron. It is often considered to be a physical attribute because every electron in the universe has a charge and because the charge cannot be separated from the electron. It is for this reason that charge is considered an intrinsic property in physics. Of course we cannot be sure that it is an intrinsic property because charge is only measured under interaction with another physical system, e.g. via measurement. To parallel the hard problem of consciousness, this is considered to be the hard problem of matter [2] – we do not know in of itself what matter is beyond our mathematical descriptions of its interactions.

These mathematical descriptions have a different property than the systems they are intended to describe. They can readily be copied between different physical media. Whereas I cannot take charge and separate it from an electron, I can take my description of its behavior, say in the form of Coulomb's law and formulate it in my mind, write it on a piece of paper, or type it on my computer. Those are three very different physical systems – the wet chemistry of my brain, the more solid chemistry of the paper, or the silicon in my computer. Each has a different awareness of the significance of the mathematical statement, arguably I have some 'awareness', whereas the paper and computer do not – but that is not the important point to be made. The more important point is that if I delivered that information to you (another physical system) via telling you, giving you the paper, or emailing you my typeset version, you could do roughly the same things with it as I could. For example, you could go and build a van de Graaf generator to test aspects of the theory if you were so proficient and inclined.

Mathematical statements are powerful descriptors of the natural world, not just because they capture its regularities, and allow prediction, but more importantly because they are a kind of representation that can be reliably copied between different physical media and retain many of their same properties – *particularly associated with the transformations they can cause*. The power of mathematics is precisely in that it is information that can be robustly copied, meaning we can readily see its structure across very different systems, giving it's umph in the scientific arena. (If mathematics were not copiable there would be no way to build mathematical models of any physical phenomenon) This can be contrasted with language – another human abstraction but one that is much less reliable in the transformations it can cause to occur: if you tell me a statement I am much more likely to misinterpret (change its downstream causal consequences) than if you present me with a mathematical formula (pending I am a sufficiently trained physical system that knows mathematics). The unreasonable effectiveness of mathematics in describing physical reality may stem from the fact that mathematics is the most reliable kind of information that can be copied between different physical systems (at least discovered by our biosphere so far). It is only unreasonable if you do not recognize that mathematics and our ability to use it is a manifestation of the physics of information.

If information is physical, as Landauer once argued [3], mathematics as a kind of information should be no exception.

Do the limitations of mathematics then impose limitations on reality itself?

Probably.

It is perhaps most straightforward to see this in the following sense. If mathematics is concerned with the interactions between physical structures and their transformations, there is a direct analogy to be made between computation and physical transformations. In fact, this is not a new insight at all but goes back to John von Neumann and his formulation of the idea of universal constructors, meant in some sense to be the physical embodiment of universal computers. Computation is not necessarily physical, but construction - the process of causing a specific transformation to occur - is. In contrast to the abstraction that is a computer, in the case of constructors, the input and output are not numbers but instead physical objects. A transformation that cannot be caused to occur, either spontaneously (by a timer) or by another physical system (by a constructor) [4] amounts to the physical equivalent of an uncomputable function.

We already know there exist transformations that are unconstructable by a machine with knowledge of our laws of physics, which operates according to those same laws. For example, our laws must obey certain symmetries. Charge, parity and time reversal symmetry (CPT) is one such symmetry. In our universe, a transformation on T is not possible without a corresponding transformation on CP that preserves CPT invariance. We also cannot build a perpetual motion machine as it is forbidden by the laws of physics. These are in some sense not such interesting impossibilities because they are ones that are imposed by the laws of physics on possible constructors.

But this logic can't be quite right because the laws of physics are formulated by us. And by the above considerations we are arguably constructors. Constructor theory separats 'programmable' constructors as ones that can interact with another physical system viz a program that specifies which transformation the constructor will perform [4]. This necessitates that the transformation be encoded in an abstraction. A question is whether it is possible, if only in principle, for a universal constructor to exist, where a universal constructor is a system that can, with the appropriate program and resources perform any possible transformation whatsoever. Currently the best approximation to a universal constructor in the known universe is our very own biosphere with the technological civilization it supports. Understanding the connection between mathematics and the transformations mathematical statements can cause allows us to flip the logic. If universal constructors are possible in our universe, then any transformation that cannot be represented mathematically is *not* a possible transformation – that is, a transformation cannot be caused to occur by a general-purpose constructor if there is no possibility to build an abstract representation of that transformation. It could still be the case that some transformations that can be mathematically described (such as inverting C and P) are still not possible in isolation, but if the inverse logic could be proven it would preclude non-abstractable transformations from *ever* being possible. It may be that the boundary of our ability to describe the universe mathematically (or at least abstractly, e.g. through language or other representations) defines the boundaries of what is physically possible.

It should be noted that the downward constraint mathematics imposes via an assumption of universal programmability is scale dependent: some scales of reality are more amenable to mathematical representation than others, meaning more transformations are possible at those scales. We happen to live at a particular dense scale as far as the permitted possible transformations among objects. These are possible because they can be abstractly represented (via human minds, computers, bacteria, etc) and therefore can be caused to occur.

One of the reasons that the correspondence between math (here codified as our descriptions of the world that can also do causal work) and reality is not better understood is because modern science consists of two fundamentally distinct descriptions of the natural world. The first emerged in the scientific revolution of the 17th century with Newton's mathematical formulation of laws of physics in terms of an initial state and fixed, deterministic laws. Here the laws/math is immutable and unchanging and are not part of the structure of reality. The second emerged in the 20th century with an algorithmic view of nature, where the "laws" often depend, in part, on the current state of the system in a computational view, or the current objects in a more physical one. Thus far, these two perspectives have been applied to different domains of science: the Newtonian legacy for physical systems, and the more recent "algorithmic" or "state/object - dependent" view for complex biological and technological systems. However, ultimately these two modes of explanation must describe the same physical reality, albeit possibly at different scales. While unification of these two formalisms is not necessary for the domains of science where each is independently valid, it is essential for some of the most difficult frontiers in science such as the emergence of life, which arguably occurs when our traditional Newtonian approach to physics based on initial states and fixed, deterministic "laws" of physics must yield the to path-dependent, historical narratives characteristic of the object-dependent dynamics of the biosphere. We need to explain a transition from a physics where abstractions and relations are descriptive to one where they are also causal.

She expounds on all of this to me and then asks "When can you do the math and explain this to me?".

1. Walker, Sara Imari. "The descent of math." *Trick or Truth?*. Springer, Cham, 2016. 183-192.
2. Mørch, Hedda Hassel. "Is matter conscious?" Nautilus (2017).
3. Landauer, Rolf. "Information is physical." *Physics Today* 44.5 (1991): 23-29.
4. Deutsch, David. "Constructor theory." *Synthese* 190.18 (2013): 4331-4359.