The physics of sound, acoustics and music and many false assumptions

There are possibly more incorrect assumptions in the area of sound and music than any other area in physics. Why should music assumptions be in a scientific essay? Music today has been so trivialized that we forget that it is an area of physics in its own right. It has its own mathematical system and with this system everything in music can be defined with nothing more than positive integers. It also provides much valuable information as to vibration and resonance.

We can begin with the difference between vibration and resonance. Most people seem to either ignore this difference or are totally unaware of it. Vibration and resonance are two related but quite different phenomena. The terms are often used indiscriminately, which is completely wrong.

Resonance is initially the result of the sympathetic response to a vibration. A resonance can also respond to a resonance. A vibrating body will only vibrate to the frequencies of its nodal structure whereas any material with the proper elasticity will resonate to any frequency fed to it. A good example is the tuning fork. The only vibration on a tuning fork occurs in the tines.

When you strike the tine of the tuning fork the tine bends. If the material of the fork was compressible the mass on the side away from the force would become compressed and the side toward the forced would become decompressed. In an incompressible material such as the steel of the tuning fork something different happens. The shape of the tine changes from rectangular to trapezoidal, the longer base being toward the bend and the short base away from it. Then the restoring force[1] (a scientific deus ex machina if there ever was one) brings it back to the rectangular shape and the momentum of the tine cause the tine to bend the other way creating a simple oscillation.

Strike the tine of a tuning fork against a solid object and you get a sound you can scarcely hear. This sound, as is most sound, is the result of a resonance, not a vibration. Touch the end of the tuning for to a tabletop and the sound elicited is amazingly strong. You can use a tuning fork of any pitch and the tabletop will produce a sound with similar amplitude. The resonance in the body of the tuning fork creates the sympathetic resonance in the tabletop

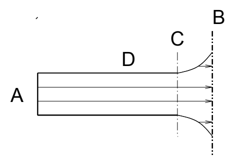
This deformation of the tine causes a powerful wave to form. Whether or not it is a P-wave or an S-wave is not important to this discussion. This wave travels into the base of the tuning fork. When it reaches the end of the base an inverse conjugate is returned which travels into the opposite tine. This induces a mirror movement in the second tine which is the opposite of the initial movement. Again the deformation of the second tine cause a traveling wave that again travels through the base and an inverse conjugate is again returned now affecting the first tine. This is a common occurrence and is known as a feedback loop. This is what keeps the tuning fork vibrating for such a long time. Since it takes a finite amount of time for the wave to travel from one tine to the other a cumulative lag develops which would in time dampen the vibration itself. This may have as much if not more to do with the dying away of the sound than the friction of the motion within the atomic structure.

This traveling shear wave creates a standing wave in the base of the tuning fork which is longitudinal and is the nature of resonance. We do not hear the sound of the base of the tuning fork very well because of the impedance mismatch between steel and air. The area of interface between the base and the surrounding air is too small to create a very loud sound. To understand why let us look at a different instrument, a woodwind such as a clarinet.

In an article in an earlier edition of Scientific American (July 1948) Frederick Saunders, a respected Harvard physicist and acoustician, put forth several false assumptions, most of which are commonly held. When you depress a key on the clarinet you open a hole in the side of the instrument. When the player blows into the instrument the air from this blowing, which in an instrument with no open keys, would travel to the end of the instrument where a conjugate is returned to the mouthpiece. If a key is depressed the conjugate is returned from the location of the opened hole.

This causes Saunders to state that only in the fundamental does the sound emit from the end of the instrument. According to Saunders when a key is depressed the sound is emitted from the air hole. This is a commonly held belief. If this were to be true we must ask why we would put a bell on the instrument if it only affected the fundamental. Any oboe player will tell you that if you stuff a piece of cloth into the end of an oboe it mutes all of the tones of the oboe, not just the fundamental.

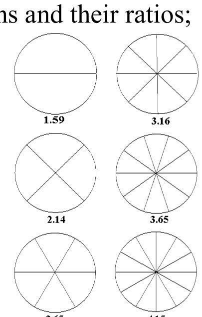
When a player blows into an instrument, such as a clarinet, the reed vibrates. The vibrating reed is the only vibration that occurs in the sounding of the instrument. This vibration opens and closes the route of the air passage into the body of the instrument creating a timed series of pulses which are traveling compression waves. These particular compression waves do not create the sound; they merely time the vibration of the reed. The action forms a stationary wave that is part of the series of events that create the primary standing wave that actually creates the audible sound[2]. This particular standing wave does not create the airborne sound wave. Fluid Dynamics treats a gas[3] such as air as a liquid and this is easily observed in the standing wave that forms in the entire body of a wind instrument. The drawing below is a longitudinal cross section of a cylindrical bore with a bell, such as exists on a brass instrument.



If a force is applied at A in the direction of the arrow we know that the force per square inch of a transverse cross section at C is equal to the force per square inch of a transverse cross section at A. The total amount of energy is proportionate to the ratios of various cross sections, i. e., if the area of a transverse cross section at B is four times that of one at A or C, the total amount of the force is four times as great.

It is in the reed where all of the magic of the sound of the instrument happens. As with the tine of a tuning fork the bending of the reed causes a fairly powerful transverse shear wave to feed into the mouthpiece and the sides of the instrument. This action creates a totally different type of standing wave in the body of the instrument. This standing wave is totally different from the one created by the moving compression wave. If a finger hole exists at D the pulse of the air crested by the blowing of the performer goes only as far as D. A conjugate is thrown back and a standing wave is created.

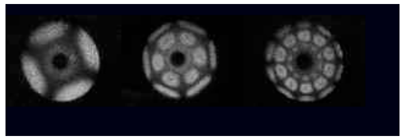
The first clue to the nature of this standing wave is found on the surface of timpani. The drum heads of timpani are rather unique as it is both the source of the vibration and the interface of the resonance to the air. The well known Chladni patterns illustrate this very well. There are three types of patterns, one that consists of equal pie-shapes segments, one that consists of circles and a third which is a combination of the first two. The pie-shaped segments are the result of the resonance. The circles are the vibration. The third is a composite and we can ignore it for this discussion. Here are the diametric patterns and their ratios;



These are the partials that are part of the resonance that creates the audible sound. It is stated that the partials of the timpani are not harmonic. This is not true. The reasons are contained in my Structural Resonance papers which can be found on my Website: wropera.com. The diametric patterns occur of course only in circular areas but the patterns are present on any resonance and take their shape from the body that is resonating[6].

Holographic pictures show that the standing wave in a wind instrument is also composed of pie-shape segments which run the length of the body of the instrument. Here are three examples of this:

The first from a student project at Rollins University[4]



The second from the Acoustics 2012 Nantes Conference[4]

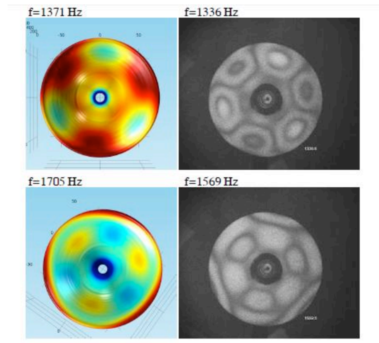
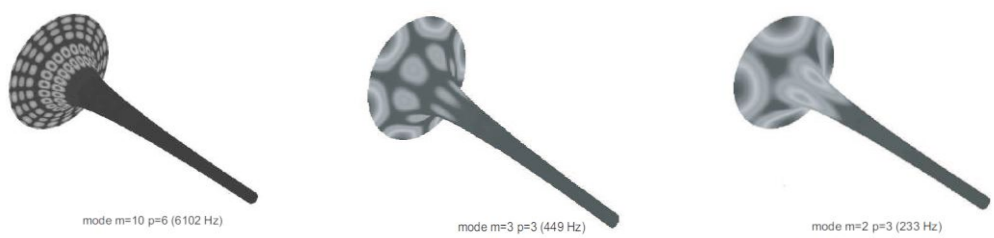


Figure 2: Elliptic modes of the vibration of the trumpet bell, as calculated from a 3D simulation (left) and observed using electronic speckle pattern interferometry (right).

The third from Acoustics '08 Paris[4]



These pictures indicate that the structure is diametric and that it runs the length of the body of the instrument.

This same structure appears in the splash of milk drops. This is the nature of all resonance. Almost all instruments have an air to air interface to the airborne carrier wave. The sides of the instrument create some of the sound but the impedance mismatch between solids and the air is large so that in most cases the sides add little to the sound. Because of the longitudinal nature of the standing wave in the body of the instrument little sound is emitted through the finger holes. In string instruments the sound emanates primarily from the f-holes.

I have a friend who makes bass viols. He once drilled a hole near the top of the front plate and was amazed at the increase in the loudness of the sound. The timpani require a very large area because the interface to the air is from a solid.

Bells have an air to air interface. If you stand in line with the swinging of a cathedral bell you will notice that when the bell swings toward you it is considerably louder than when it swings away. The lowest note on a Glockenspiel is the G an octave and a half above middle C (768Hz in the scale of C). A cathedral bell that sounds that same pitch weighs six hundred pounds. Since every doubling of the mass lowers the pitch an octave the fundamental of the cathedral bell is way below the range of human hearing. The pitch of the bell is created by the standing wave that forms within the air chamber of the bell.

Loudspeakers invariably have an air chamber surrounding the vibrating membrane, sometimes a simple bell-like extension from the diaphragm but often a fairly large resonating chamber. As with almost all other instruments the loudspeaker has an air to air interface.

Much of the sound of a piano comes from the surface of the air in the space above the sounding board. When the lid is closed the interface to the standing structure is blocked and the piano is much softer. Again, the large area of the surface of the piano plus the fact that the sounding board is open underneath the piano allows the piano to still sound quite well even with the lid closed. Only the strings are permitted to vibrate. Heavy clamps are placed on the sounding board to insure that it does not vibrate.

If we hold down the loud pedal and sing into the body of the piano *all* of the strings resonate. Only those whose nodal structure is at or near the nodal structure of the sung sound with vibrate. This vibration creates a resonance that is fed to the tuning pins, then to the hardwood block holding the tuning pins, then to the iron frame and finally to the sounding board. The sounding board has good elasticity and resonates very well to the resonance created by the strings. This resonance in turn is fed back into the body of the piano and finally back into the strings. Much like with the tuning fork this creates a feedback loop. In a good piano this sustains the sound a very long time.

Even the sound of a drop of water is caused by a diametric structure. A resonance in water has almost no effect on the surrounding air. Edgerton's high speed photos of milk drops often show a diametric structure forming in a cavity formed by the drop when it hits the milk. Here are two high speed clips that show the evolution of the diametric structure quite clearly[5]. The drop displaces the milk and a corona is thrown up around the depression. Right at the end we see the pie-shaped sections of the diametric structure form. This creates a standing wave in the volume of air within the corona. This in turn interfaces with the air to form the carrier wave that carries the sound. Even a milk drop is a wind instrument.

The compression wave that the ear responds to is simply a carrier. It is created by a resonance and has nothing all to do with the generation of a sound and yet that is what we usually refer to when we speak of a sound wave. The nature of the sound wave that occurs in areas such as the body of a wind instrument and is exemplified by Fluid Dynamics is quite different. A clue to its makeup can be found in the cochlea.

The cochlea developed very early in the evolutionary history. It appeared early in marine vertebrates. In marine vertebrates there is no middle or outer ear[6]. The resonance that travels through the water is easily transmitted to the body of the vertebrate, such as a fish. There is very little impedance mismatch between the water and the body of the fish. It then travels through the bones into the cochlea. The cochlea of mammals is quite similar and, as with the marine cochlea, the nerves of the Organ of Corti terminate in the liquid filled Cochlear Canal. In the final analysis we hear under water just as do fish.

Rather than change the design of the cochlea evolution used the add-on approach and created the middle and outer ears. The viscous nature of air (its takes about six full cycles of the airborne carrier to respond to middle C) causes the airborne sound wave to contain a more complex version of the sound than that presented by the standing wave that creates it. Tones are sustained and sound with other tones creating differential tone that are not present in the initial structure. Transferring the resonance of the eardrum (*resonance, not vibration*) to the cochlea is a very easy thing to accomplish and would not require the bizarre shapes of the auditory ossicles. What the middle ear quite likely does is return the structure of the carrier wave to that of the standing wave that created it so it can be understood by the Organ of Corti, the nerves therein and ultimately the brain.

This is the reason that public address systems do not sound real. A microphone picks up the airborne carrier wave created by the initial sound source. This is corrupted by the viscosity and hysteresis of the air. This corrupted version is then fed to an amplifier and finally to a microphone which corrupts the sound a second time. This double corruption is what is picked up by the ear. The ear is designed to de-modify one corruption and thus the signal that enters the cochlea is still corrupted causing an unreal sound to reach the brain. The exact same problem occurs in recordings, the corrupted version is that which is stored and is double corrupted when fed to a speaker.

This is why, even when created by the most skilled engineers, recorded sound does not sound real and until the nature of corruption of the initial wave is understood it never will.

The information that is sent to the brain by these nerves is in the form of discrete pulses which are in effect, binary bit patterns[6]. This must be the construction of the standing waves that appear in incompressible media and also in air (when it behaves as a fluid) and fluids themselves. Instead of the fairly complex compression wave in the airborne carrier the resonance in solids and liquids consists of a series of discrete frequencies.

Another significant incorrect assumption has to do with tempered tuning. There exists no mathematical justification for the tempered scale. Schoenberg defined his twelve tone music system as using twelve tones related only to one another[7]. The truth is that the twelve tones of the tempered scale are not related to each other in any way. They exist merely as a rather poor approximation of the true properly tuned system (there is only one). This notion grew out of the chromaticism of the late nineteenth century and was partially an attempt at atonality. Atonality is physically impossible as any group of discrete frequencies will always form a fundamental no matter how remote it might be[8].

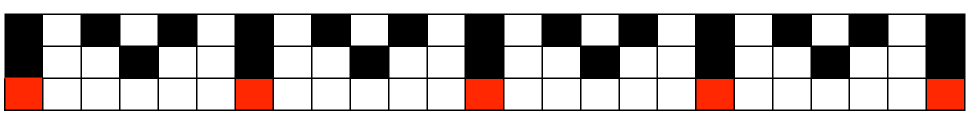
For those unfamiliar with tempered tuning here is a brief description. The natural scale is a series of discrete ratios. It is definitely a non-linear scale. The tempered scale attempts to linearize this scale by making each half step (the smallest interval in the scale) equal. Mathematically this requires each half-step to be equal to $\sqrt[12]{\mathbf{2}}$. Thus not only is the tempered scale musically meaningless it cannot be perfectly tuned as a natural scale can be. Physically speaking there is no twelfth root of two.

The dodecaphonic system has given birth to the tonometric system. By linearizing a non-linear system, in this case the chromatic scale, we have the notion that musical intervals can be measured and the elements on this linear yardstick are the cents of the tonometric system. The cent is the hundredth root of the twelfth root of 2, $\sqrt[100]{\sqrt[12]{2}}$. A tonometric cent then is: 2

$$\sqrt[1200]{2} = 1.0005777895065548592967925757932$$

Now we have a very elaborate system that measures absolutely nothing. All it does is take a very precise, perfect system and turn it into a mathematical mess. Weights and lengths can be made metric as they are quantities. A ratio is not. A major third is defined by the *ratio* $\mathbf{5}/\mathbf{4}$. This provides us with the decimal value. 1.25. This is the only valid definition of a major third. The tempered major third is defined by $(\sqrt[12]{\mathbf{2}})^4$ (there are four half steps in a major third). This gives us a decimal value of 1.2599210498948731647672. This isn't even close to the proper major third. In fact this third this replaces is the Pythagorean third, a definable interval expressed by the ratio $\mathbf{81}/\mathbf{64}$. This version of the major third is never used in proper harmony.

The reason properly tuned music sounds so good is the result of a phenomenon known as the differential tone. The differential tone is both conceptually and mathematically very simple. Rhythm and harmony are actuality the same thing. Rhythm deals in cycles per minute and harmony deals in cycles per second but the math is the same. Consider a very common rhythmic device in which two rhythms are sounding together. In this structure one rhythm is sounding three beats while the other is sounding two. Musicians refer to this simply as 'two against three'. Technically it is called a hemiola.



After every third beat of the top row and every second beat of the second row the beats line up. In any sound structure this doubles the amplitude of the beats where they line up creating a third rhythm. This third rhythm is the differential. If this represents rhythm and the second row is beating 120 beats per minute the top row would be beating 180 beats per minute and the bottom row would be a new audible rhythm beating 60 beats per minute.

The same thing happens if the structure is sound. $\mathbf{3}/\mathbf{2}$ is the ratio of the perfect fifth. If we let the second row be 440A the top row would be $\mathbf{3}/\mathbf{2}$ x 440 =660. This is the E above the A of 440. The math of the differential tone is quite simply. From the larger note of the ratio subtract the smaller note and the result is the differential tone. 660 – 440 = 220 and 220 is exactly one octave below 440A. The lower note supports both the upper A and E and it if for this reason that a consonance is such a powerful sound.

Things are quite different when we use tempered tuning. In this case the E above 440 A is 440 x 1.498307076876681 (the tempered perfect fifth) = 659.25511382573. This produces a differential tone of 219.25511382573. While this is close to the lower octave is it still out of tune sufficiently to cloud the perfect consonance. The major third is much worse. The major third is $^5/_4$ or 1.25. The major third above 440A is 550 and the differential tone in this case 110, two octaves below 440A. The tempered third above 440 is 440 x 1.25992104989 = 554.36526195374 and the differential tone is 114.36526195374.

The differential tone, being a real, audible tone reacts with the other two tones of the interval forming secondary differential tones. When the differential is an octave of the fundamental it adds nothing to the structure but when the differential is the result of the tempered interval the differential reacts with the other tones and then with those results as well. The result is tonal chaos and what is known as a beat structure develops. In the case of the tempered third the beat is about six times a second. Traditionally piano tuners use this beat structure to test their tuning.

A thorough explanation of the mathematics of the enharmonic system (there is only one such system), the system that allows for perfect harmony in all related keys, can be found on my Structural Resonance papers on the Website; wropera.com

Even timbre is the result of differentiation. The partials of a sound are real tones in every way and when they sound together they create differential tones. This is why some partials are louder than the fundamental. While it is too complex to be discussed in this essay it appears that the first and second overtones do not appear naturally in the series but rather are created by the complex differentiation of the entire overtone series. In most case the series starts with the third overtone. It is difficult to ascertain which partials are initially parts of the overtone system of a tone and which are differentially produced. It is for these reasons that creating timbre by a complex of sine wave does not work

We are now experiencing an epidemic of hearing impairment from listening to loud music. This is not really so much the result of the high volume as the ear is very well adapted to dealing with loud sounds. What is the destructive element here are those dreadful beats. They are always annoying and often dangerous and are ever present in tempered tuning and analog timbres. This is a false assumption that requires some attention.

Musical theory can be difficult enough to study without the tonometric system. Many elaborate theories have been proposed based on this system but the mere acceptance of tempered tuning obfuscates any musical aspects and makes the whole thing useless. Alexander Ellis in his translation of Helmholtz' On the Sensations of Tone put every example in the book into the tonometric system. There is not one indication in the entire book as to how this should be used or what purpose it serves.

The final false assumption we will look at in sound concerns concert halls or listening areas in general. In a November 1963 article in Scientific American Vern Knudsen, a prominent architect stated that there was no longer any mystery as to the building of concert halls. They now knew how to build acoustically true halls. Then a few years later Mr. Knudsen was part of the team that designed Avery Fisher Hall, possibly the worse concert hall ever built.

Apart from the audience experience, where what you heard depended on where you sat, what happened on stage was the real problem. Performers on stage could not hear each other even when they standing next to each other. When they attempt to 'fix' such a hall one of the procedures is to remove 'excess sound'. While this cuts down the echoes, it also deadens the hall and allows no feedback of sound to the performers, something very necessary to good performing. It is also require a lot of energy to sing in such an acoustical environment.

Some years ago I had an opera company and we debuted in The Cathedral of Saint John the Divine in New York City. The only term to describe the acoustics of St. John's is ghastly. As with Avery Fisher Hall the singers could not hear each other. Even very small groups had to be conducted. Sometime later we performed the same work in The Chapel of the Resurrection in Valparaiso, Indiana. The Chapel was very large and was all stone and glass so we expected another acoustical nightmare. What we did experience was nothing short of miraculous. The sound was glorious and was exactly the same anywhere in the Chapel, even when the Chapel was empty. Singers could be as far away as physically possible and they could hear each other just as if they were standing next to each other. In a properly tuned hall the last thing we wish to do is remove any sound whatever.

Neither I nor any of the performers involved had ever experienced anything like this. All halls should react like the Chapel of the Resurrection. Good acoustics is much more than just being able to hear what is happening. Hearing music inside a standing wave is an experience that is incredible and is never forgotten

What happens in a listening area is the same as what happens in the body of a trombone. The Chapel had a definite nodal frequency, as does all such areas. By a simple stroke of luck the pitch we were using was very close to that nodal frequency and a powerful standing wave was created filling the entire Chapel. Thus the defining factor in a properly tuned hall is it dimensions. Vitruvius describes this at great length two thousand years ago in the Ten Books of Architecture. These books, which survived because of the Scriptos of Charlemagne, were the bible of architects throughout the Middle Ages up until the Renaissance. He detailed plans for various buildings are in the first four books. All of his proportions are composed of simple integral relationships, the simple ratios of music, even when the effect is for the eye.

He was very meticulous as to the dimensions not only of the theater itself but most of the interior sections as well. He also describes the use of bronze vessel located in the back of the theater to resonate and reinforce the sound.

> On the foregoing principles, the brazen vases are to be made with mathematical proportions, depending on the size of the theatre. They are formed so, as when struck, to have sounds, whose intervals are a fourth, fifth, and so on consecutively to a fifteenth. Then, between the seats of the theatre, cavities having been prepared, they are disposed therein in musical order, but so as not to touch the wall in any part, but to have a clear space round them and over their top: they are fixed in an inverted position, and one the side towards the scene are supported by wedges not less than half a foot high: and openings are left towards the cavities on the lower beds of the steps, each two feet long, and half a foot wide.

He states that the mathematical proportions of the vessels are dependent on the *size of the theater*. This mean that the theaters were tuned, just as was the Chapel of the Resurrection. Even many of those who have taken the trouble to read Vitruvius apparently do not agree with him.

In an article in the November 1963 edition of Scientific American Vern Knudsen wrote:

> Vitruvius, the first-century Roman architect and engineer, wrote that large vases tuned as resonators were often located in the seating area to reinforce certain sounds. Whether or not such vases were actually used is uncertain, but in any case they could only have absorbed sound, not reinforced it.

How could a scientist of Knudsen's stature make such an absurd statement? The vessels would not only resonate but when the pitch presented to them was in the nodal structure of the vessels it would vibrate reinforcing the standing wave already present in the theater itself. This is the way vibration works.

Vitruvius was very specific as to where the vases should be placed and of the pitches. To the ancient Greeks the only consonances were the perfect fourth, the perfect fifth and the octave. They did not think in terms of superposed intervals so they listed the perfect eleventh, the perfect twelfth and the perfect fifteenth as well. These six intervals were the framework of the music of that time.

Their tuning system was somewhat different. Vitruvius said that in small theaters a horizontal range halfway up the theater should contain thirteen equally spaced niches each with a specific frequency.

In larger theaters he specified it be divided into four horizontal areas, the top three all with thirteen niches and vases. He described the tunings of the three ranges in the terms of Aristoxenus and the Greek methodology. The bottom layer should be the enharmonic system, the middle range the chromatic and the top the diatonic, the same range as the single range of the small theater.

It is difficult to know what he meant by enharmonic and chromatic as the terms are different those we use today. Here is an example of the tuning of the three ranges that gives a good idea of the complexity of the tunings of the thirty-eight vases[9].

Even in prehistoric times music played a significant role in the indigenous cultures. French researchers Iegor Reznikoff and Michel Davois have recently shown that cave art may well have been used in rituals accompanied by songs or chants. The two studied the acoustic resonances of three caves in the French Pyrenees by singing and whistling through five octaves as they walked slowly through each cave. At certain points the caves resonated in response to a particular note, and these points were carefully mapped[10].

When Rezinikoff and Dauvois compared their acoustic map with a map of the cave paintings, they found an astonishing relationship. The best resonance points were all well marked with images, while those with poor acoustics had very few pictures. Even if a resonance point offered little room for a full painting, it was marked in some way - by a set of red dots, for example.

Since virtually all of the great European cathedrals were built to specification set forth by Vitruvius we can assume they were ‘tuned’. Since we invariably invade these structure with even tempered organs tuned to 440A we have no idea of how they were designed to operate. The odds of 440A being the pitch of any one of these cathedrals is very remote.

The simple, precise ratios of music do more than just create pleasant sounds; they are a valuable tool for the study of acoustics, vibration, resonance and sound. These areas of science need a truly fresh look. Let us once again approach music as the science that it is.

References

1. "restoring force," Encyclopedia Britannica online. Accessed Sept. 4, 2012 http://www.britannica.com/EBchecked/topic/371907/mechanics/77538/Simple-harmonic-oscillations#ref612064
2. "standing waves," Georgia State University. Accessed Sept. 4, 2012 http://hyperphysics.phy-astr.gsu.edu/hbase/waves/standw.html
3. "Fluid dynamics treats a gas as a liquid," Uniten, Department of Mechanical Engineering. accessed Sept 4, 2012 http://metalab.uniten.edu.my/~kannan/MEHB223Intro_MEHB223.pdf
4. "Vibrations in a trumpet bell." Rollins University Student Research. Accessed Sept 4 2012
   http://vanadium.rollins.edu/research/students2000.html
   "Acoustics of the trumpet bell," Acoustics 2012 Nantes Conference accessed Sept 4 2012
   http://iwk.mdw.ac.at/lit_db_iwk/upload/16346_a.pdf
   "External Sound Radiotion of vibrating trombone bells," Acoustic '08 Paris. Accessed Sept 4 2012
   http://webistem.com/acoustics2008/acoustics2008/cd1/data/articles/001911.pdf
5. "Edgerton's milk drop coronet," YouTube accessed Sept 4 2012
   http://www.youtube.com/watch?v=86zIqI3SVIY
   "Milk Drops." YouTube. Accessed Sept 4 2012
   http://www.youtube.com/watch?v=AXOjZGScfH4
6. "Musical Engineering," Harry F. Olsen McGraw-Hill Book Company 1052
7. "Physics of Music – Notes," accessed Sept 4 2102
   http://www.phy.mtu.edu/~suits/scales.html
8. "The Craft of Music Composition," Paul Hindemith. Associated music publishers, Inc. 1942
9. "Ten Books of Architecture – Vitruvius," Sounding Vessels in the Theater – Chapter five accessed Sept 4 2012
   http://www.bostonleadershipbuilders.com/vitruvius/book05.htm#5
10. "Tunes for Painted Caves," Iegor Reznikoff and Michel Davois accessed Sept 6 2012
    http://dspace.dial.pipex.com/town/avenue/pd49/pockets/painted/portel.html