Relational Resolution of Time needs Many Worlds Interpretation with Small Probability Cutoff

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Most once mysterious questions concerning the concept of time have been resolved satisfactorily. While reviewing the progress, many well known issues are presented in refreshingly new ways: Zeno's arrow paradox, time in relativity theory, etc. The still unresolved issue is a fully relational resolution from which time is emergent. The concept of time is closely linked with Many World Interpretations (MWI). Reviewing problems of quantum mechanics (QM), unitarity is stressed to be the most severe, because it gives rise to religious interpretations. Extremely low probability of after-live is not enough suppression, because the "dead fraction" does not count towards the statistical ensemble in a many minds MWI. The proper decoherence mechanism will cut off extremely low probabilities, resulting in zeros (impossibility). A simple physical system, e.g. decaying nuclei, shows that these problems are closely related with the issue of synchronization and emergence of time. In the same spirit as describing a quantum computation as put together from computations performed in parallel universes, similarly time is arising from the interference of MWI branches. While trying to let time emerge in the multiverse structure, the extremely low probability cutoff directs from the outset only towards extremely small corrections to QM that do not conflict with current observations.

1. Time is not mysterious

Time measurement is no different from less mysterious measures: objects or circumstances are compared with some convenient standard via "arranging" it close to them and quantifying the differences between them. If standard meter sticks happen to fit a thousand times end to end on a path, that path is measured to be one kilometer long. Time is no different; one uses some convenient standard of regular change, the "clock", and holds it close to the process to be timed, meaning the clock is let run while the process of interest occurs. The clock, like the sticks, better have somewhat regular repeating changes to be useful and the "process" should have at least changes at the beginning and end so that we can start and stop the clock at those and meaningfully count the standard changes our clock repeated in the meantime. Time is a measure of change as measured relative to the changes of a clock!

Time, like color, first of all appears to us. Many still wrongly identify color with wavelengths of light, although there is no wavelength for purple[b], perceived color depends on surroundings, and so on. Nevertheless, only few take color as something existing outside of them. Two aspects support this state of affairs: Most are at least aware of the existence of a propagating signal with properties like wavelength, while space-time is mostly thought of as pure emptiness, and so nothing underlying is expected. Secondly,

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[b] Generalized "purple" is due to long and short wavelength receptors being excited, but not ones accepting wavelengths in between.

apart from spotting red fruit in green foliage, color was not too important in human evolution. We are often unaware of color; many do not remember dreamed color. Space-time though is extremely important at every turn for hunters and hunted gatherers. Therefore, we are completely engulfed in our models of space and time. We might forget whether there even was any smell or sound in a dream, but never whether there was not any time. Most humans believe in the reality of physical time as being as perceived, and attempts of resolving this are greeted with difficulty, if not plain resistance. Yet already about 2500 years ago, Parmenides of Elea recognized time's flow to be illusionary. Today, we merely use more precise language to express this. Basically, when using models where the time-axis is already incorporated, "put down" or "drawn on paper", all (should) agree to model-appropriate language, and then "flow" is not appropriate. There is no other, second parallel[c] time that allows time for flowing to occur. The issue is about proper use of language and not so much about illusion[d]. None of this involves relativity theory, which Parmenides could not have known. Space-time diagrams with x- and t-axis can be fully classical and yet Zeno's arrow paradox is already understood. The moving arrow indeed stands still! If the arrow is taken to be its whole history traced out in space-time, it stands still. If the arrow is to be identified as space-like slices through that history, those sections also stand still[e] in space-time.

The perceived directionality or "arrow" of time is also not mysterious anymore. Especially perceived time via set down memory (short or long term) is satisfactorily identified with the direction of entropy rise. There are details to be cleared up, like for example whether the quantum mechanical (QM) basis and statistical mechanics are responsible for the emerging thermodynamics; Ilya Prigogine and many philosophers interested in consciousness prefer thermodynamics as the fundamental theory. Also, how the entropic and the cosmological time arrow are linked (say via Roger Penrose's Weyl curvature hypothesis), is still under debate. Nevertheless, such are almost purely problems about entropy rather than about the direction of time, which has been "dissolved" into the entropy.

Any mentioning of time travel should immediately raise questions like: Into which of the many possible tomorrows or yesterdays would you travel, which of them will send you back, and how many of you will arrive here returning? Nevertheless, time travel still is discussed as an intriguing technical issue, one of whether we would be able to invent the means. The feasibility of it is often thought of as an open, meaningful question that can be decided in the field of general relativity (GR), rather than a question that makes fundamentally no sense. Yet the problem is resolved already in the same way the flow-feeling is: By proper use of language, it disappears.

Lastly, time has already often and widely been understood as relational. There is no option, no choice here, since any explanation of time that is not relational just does not really resolve (dissolve) time, however useful such explanation might be. In conclusion,

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[c] A second time dimension as described by Itzhak Bars would also not allow any further flow along the firstly considered time direction, because they are orthogonal.

[d] Optical illusions for example are not resolved by proper use of language.

[e] Maybe a still controversial standpoint, since in the past Zeno has been "refuted" with infinitesimal mathematics. It should not be still consensus outside high school didactics though; Zeno must be understood via taking into account that he tried to defend his teacher's (Parmenides) insights via the paradoxes.

the only real mystery left in the understanding of time is that we as yet have few clues about how this final relational resolution looks like. The aim of this essay is to clarify this fact and to give some hints on how to tackle it.

2. Relativity almost resolves Time

Relativity has helped to demystify time. When doing kinematics, the special relativistic aspect is the addition of simple rules about how to draw light paths and spatial and time like directions through an otherwise classical space-time diagram. One can understand most about the theory of special relativity (SRT) just drawing without need to calculate. The twin paradox can be understood in this way for instance. Looking back from our vantage point in history, nothing is surprising about twins ageing differently when going different paths through space-time; one should be rather surprised if they did not.

It is worth to look into how SRT uses light to measure in order to get a flavor of what a satisfactory relational resolution needs to achieve. It has not been stated in this way before: SRT in a way gets rid of space and time. Because light travels with light velocity, *it has no time to exist* and there is no space in-between emission and reception of it! If we want to attain the point of view of the used light, namely see the world from the light's own rest frame, we will find that the more we accelerate to travel along with the light, the more the travel time gets shorter due to time dilation, the traveled distance Lorentz-contracts further and further and the light's energy red-shifts away to being undetectable. This is remarkable: All that happens on the fundamental level, is that two objects exchange an interaction quantum without there being time or space involved, but if one takes all these interactions between objects and observers together, than from the observer's point of view, the light may have traveled millions of years over vast stretches of space, and so space-time emerges. This is how satisfactory resolutions taste: The resolved issue must have totally disappeared into something other that has no likeness with the original problem. Substituting for the empty box that continuously exists, the space-time, one encounters a vastly interconnected web. Events are connected by lines that do not represent length but only the plain fact of two objects exchanging a quantum of interaction. Of course, SRT does not straight forwardly also resolve the time that the objects "undergo" in between interactions, yet though SRT does not fully explain time, we learn two things from the expose: Firstly, SRT is a role model for any resolution of time, in that the aspect that is being measured is not implicitly involved in the way one measures, or worse, part of the measuring device. This is the very reason that SRT is so successful: It measures lengths in space-time with the best standard possible on principle, namely one that cannot change its length, because it actually has no length. Secondly, we learn to expect a net- or web like relational resolution to replace a fixed concept of outside existing, continuous time, in a way that totally rips it apart from its previous order, like when the spatial order completely falls apart if one tries to imagine that everything we see, even far away stars, is actually right on top of our retinas as according to the point of view of the light that is exchanged.

Let us digress some in order to reveal the power of the abstract relativistic view of space-time: When considering a homogeneous universe, classical expansion through space and GR's global Friedmann-Robertson-Walker (FRW) description fit together seamlessly. The way in which a local cloud of Newtonian dust expands *through* space is consistent with the global FRW picture of the universe expanding as a whole, expansion *of* space. This seems paradoxical: in the Newtonian/SRT description the underlying space stays the same, uninvolved stage, while in the GR picture the space seems to expand in the concrete sense that there is more of it than before; this is obvious when considering closed or compactified open universes. Space is expanding globally; there is more of it after a while, while locally it seems to actually nowhere expand! GR resolves the paradox via space-time being one continuum. The four dimensional (4D) whole is one unchanging consistent arrangement where time is already taken care off. The smaller space in the past is simply a different region of the whole. It did not grow into the larger space of today; it is still in the past! Some cannot accept 4D space-time as more than a picture that still tells us that space is actually growing, but those should explain why they accept 3D space as more than just a model. One reason is the misleading feeling that only a "real" flow of time allows "actual" dynamics that develops the future according to physical laws, so that it actually may end up consistently constrained in the picture. Another reason is that the classical GR 4D continuum only contains one possible past and future, and especially the future we do not like being already taken care off. MWI[1] and especially many minds interpretations are helpful to overcome both hurdles. With this in mind, one may as well accept the 4D continuum just like the 3D one, and then the expansion of space is a non-issue.

However, observed relativity could be effectively due to all objects, including observers, being made out of the excitations of a medium with a constant propagation velocity of low energy waves. An absolute background coincident with the cosmic microwave background (CMB) can give rise to effective relativity. Especially with the advent of by string theory inspired universe on a membrane models[2], entertaining Einstein-Aethers[3] is not career suicide anymore. Space might be quite tangible as an actual web of strings, the inside of a droplet of super fluid Helium III[4] or the surface of a large pond of some fluid; indeed it was already shown[5] in 1945 that a crystal-like Dirac-sea mimics SRT contraction and mass-energy increase[f]. Even GR may well be due to a medium[g] with the right properties (e.g. tension of space) and then our time is not the fundamental time that the medium "lives through". Such models are popular with "crackpots", and because of the danger to be cast as one, disproportionately many references were just provided. Such models cannot explain time fundamentally, because they only start an infinite regress: If space is some ether, where is the ether? If in another space, then where and what is that other space? And so on. Ether space-times favor considering their perhaps higher dimensional embedding. Especially early on, this was perceived as a nuisance rather than an opportunity. A fundamental theory must explain space-time without it being inside other, more fundamental ones. With insisting on this, an infinite regress is cut short, but although this is attractive, one needs to be open to the

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[f] A moving Burgers screw dislocation in a crystal contracts $L' = L\,[1-(v/c)^2]^{1/2}$, where c is the velocity of transverse sound. The energy is the potential energy of the dislocation at rest divided by $[1-(v/c)^2]^{1/2}$.
[g] It is beyond the scope of this short essay to explain the fact that even expansion of space as fast as during inflation and also black holes in as far as we observationally know about them can be consistently reproduced in space-substance models.

possibility that it may be cut one or more steps too short. If time and space are truly unified, why are we even discussing the nature of time and not that of space-time? The unification of time and space via relativity might be an illusion and not fundamental on the level we reside[h]. Even much of quantum mechanics could be due to an underlying medium having a temperature proportional to the Heisenberg constant h, yet *not* quantum entanglement, and this fact redirects focus away from space and relativity towards time and quantum aspects.

3. Similarities of modeling Time and modeling Possibilities

Properly understanding a time diagram, even just a classical single-axis graph, is burdened by the same difficulties the MWI is hindered by. It is hard not to imagine some flow, a sort of divine finger that first points to one point and afterwards to the next point and so on. It is like the point "Now" should glow, signifying that I only ever perceive this "Now", and that glowing point should move with time along the axis. But of course there is no other parallel time in that model that would allow the glowing to wander forth or a divine finger to point first here then there. Some fear this; all seems dead. Actually though, if conscious perception of "now" deserves a happy glowing, the time line is warmly glowing everywhere it corresponds to our being aware. The t-axis model emancipates all the points in time. Some might raise that it is matter of opinion or not proven that also all the other points in time are just as "existent for themselves" as for example your perceived "now" is as you read this. This is just not correct though. We always use a model to communicate. Thinking itself is to internally communicate verbally or otherwise. In everyday live, our model of time is a short stretch or point that is lit up before we have moved over to the next one, leaving the one before to dim away. We in a sense correctly talk using terms corresponding to that picture. But as soon as one employs a formal time axis diagram, one by that choice alone decided to adopt that new model instead in order to think and communicate. With no other parallel time dimension left anymore to allow any change, like brightening or dimming, all points in time, and later we will see also all possibilities, glow the same. That is not opinion or awaits any proof; it is plainly sticking to making sense and not flip-flopping on one's own assumptions.

We expanded on this resolution of the "flow of time" so much because circumstances are precisely the same with the most advanced and consistent model to consider other possibilities, like you reading a novel now instead of this. The MWI is often presented as an interpretation of QM, but just like space-time does not necessarily imply relativity, so also the MWI can be classical without involving QM. The MWI basically just draws down representations for all the different possibilities that one would

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[h] A deeper reason for the failure of too abstract views like bootstrap models may be that our resolution remains too low. We may still discover many strata inviting to ask "what *things* is it made out of?" (…, atoms, nucleons, quarks, ???, strings/loops, …) before an abstract relational fundament can be resolved. Sciences (e.g. sociology/biology) suggest emergence from lower layers every few orders of magnitude. It has been argued that the Δx versus Δp tradeoff or the fact that for high resolution needed large energy hides itself behind an event-horizon prove that the lowest stratum has been identified (Planck level).

like to consider. Employing the QM-MWI additionally merely implies that one accepts certain rules concerning how the different possibilities are arranged and can "interact" with one another. For example, the possibilities may correspond to mutually orthogonal quantum states of a basis in Hilbert space. The term "parallel worlds" for all the different possibilities, for example if we consider drawing a diagram representing different possible universes, comes from the picture that these different possibilities do not interact and just evolve parallel next to each other along in time. It is the giveaway for this being in substance a classical theory, because "parallel" is the very opposite of "orthogonal", and the states in the QM-MWI actually interfere.

The important point here is that the MWI presents the same problem we encountered before: One easily feels there should be a special status to the actually realized possibility, some sort of glow again only at the actually observed, existing possibility, and that all other possibilities are dead. Of course we may talk about what to do if we find ourselves at a *certain point* in time in a *certain situation* on the model, and thus we should employ all the three categories, namely that Necessity implies Existence implies Possibility and not the other way around. Yet the MWI idea to grasp is that there is no divine finger pointing to only the existing possibilities. All the possibilities "exist for themselves" in precisely the same way that your situation right now is possible and existent "in and for itself" only, especially when the worlds go on in parallel. It is unreasonable to think any other way about this: I toss a coin and it lands heads up. If this were so special that there is no Universe possible where I just wrote "down" instead of "up", than from the very beginning of time all physical laws conspired to let some guy some Tuesday afternoon throw a coin heads up, in spite of that the story is consistent with "down" replacing "up" everywhere. The parallel universe where I wrote "tails" instead of "heads" certainly is possible and "exists for itself". Even if some ghost actually actuates the existent possibilities by pointing to them on a divine board, a model of totality must contain all the different possibilities of that ghost, all the possible ghosts, and so forth.

The MWI is one that basically the old Greeks should have come up with more explicitly than one may claim Parmenides and Zeno understood. Nevertheless, it is nowadays the most insightful QM model. It can do without the ugly collapse of wave functions. If there should be something like a collapse at all, it may be "accidentally" rather than necessary to understand all the issues that the collapse was supposed to explain. Of course, MWI is not problem-free. The two problems most often raised are: It needs to have a preferred, or "interpretation basis", as David Deutsch for example has argued and helped to provide. Secondly, the Born probability law has not yet been derived from the model in an as water-tight accepted way, though good suggestions exist[6]. The most problematic though has as yet never been raised: the by suggested de-coherence mechanisms[7] still un-abated unitarity. It takes all micro states to be possible even if the corresponding macro state should be considered impossible, like a world were all cars stay upside down for a week or whatever. As of now, MWI still introduces no cutoff to QM that would leave extremely small probability equaling zero. It needs to be pointed out, without having here space to elaborate much on it, that this is in danger to be taken as a religious theory[i]: It seems possible to tunnel into a situation were after biological death one wakes in front of a guy with a long white beard (maybe due to labs with virtual reality suits appearing, those details are immaterial). Especially in a many *minds* interpretation, the statistical weight of all the enormously more probable "stay-dead" micro states bears actually no weight. Much worse: having believed beforehand renders the situations before and after death more similar. Some may argue the probability of "tunneling into heaven" indeed higher due to holding a belief. These are of course unpleasant aspects counter to science's aim to cure us from the destructive religion-virus. Anyways, the problems enumerated are resolvable since there is no alternative to MWI that would be less religious. In the context of this essay, it is most interesting that the consideration of a simple system, namely unstable nuclei for example, relates the extremely low probability cutoff problem with the issue of how time emerges.

4. Relational Synchronization

Unstable systems like an excited atom or a radioactive nucleus, behave entirely statistical. The system will decay with a certain probability, *P* of *dt*, during a time *dt* that we wait, say for example *dt* being the next minute. This process is statistical, i.e. the system *does not age*: It does not matter how long we have already waited for the observed system to decay, even if we waited for years. The probability *P* does not grow with time *t*; it only depends on the time *dt* that we will further wait into the future.

How can such a system look like internally? Any system that proceeds deterministically from one internal state to others, like a classical gas of many molecules, could mimic such a system only for a finite while. Even if it is a very complicated system with many hidden variables, at some point it would exhaust its number of internal states and be due to get to the internal state that makes it decay. The observed radioactive decay law would not result over arbitrary amounts of time; the systems would age. The QM description of the isolated system is a stationary state, i.e. stable, because an energy eigenstate has no uncertainty in its energy and thus infinite uncertainty in the time variable; it never changes at all and therefore does not age, but neither does it decay. There are two equivalent ways to think about an atomic electron orbital: One is to regard the electrons as indeed smeared out all over the orbital. This picture is very tranquil, nothing moves, it does therefore obviously not age or decay. Equivalently one may think of electrons actually moving but the different states of moving around differently are all there at once in superposition. The latter view is somewhat more open to allow decay and has time emerging from the interaction of electrons and nucleus; one synchronizes the other, but as long as they are not interacting with the rest of the universe, they might proceed with any speed relative to not interacting clocks. The decay rate is not so easily emerging.

That the system nevertheless decays *principally must be* and is described as due to an interaction with something outside of the otherwise maybe isolated system. A totally

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[i] Frank J. Tipler for example went indeed religious instead of holding on to scientific rationality. Proper procedure prescribes carefully judging a theory's domain of applicability, not going off on exactly those aspects, singularities and the like, were the theory actually breaks down.

isolated system has no synchronization with the time experienced by observers and therefore there is no way for it to lead to an observed decay time constant. In the usual picture, the system interacts with the background field, like the electro magnetic (EM) field around the excited atom. Even in its vacuum state, the field is still having vacuum fluctuations. These are just the virtual photons that come along statistically at a certain rate and interact with the otherwise lonely atom to make it decay. Since virtual particles are un-observable, the atom radiates “spontaneously”. The main point to consider is *virtual* photons coming along statistically at a certain rate. Something principally unobservable that does not really exist (in this universe) synchronizes the otherwise timeless system.

However, just like before with the electron and nucleus holding each other in tact without synchronizing the waiting observer, now the unstable system plus background field inside an isolating box would together again represent such an unstable system. Here is where some accept QM time to be absolute. The more sophisticated way to proceed is to argue that time still emerges from interactions because one can on principle not isolate systems sufficiently: At least gravity will overcome any isolation efforts. The systems are not only in specific background fields, like the excited atom considered inside the vacuum EM field, but they are also inside the gravity field that lets all objects in the universe interact. Although this seems true, it is far from clear that this alone is the right or even just the simplest way to regard the emergence of time.

The background-interaction picture alone does not immediately connect to the extreme low probability cutoff problem that might actually be observable in advanced future tests of the decay law. Observing a sample of 40 neutrons for 10 minutes and having none of them decay has similar probability to being the chosen one among an advanced future population of 10^{12} people and being allowed to wear the lucky hat. 100 neutrons being stable for a year though could violate emergence of time physics and fundamental limits may have to be considered[j]. To put it humorously: There is a limit on how many sentient beings can evolve in the same Hubble volume and play the lucky hat lottery together. The background-interaction picture alone does also not take into account the fact that the MWI will at some point emancipate *all* possibilities, i.e. identify indistinguishable situations at other times far in the past or future.

In our flat, expanding and maybe infinite universe it is difficult to argue expansion *of* space, because cosmological time t_c is determined by the CMB and average star background. The cosmological principle demands the CMB to be isotropic and homogeneous, its temperature T changing only with t_c. However, if two regions far from each other experience different temperatures, it does not violate the principle, it only implies that they are at a different time t_c. Worded as above, the principle misleads. The statement that decay probability P increases only with dt may mislead similarly! Both need to be turned around to actually “tell the time”. We cannot simply observe P like we can T, nevertheless, in the MWI, P has concrete meaning, being the ratio of the number of worlds were decay will have occurred over the number of all branches that split off from the initial situation when we started to wait.

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[j] Nowhere here is argued that the unstable systems are actually isolated from the observer, or that their sampling is not affecting them, because a macroscopic sample of uranium contains far more than 100 nuclei that have not decayed in thousands of years.

MWI is a classical viewpoint, and if not QM interaction between parallel branches is switched on, also the parallel branches cannot be synchronized with regard to one another. One has to include the interactions of the systems in and with the other branches. An electron for example does not know which nucleus it circles; it kind of circles all identically circled nuclei of atoms in similar situations everywhere in space and time at once. Such ideas are also inspired by quantum computing, where the power of a quantum algorithm is said to be due to many classical calculations going on in parallel (worlds) while still influencing each other (interfering). That is why quantum and parallel computation are equally powerful, with the caveat that one never can have as many parallel processors as one can have parallel worlds. Radioactive decay rates may be due to taking correctly into account all the things that happen to the nucleus in parallel worlds, with the seemingly spontaneous decay being triggered by real interactions in parallel worlds.

Using different words for clarity: One may suggest the way forward is to ask if not the discussed virtual photons are actually somehow connected to real particles interacting with the atom in parallel worlds. Virtual particles are due to time-energy uncertainty, which in turn can be interpreted to exist due to there being parallel, classically based descriptions with different energies, i.e. a statistical ensemble of atoms that have actually all slightly differing energy values and are therefore unstable. Of course, the observer's ignorance about hidden variables (the "real" dE) is not sufficient, because such a view would be a classical MWI, but branches can be in coherent superposition as long as the correct de-coherence mechanism has not taken place. In first approximation, one does not worry about this interpretation and just calculates usual cavity quantum electro dynamics. This yields the decay law as far as we have observed it, i.e., none of the above is actually a new or crazy idea, just a reinterpretation. The difference comes only with allowing corrections to the current picture, and those need to be inspired by a progressive interpretation. Here it is that the above interpretation can be expected to be helpful, because corrections to QM have to be extremely small in order to not be ruled out by experiments. But we saw that the suggested decay system very closely relates time synchronization with the extremely low probability cutoff issue. In other words, trying to correct QM-MWI in a way that tackles that particular problem is from the outset only going to lead to corrections that are indeed extremely small!

5. Conclusion

Time is related to entanglement between quantum systems[8] and different times are cases of different universes in the MWI setup, as time can be resolved only in the theory of the "structure of the multiverse", as Deutsch put it. It is expected that proper decoherence models are closely related to the emergence of time and that in result quantum states lose coherence naturally, yet at a very small rate; as has also all been suggested before[9]. The novel ingredients here are firstly to take interaction between MWI branches much more seriously and secondly a cutoff on the smallness of probability: extremely small probabilities should turn out to be actually zero in order to give a non-religious theory. Seeking of models directed along such lines ensures from the outset small corrections not contradicting experiments.

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