Physics as Mathematics of Information

Gerald Vones

Federal Ministry of Science, Research and Economy
Vienna, Austria, European Union
gerald@vones.eu

Abstract

It is argued that physics is mathematics of information, hence any physical entity is mathematical, but not vice versa. As a necessary basis, the concept of "information" is discussed. As a result, there is no freedom in setting physical scales, rather the Planckian scales are to be used according to the density at which nature stores information.

In the context of reductionism it is argued that all natural phenomena can be reduced to the laws of physics [1]. As the clearest example, *chemistry is physics of atomic/molecular bonds.* So, any chemical entity is physical, but not the other way round. However, in principle chemistry is interrelated with any physical aspect. For the example of gravitation, which certainly is not a chemical phenomenon as such: It makes a difference for a chemical reaction whether it takes place in a laboratory on Earth or whether in takes place in the microgravitation on board of a spaceship.

Reductionism has a variety of facets and it is doubtful whether it will survive in its entirety. But what regards the relation between physics and mathematics, a reductionistic step after the above example very clearly suggest itself. Tegmark [2] argues that the physical universe is mathematics in a well-defined sense, and I propose the following substantiation: *Physics is* *mathematics of information.* So the connection between physics and mathematics neither is a trick nor is it a mystery. Rather, any physical entity is mathematical, but not the other way round. I recommend reading of the book by Dewdney [3]. One can hardly do it better than those reference.

The above is to be taken with a grain of salt. Despite of its omnipresence in modern physics [4], the concept of "information" has remained uncomprehended. On the one hand, the concept of information is absolutely simple and transparent: *Information alias entropy is a pure number, namely the number of degrees of freedom of a system.* Like any pure number, it cannot be rescaled, rather is completely determined. For a number stored digitally in a computer, the respective information is the number of bits needed. For a number written in digital form, it is the number of digits. For a screen, it is the number of pixels. For a text, it is the number of characters. For a system described by classical statistical physics, it is the logarithm of the number of elementary cells of phase space the system sweeps out over its time evolution with equal probability. For a Schwarzschild black hole, it is the area of the 2-dimensional horizon expressed in units of the Planck length squared (throughout, I leave away numerical factors of order of magnitude unity). When exponentiated, it gives the number of possible states: For a computer memory, this is the number of different numbers that can be stored. For a screen, this is the number of different images it can display. For a black hole, this is the number of different histories it can have. For most of these examples, the bases of the logarithmic as well as the exponential functions are different. But this is a minor aspect and one can always go to the basis of 2, as any digital number can be written in a binary representation. A "bit" is the natural unit of information. Another aspect is a biased probability distribution of the elements. For example, the probability for the appearance of the letter "x" is lower than those for the letter "e" in a text based on a natural language. This leads to the so called Shannon entropy - what neither makes a crucial difference.

Physics taught, to the dismay of Planck himself, that the phase space volume as the number of possible states of a system, is countable. Meanwhile it is known from the work of Bekenstein [5], that the entropy of a black hole equally is countable. This makes sense: The number of degrees of freedom of a system as well as the number of possible states are natural numbers. In this sense, the exponential and logarithmic functions are not smooth. Representing the numbers 4 to 7 equally needs 3 bits - another minor aspect.

On the other hand, there are quite intriguing facets of the concept of information. From the above emerges a tension between factuality and possibility: Given a factual number, one can estimate its information by counting the number of digits. The other way round, exponentiating the number of digits yields the number of possibile patterns, any of which can be interpreted as a different number. The second even more confusing aspect is those of semanticity. One associates information with something a receiver can make sense of. An artpiece of Picasso displayed on a screen is felt as making some sense, irrespective of whether or not one likes it. But any object has a primary semanticity, that is the number associated with it. When the New Horizons probe hopefully will send images from Pluto this summer, any such image will be transmitted as a chain of 0s and 1s, thus a number. Concedetly, bringing the entirety of aspects related to the concept of information under one hat is not straightforward. As I shall argue elsewhere, rather important - not yet realized - conclusions follow from doing so.

One can raise strong objections against the view that physics in fact is mathematics. One can argue that physical objects or processes are concrete, one is able to see, feel, smell, taste, hear them -, while mathematics is completely abstract, nowhere at home but in the imagination. But wait a minute! Physics does never deal exactly with those aspects which are the immediate elements of reckognition. For example, what I actually experience when seeing the color called "green", is outside of any physics. The reason is that the physical world is invariant under such reckognitions. If I saw red in place of green and vice versa, nothing would change. It may appear that seeing too much of red would make people aggressive, but in any case evolution would have selected those staying calm when seeing the dominant colors of our natural environment. The elements of immediate reckognition are purely private.

The private such sensations are, the public is their occurance. Where one emmetropic persons sees green, any other does as well. Else, traffic lights couldn't do the job. And there are a lot more public facts in this case, namely the entire electromagnetic spectrum radiated off by the viewed object. While there is almost a continuum of possible frequencies and related intensities, our eye only extracts three weighted integrals of the intensity what results in the 3-dimensional space of colors. The similar number of dimensions with 3-dimensional configuration space (ordinary space) is purely accidental. Rather, there is a fundamental difference. While the 3-dimensionality of the space of colors results from human anatomy, namely the three species of retinal cones, 3-dimensionality of ordinary space is objective. One can estimate the static potential generated by an electrically charged point source via measuring the force it produces on a probe, and finds that the potential goes inversely proportional to the distance from the source. Theoretical physics and mathematics teach that this is the case in 3 and only in 3 dimensions.

Physical entitities usually are dimensionful, not just pure numbers. A length is something that can be measured by a yardstick in units of yards, equally sensibly in units of meters or so. But the Bekenstein-Hawking entropy teaches that actually a length has to be measured in units of the Planck length. Only then it is correctly connected to the concept of information. Quantum physics had yet taught that space and momentum have to be measured such that their product is measured in units of Planck's constant, only then the phase space volume is correctly connected to the number of possible states. What happens if I decline doing so and insist on measuring length in yards? Well, then I run into troubles. Regard Special Relativity. Over history, spatial and temporal intervals only had been divided by each other to arrive at velocity. Multiplication equally would have brought no problems. But with the insights gained by Maxwell and Einstein, it came to addition. One cannot add apples and oranges, says an idiom. This is not quite true. One can regard both as representatives of the more abstract entity "fruit". In the case at hand, a 1:1 count sounds quite reasonable, but if one adds up apples and cranberries, some different weight should be assigned to these. The same with space and time. They are different, but Special Relativity clearly says that (the squares of) spatial and temporal distances are to be subtracted to arrive at the invariant distance in spacetime. The relative weight is the velocitiy of light. In other words, if one measures time in seconds, then one is forced to measure space in lightseconds, else one cannot perform subtractions or additions.

When it comes to information - which I repeat is a pure number, namely the number of degrees of freedom of a system - then the interrelation between mathematics and physics unfolds: To add or subtract a physical quantity like the area of the black hole's horizon to a pure number with the meaning of information, it necessarily must be measured in Planckian units. This still may sound a bit strange, but imagine a number written down on a piece of paper, what acts as a primitive memory. If all the digits are written of equal size and with equal spacings, then one can use a yardstick to estimate the information of that number. In the essence, one just has to measure the length of the number. There is one overall degree of freedom. It is the size of one digit relative to the scale of the yardstick. How must this scale be set to get the right result? Since the information of one digit is the logarithm of 10 to the basis of 2, the scale must show this result for the size of one digit. The fundamental insight gained is: *There is no freedom in setting scales, rather the identity of physics with parts of mathematics requires that the scales are* *set according to the density at which nature stores information.*

A priori, not even the basic physical entities like space or time are known to mathematics, rather mathematics is prior to physics. 11 - 7 has not been 4 or will be here ore somewhere, it simply is. While this is fully in line with the ideas of reductionism, it nevertheless feeds another objection against the identification of physics with parts of mathematics. Prior structures are regarded as unphysical since the success of the Riemann-Einsteinian approach to gravitation. But, with all respect for the unbroken quantitative success of General Relativity, caution is justified as long as this approach has not yet led to a quantizable phase space for gravitation. Mathematics definitely knows entities that are not physical: Infinity, the continuum, cardinality and so on. Even such an elementary mathematical object like an isosceles right triangle cannot physically exist, since its sides are incommensurable. But isn't this just like the connection between physics and chemistry? For a chemical process to take place, there must be some physical environment. A reaction vessel is made from some stuff, obviously, but this stuff shall not chemically react with what is inside. Rather, it is there to confine the reactants and to act as a source or sink for heat, at most as a catalyst as well. For physics, which is countable, to take place, there must be a mathematical environment. Although currently regarded as irrelevant, there are prior structures especially for an intrinsically curved metric manifold as it is at the heart of General Relativity. It is a proven mathematical theorem, that given any intrinsically curved manifold, this can be produced from embedding in a flat space of sufficiently many dimensions, the metric being a purely secondary quantity derived from Gauss' formula for an induced metric.

Although the formula for an induced metric remains almost unchanged if the embedding space itself is curved, Gauss started from a flat space as a prior entity. In union with the concept of information, a flat space clearly is preferred. It is the zero point for the Bekenstein-Hawking entropy. Furthermore it has the maximum of global symmetries, these even trivial. Such prior entity characterized by global symmetries renders the questions about units, which has been highly controversial [6, 7], simple. In a flat metric space, the generalized yardsticks behave as vectors under rotations (boosts) and trivial under translations. In addition, the lengths of the devices are absolutely gauged from the requirement discussed above. This brings the velocity of light back on the scene. The opening angles of null cones cannot be globally equal to 90 degrees in a flat embedding space, and equally have that value everywhere inside an embedded intrinsically curved manifold. Rather the embedded manifold experiences a variable speed of light - which is a pure number, of course. To avoid misunderstandings, it shall be said that actually the situation is more subtle. The global symmetries cannot be restricted to spacetime quantities, they must comprise all physical entities - energymomentum in particular. Currently one faces two symmetries which connect spacetime to energymomentum. The one is the symplectic structure of phase space, involving Planck's constant. The other is General Relativity whose foundations in terms of fundamental symmetries are much less clear, involving Newton's constant. Reference [8] strongly argues that the missing link between them implies a collapse of the space of theories.

The next glaring question about information is: Is the above picture, seemingly based on classical physics, to be modified in the light of quantum physics and the yet undiscovered quantum gravity? To my understanding, the answer is "no". First, the Planck lenght - the scale at which nature stores information - is the square root from the product of Planck's constant with Newton's constant (with the velocity of light set unity). So both are yet implicitely at work. Second, a wave function can be characterized via the associated set of quantum numbers. If it should actually be that a collapse of the wave function occurs, then a new wave function is created, again unambigously characterized by its set of quantum numbers. In this context it should be mentioned that given the strange aspect of a collapse of the wave function, there are considerations on whether factually a process like cellular automata does underly [9]. Third for gravitation, it rather seems that the black hole entropy is the starting point for a comprehension of gravity and its entropic character [10], which eventually will guide physics into the right way to quantum gravity. Penrose conjectures that quantum gravity could bring an objective reduction of the wave function. But he also brings this in connection with conscious reckognition [11]. Von Neumann early had proposed that the conscious reckognition by an observer produces the collapse of the wave function. The relation to consciousness - not only what regards the wave function, but also what regards the existence of mathematics - is extremely interesting, but is much more sophisticated than what I discuss here. I simply take for granted that mathematics as we know it exists.

Reference [11] furthermore focuses on the question of computability. In fact, the halting problem is where information meets (countable) infinity. From the standpoint of physics, the primary question rather is: Does infinity exist in the physical sense, in particular via an unbounded and/or continuous time? Two thirds of the question appear as yet answered: Time begins with the big bang and goes in steps of the Planck time. Should time have an end as well, then the halting problem is none of physics, it is one of pure mathematics.

In any case, what an algorithm can do sure enough, is to produce illusions. Since the concept of information is uncomprehended, one is in danger of falling prey to such illusions. In particular, the increase of entropy over time, is it real or is it fallacious? It is a remarkably self-contradictory though successful exercise to write a computer code producing a series of "random" numbers. When done in a good way, the pseudo-random sequence is exponantially longer than the code. Many of the computations today, from fundamental physics to almost any field of application are based on such pseudo-random sequences [12]. Weren't nature itself like that, there would be a threat that all these computations are substantially misleading.

References

[1] 't Hooft G, The Obstinate Reductionist's Point of View on the Laws of Physics, Lecture given at the 2001Technology Forum, Alpbach, Austria, August 23-25, (2001)

[2] Tegmark M, *Our Mathematical Universe - My Quest for the Ultimate Nature of Reality* (Random House LLC, New York, 2014)

[3] Dewdney AK, *A Mathematical Mystery Tour* (John Wiley and Sons, New York, 1999)

[4] As one representative of a vast literature: Bub J, Quantum entanglement and information, *The Stanford Encyclopedia of Philosophy* (Zalta EN ed., Winter 2010 edition)

[5] Bekenstein JD, Phys. Rev. D 7, 2333 (1973)

[6] Duff MJ, Okun LB, Veneziano G, JHEP 03:023 (2002)

[7] Duff MJ, arXiv:hep-th/1412.2040 (2014)

[8] Matsas GEA et.al. arXiv:physics.class-ph/0711.4276 (2007)

[9] 't Hooft G, arXiv:quant-ph/1405.1548 (2014)

[10] Verlinde EP, JHEP 1104:029 (2011), arXiv:hep-th/1001.0785 (2010)

[11] Penrose R, *The Emperor's new Mind* (Oxford University Press, 1989)

[12] Lang CB, Pucker N, *Mathematische Methoden in der Physik* (Springer Spektrum, 2005)