Quantum Measurement as an Arrow of Time

C. Vinson

December 1, 2008

In this essay, I wish to provide some reasons to take seriously the possibility that physics is neither symmetric under time-reversal nor deterministic. I want to aim for accessibility to readers outside physics, so I include some material that will be introductory for specialists, I hope at least to offer an unorthodox perspective. I will start with an examination of how the formalism of quantum mechanics relates to the arrow of time.

1 Introduction to Quantum Formalism: U, R, and Density Matrices

Quantum mechanics' formalism has two distinct parts that I will, following Penrose (2004), call **U** and **R**. **U** stands for "unitary evolution" and **R** for "state reduction." **U**'s name comes from the uniqueness of its evolution: given some initial conditions, there is only one solution under **U** that will satisfy them, or in other words, **U** is deterministic. **R**,in contrast, is nondeterminstic.

1.1 U: Unitary Evolution

U's most familiar formulation is the Schrödinger equation, but there are many other more or less equivalent variations, including the Heisenberg picture, the de Broglie-Bohm pilot wave, path integrals, etc. While both the mathematical structures and the physical interpretations of the mathematics vary among these different perspectives, they all lead to the same observables. Most of the effort in physics has gone to understanding **U**, as for instance in describing forces and particles using quantum field theories.

Is **U** invariant under time reversal? This is a subtle issue. The Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi$$

is not invariant under the obvious transformation $t \rightarrow -t$ because the classical Hamiltonian H, which turns into an operator in quantum mechanics, is invariant under time reversal, but the time derivative on the left-hand side is not invariant. However, note that combining time reversal with complex conjugation acting on the Schrödinger equation gives

$$i\hbar\frac{\partial\Psi^*}{\partial t} = \hat{H}\Psi^*.$$

This is the same equation, only with Ψ^* taking the role of Ψ. Observables in nonrelativistic quantum mechanics always take the form $< \Psi|A|\Psi >$, as this provides the probabilities with appropriate normalization, so it's not possible to distinguish between conventional quantum mechanics and quantum mechanics with all wavefunctions and operators replaced by their complex conjugates. Because quantum operators are self-adjoint, $A^* = A$, so for operators this holds in the most obvious way; for wavefunctions, complex conjugation is an invertible map from the Hilbert space to itself. Thus, given a description of some physical situation in terms of wavefunctions defined with respect to the conventional direction of time, there's some other description using the reversed direction of time, only one also has to replace the wavefunctions with their complex conjugates. In other words, in nonrelativistic quantum mechanics the choice of the direction of time and the roles of ordinary and conjugated wavefunctions are just conventions, but the conventions are linked so that changing one requires changing the other.

I've presented this in some detail because the situation in relativistic quantum mechanics is analogous. The CPT Theorem, the acronym standing for the three discrete symmetries charge conjugation i.e. replacing particles with antiparticles, parity, and time reversal, states that all local, Lorentz-invariant quantum field theories (QFTs) with Hermitian Hamiltonians must respect the combination of these symmetries. Unlike nonrelativistic quantum mechanics, where time-reversal is linked to an unobservable symmetry in the mathematics, charge conjugation and parity are both also physical symmetries. Known physical processes violate all of the individual symmetries and their pairwise combinations, but no known process violates all three at once, so physics seems to respect the CPT Theorem. This is the most general sense in which **U** is time-reversal invariant: while, strictly speaking, it is not, reversing time just corresponds to changing conventions about particles and antiparticles and handedness. This kind of time-reversal asymmetry seems to have little to do with the other kinds of time-reversal asymmetries I will consider.

1.2 R: Reduction of the Wavefunction

R refers to the interpretation of the moduli of the complex amplitudes that evolve according to **U** as probabilities of measurement outcomes; a traditional term for it is "collapse of the wavefunction." **R** is just as critical as **U** for retrieving useful predictions from quantum mechanics, and the nondeterminism and randomness in quantum mechanics all comes from **R**. There is a long tradition in physics of attempting to "explain" **R** as a manifestation of **U**; I will first take **R** as a real physical event, considering the various attempts to derive **R** from **U** later. **R** has a number of curious features besides its indeterminism, including its apparent discontinuity, but the principal one that will concern me here is that it's not invariant under time reversal.

The nature of the invariance is subtle. I'll illustrate with an argument that doesn't show the invariance. Consider a series of measurements on some quantum system such that eigenstates of the operator corresponding to the measurement are not also energy eigenstates. Using quantum mechanics in the conventional time direction, each measurement corresponds to an operation of **R** that puts the system in some particular state, followed by time evolution under **U** until another measurement is made, at which time the system discontinuously flips to some state, which may or may not be the same state as the previous measurement. Can one distinguish this picture from its time-reversed complement? I could argue that it is by observing that *before* a conventional-time measurement, the quantum state may be in a complex linear superposition of eigenstates of the measurement operator, but *after* it cannot be in such a superposition. The problem with this line of reasoning is that quantum states themselves are not observable, but only the probabilities that certain measurements will have certain results; thus, it's possible to construct a time-reversed picture where measurement puts a quantum system in some state *before* the measurement occurs and the state evolves back in time according to time-reversed **U** (Aharonov, Bermann, and

Lebowitz 1964).

How, then, is **R** time-reversal asymmetric? There are physical situations where **R** provides correct predictions, but not correct retrodictions. A physical example of this, based on Aharonov, Bermann, and Lebowitz (1964)'s discussion of a Stern-Gerlach experiment, would be two spin measurements of a beam of spin-$\frac{1}{2}$ particles in different directions, for instance $|\Leftarrow>$ and $|\Rightarrow>$ followed by $|\Uparrow>$ and $|\Downarrow>$. A preselection of particles based on the first measurement, either all $|\Leftarrow>$ or all $|\Rightarrow>$, gives equal probabilities for the outcome of the second measurement, as usual; and moreover, the standard procedures of quantum mechanics provide well-defined probabilities for all subsequent measurements on that same ensemble of particles. Applying **R** to create a retrodiction using a postselection based on the $|\Uparrow>$ or $|\Downarrow>$ measurement would suggest that the probabilities of $|\Leftarrow>$ and $|\Rightarrow>$ are equal, but if an even earlier measurement put all the particles in one state or the other, this retrodiction doesn't hold! This represents a genuine asymmetry in **R**. I will return to the question of the origin of this asymmetry later.

1.3 Density Matrices

I want to introduce one final element of the mathematical formalism of quantum mechanics: quantum statistical mechanics requires a kind of generalization of the probability distribution over classical states called the density matrix. For a closed system, it describes a probability distribution over some set of possible quantum states, in effect multiplying each state by the probability it occurs in the mixture. The mapping between mixtures and matrices is not unique, so a general density matrix may be expressed as any of an infinite number of different mixtures. Formally, a density matrix in a finite-dimensional Hilbert space is the outer product of quantum states multiplied by probabilities p_i:

$$\rho = \sum_i |\psi_i > p_i < \psi_i|.$$

If the density matrix is diagonal in some basis, the probabilities represent the classical probabilities to find the system in that state when measuring it. In an open system, the density matrix may also represent correlations with the external environment. Using this interpretation, density matrices play a role in the decoherence approach that attempts to explain **R** in terms of **U** (see 3).

2 R, Boltzmann's H-Theorem, and the Second Law

The second law of thermodynamics started as a phenomenological observation about thermal energy and chemistry, that a particular property termed "entropy" never decreased in any spontaneous change. Boltzmann introduced the statistical interpretation of thermodynamics, reinterpreting entropy as a measure of how "unlikely" it is to find a system in a given state; he then proceeded to "derive" the second law by arguing that a system evolves towards states that are more likely, so entropy should increase rather than decrease with a probability depending on the relative unlikeliness of the possible transformations. Boltzmann wrote before the discovery of quantum mechanics, so he based his arguments on Newtonian mechanics, which like **U** is time-reversible and deterministic. How is it possible to derive a time-asymmetric conclusion like the second law from a time-symmetric dynamics? Loschmidt and Zermelo asked this question about an early version of Boltzmann's argument called the H-Theorem and pointed out that nothing in it specifies a direction of time, so it applies just as well into the past as the future Sklar 2008. This does not agree with anything like the usual picture of the physical world, because it implies that entropy should be higher in the past as well as the future. Thus, the most probable explanation for the current universe is as an unlikely fluctuation from a high-entropy state, with memories and planets created as part of this fluctuation only a few moments ago!

Notwithstanding this problem, for the most part Boltzmann's technique of treating entropy as a statistical property of the microscopic components of a macroscopic system works, in that it makes predictions testable and borne out by experiment. Thus, the question becomes why Boltzmann's approach works despite its formal problems. The traditional tack has been to rephrase it and ask why entropy is so low in the past, i.e. towards the beginning of the universe. Unlike Boltzmann's H-Theorem, which explained entropy increase as part of the dynamics, this perspective envisions it as a kind of global boundary condition.

Note, also, that low entropy alone is not enough: it must be in one of the, admittedly, much more common states where entropy will increase in the future, rather than one of the rare states where it will decrease even further. A simple physical analogy to this situation occurs with an open canister of gas, allowing it to escape into a vacuum in a typical entropy-increasing process; a classical approach models a dilute gas well enough, and as classical mechanics is symmetric under time-reversal, the time-reversed situation where a gas contracts from a vacuum into a canister is another possible situation. Without examining the microscopic properties of the gas particles, it's impossible to tell which situation is which. In a similar way, if the true dynamics is time-reversal symmetric, the universe could start out in a state where entropy will decrease. Observations in our universe show that it did not, thus this additional condition must hold. I emphasize this point to avoid a potential double standard: when reasoning about conditions on the early universe in this way, I can't appeal to the statistical arguments that the entropy-decreasing states are rare because I've already rejected the statistical argument's conclusion that the most likely explanation for the present is as an extreme fluctuation. The boundary conditions on the universe must have a different character than typical situations in the universe, because statistical arguments can't apply to the former the way they do to the latter.

A fair question to ask what Boltzmann's statistical reasoning achieves if it doesn't explain time-reversal asymmetry from time-reversal symmetric dynamics. One response points to the great practical utility of statistical reasoning. I think it also provides two other valuable insights. The first is that the difficulties Boltzmann faced highlight how hard it is to describe the manifest time-reversal asymmetric properties of the universe from a time-reversal symmetric dynamics. The second is that it moved the question to the right place: if the dynamics *are* time-reversal symmetric, then the universe's apparent asymmetry becomes a cosmological question.

From this kind of typical view that places the Second Law's time asymmetry in initial conditions, I'd like to examine the way Boltzmann's H-Theorem places the asymmetry in the dynamics. The Hamiltonian formulation of classical mechanics describes a system of N particles using a phase space of $6N$ dimensions (Γ-space), with three coordinates for each particle's position and three more for each particle's momentum. Rather than using the full space, though, the H-Theorem takes a probability distribution on a 6-dimensional space (μ-space), envisioning an N-particle system as represented by N points in this space. The problem with this approach is that the probability distribution over μ-space neglects the correlations present between the motions of particles in the full Γ-space, thus excluding the entropy-decreasing time-reversed situations (Zeh 1989).

What happens when quantum mechanics enters the picture? **U**, like classical mechanics, is deterministic, so the same arguments apply: only neglecting the correlations between the particles in a time-reversal asymmetric way leads to an increase in the entropy towards the future. **R**, however, destroys those correlations in just the way time-reversal asymmetric way the H-Theorem requires. Predicting the probabilistic outcomes of a measurement subsequent to previous measurements only requires the result of the most recent preceding measurement, not the entire history of the system. Meanwhile, a measurement on a single particle in a multiparticle system separates the single particle's quantum state from the rest of the system's, in the sense that it's possible to divide the whole state into $\Psi = \phi\Phi$, where ϕ represents the state of the single particle and Φ the rest of the system. With this division, the probability to find ϕ in some state, when measuring it, is independent of the probabilities for Φ, thus the measurement has destroyed the correlations. Together with Boltzmann's H-Theorem, this suggests a mechanism for the increase of entropy: **R**-type events reduce the wavefunctions of individual components of the larger system, destroying correlations and pushing the system to the most probable state.

3 Decoherence

Here, when I speak of "decoherence," I refer to a specific kind of philosophical perspective on the nature of quantum mechanics, which holds that **U** represents the real behavior of the universe and that **R** is a kind of illusion of perspective. The basic argument of decoherence is that treating quantum systems without considering their environments misses an essential part of the physics because of the inevitable quantum entanglements between such a system and its environment. Using the standard formalism, the density matrix of quantum subsystem coupled to an environment, where the environment has a classical-like diagonal density matrix, will evolve or "decohere" on some macroscopically short time-scale so as to become diagonal itself; this creates an apparent collapse of the wave function with similar characteristics to a hypothesized real collapse (Zeh 1989).

One obvious question to ask of the decoherence picture is what happens when one considers the universe as a whole: by definition, the quantum state of the universe has to contain everything that exists, so there's no environment to cause it to decohere. Interpretations of quantum theory like the many-worlds perspective work around this issue by trying to explain away the apparent classical nature of the macroscopic world despite the power of quantum mechanics at explaining the microscopic world. I think these ideas give important ideas in the perspective that **U** is the only part of the quantum formalism with physical reality, though, so I will offer a different viewpoint.

If **R** is not a real physical process but only the result of decoherence, I can interpret the existence of the familiar classical universe as a condition on the universe's initial quantum state. This works in a way analogous to the Boltzmann's argument placing time-reversal asymmetry as an initial condition on the universe, as it requires that the universe's initial quantum state has to be "special" in such a way as to give rise to classical-like behavior rather than the much more probable, according to statistical reasoning, outcome of an entangled nonlocal mess. In this way, then, I think decoherence provides much the same benefit as Boltzmann's original statistical arguments: it clarifies the question and places the problem at the right location in the physical theory.

4 Conclusion

My goal in this essay has been to compare a picture where there's a time-reversal asymmetry in the dynamics with the available experimental and theoretical evidence for and against that proposition. I've shown that at the least a time-reversal asymmetric dynamics is compatible with the standard formalism of quantum mechanics, as well as possibly providing a motivation for a connection between the arrow of time defined in the asymmetry of the measurement process, **R**, and the thermodynamic arrow defined by the increase of entropy. Different, more orthodox perspectives place the source of the time-reversal asymmetries in both quantum mechanics and statistical mechanics as initial conditions on the universe. At the present, however, the lack of a theory of quantum gravity makes the application of quantum mechanics to cosmology difficult, and the similar lack of a consensus ontology for quantum mechanics makes locating and understanding its time-asymmetry equally difficult. Thus, both proposals, of a time-reversal asymmetry in the dynamics or in the cosmological conditions, seem viable. The latter, however, receives much more attention when I don't see any concrete reasons for preferring it over the alternative.

I will conclude by offering one further argument in favor of placing the asymmetry in the dynamics. The broken conservation law governing entropy is a local law: if there were any significant violations of this kind of locality, it's doubtful anyone would recognize the Second Law since experiments can't verify that an entropy decrease on Earth has caused an entropy increase on Saturn or indeed anywhere else in the universe. Quantum mechanics, however, permits nonlocal correlations, and while the exact dynamics of **U** conserve the von Neumann entropy, conspirative nonlocal correlations can cause local decreases of entropy while preserving the global entropy. This adds a further stringent condition to the cosmological initial conditions: not only must the universe start out in a low-entropy state such that entropy will increase in the future, but the entropy everywhere has to obey locality.

References

Aharonov, Yakir, Peter G. Bermann, and Joel L. Lebowitz (1964). "Time Symmetry in the Quantum Process of Measurement". In: *Physical Review* 134.6B (June 1964), pp. 1410–1416.

Penrose, Roger (2004). *The Road to Reality. A complete guide to the laws of the universe*. New York: Alfred A. Knopf.

Sklar, Lawrence (2008). *Philosophy of Statistical Mechanics*. Ed. by Edward N. Zalta. URL: `http://plato.stanford.edu/archives/fall2008/entries/statphys-statmech/`.

Zeh, H.-Dieter (1989). *The Physical Basis of the Direction of Time*. New York: Springer-Verlag.