The Nature of Space

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Abstract

The role of space, whose structure and properties dictate all characteristics of matter and forces in it, has been underappreciated by physics. That empty space has a property of a rigid solid constitutes the central paradox of science; for 150 years it has remained unresolved. The work has been neglected due to physics' century-old decree that declared space empty, which was an act of frustration with ether models that could not resolve the paradox. Pressing ahead without solving the problem led to the current situation where paradoxes have become physics' distinctive feature and matter and space are fused into an inseparable totality of fields. Mathematical models attempting to describe this totality abound, and yet there is no model of space. In its place is a tradition that discourages discussion of the structure of space for fear of resurrecting old ether theories. This essay attempts to open such a discussion and offers an example of how the topic can be approached with very few assumptions in mind. It shows that, consistently applied, a simple organizing principle can lead to a straightforward solution to the old problem and reveals a dynamic, 4-dimensional, vibrating structure, in time, that conjures up an image of a 4D ocean, the 3D surface of which is the visible Universe.

1. Preamble

Once upon a time a girl sat under a tree in an old overgrown garden. She leaned against the trunk gazing at the sky. She came there often, to hide from noise and bustle. The place was always quiet, save for the chirping of birds, leaves rustling in the breeze and the buzz of insects. The trees there were big and meadows under them unkempt. At daytime she watched clouds and on a clear night, the moon and stars. Most of the time she dreamed. Of various things.

That day the sky was pristinely blue. Not a cloud in sight. And so she lapsed into a fantasy about a world she wanted for herself. A whole universe, just the way she wanted. Immersed into her visualization, trying to decide how many planets she wanted around her home star, she did not notice how a rook descended from the sky. The bird perched on the branch overlooking a shaggy meadow where tall grasses and weeds mingled with feral flowers. Having noticed her, he made a sideway step in her direction and watched for a while. Then he leaned forward and pronounced rolling his r's, "Daydreaming?"

Startled, the girl looked up. Sunlight trickled through foliage and bounced off the jet-black feathers with bluish iridescent splashes. Beautiful, she thought. The rook cocked his head to a side and pierced her with his obsidian eye. "What were you daydreaming about?" he repeated.

"Oh!" said the girl pretending to be unfazed by her encounter with a talking bird. She knew this game. She too sometimes liked to shock people. For fun. And so she replied truthfully with a note of challenge in her voice, "The world I'm going to make for myself one day. A whole new universe!"

"Ha!" said the rook, astonished. "Not bad. But… where are you going to put it?"

Bird brain, she scoffed mentally. “There!” With a sweeping gesture she pointed to the sky, undoubtedly implying the immensity that lay beyond.

“Uh-uh,” the rook shook his head. “You can’t put it there.”

“Why not?” she objected. “There is tons of empty space out there!”

“Yes,” said the rook calmly, in contrast to her indignation, “but all that space is already occupied.”

“Occupied? By whom?” She did not believe him.

“Not by whom but by what,” retorted the rook in the tone of a middle school teacher. “It is occupied by *this* world. All the space there is belongs to *this* universe and can contain nothing else.” Seeing that he had her attention at last, he continued in the same vein. “So if you want to make a *different* world,” he raised his wing didactically pointing his leading edge primary skyward, “you have to start with the space for it.”

The girl thought his worlds over. “But… how do you make space?” she asked at last, seeming discouraged.

“This I don’t know,” said the rook. He took his time to daintily adjust a feather on his breast. Satisfied, he looked up and said casually, “But you can ask around.” With these words he flew off the branch and soared into the sky. Having reached the height where light hit his plumage at just the right angle igniting it in bluish glow, he leisurely spiraled down to the tree. He knew she was watching him. And yes, she envied his prowess in navigating air currents. “Show-off,” she muttered.

The rook circled the tree one last time and cried out in a way of goodbye, “Remember! As will be the space you make so will be the world in it!”

And that was it. The bird was gone.

Ever since, the girl was preoccupied with the idea of space. What is it? How is it made? Is it true that its structure and properties determine all characteristics of matter and energies as well as all their interactions? That’s what the rook implied. Or did he?

2. The paradox of space

In ancient tongues the word for space was emptiness. As such it was easy to overlook. What’s *in* space has always been far more interesting to us, while space itself appeared as a passive container waiting to be filled. Laymen and physicists alike, we take space for granted and treat it as given, expecting it to be continuous and invariant in all directions, scales and dimensions.

To our immediate senses space *is* empty, which goes against common knowledge that it is also the medium for electromagnetic waves, from light to warmth, to microwaves, TV, phone and radio signals, the things we experience and use every day. What’s more, for 150 years it has been known that these waves are transverse and propagate at a phenomenal speed of ~300,000 km/s, which implies that space has a property of a solid whose rigidity surpasses the strongest materials known on Earth. This incongruity between the perceived emptiness of space and its implied solidity constitutes the central paradox of science. For 150 years it has remained unresolved.

True, about half a century was spent on trying to reconcile it. The result was an array of ether models[1] culminating with what is known today as Lorentz Ether Theory[2]. It stood on a strict separation between matter and ether, whose states were described by the dynamics of the EM field it contained that also served as a mediator between the electrons. In 1905 Einstein rearranged the theory[3] and, fulfilling an old Poincaré wish[4], proposed to disregard the wave-medium property of space on the grounds that mathematics was sufficient without the need to invoke it. At the time, the frustration with the lack of a forthcoming solution to the old problem was at its height; and so when the new theory boldly dismissed ether and declared space empty, it must have come as relief. A decade later, General Relativity paved the way to accustoming us to the idea that *emptiness* can be curved, in 4th dimension. That dimension was said to be time, and so the ancient notion of space was replaced with spacetime infused with fields.

The direct consequence of Einstein's bold move is that today physics largely consists of complex calculations of fields, curvature, wave functions, energy density and fluctuations of what otherwise is dogmatically deemed empty. The situation is exemplified in a naïve layman's question posted on a physics forum recently, which provoked a bitter laugh from the residents. It was *Can you tell where matter stops and space begins?* In contrast to the macro scale of our immediate experience where it is easy to tell where matter stops and space begins, on the quantum scale matter and space are fused into an inseparable totality of fields. Today, mathematical models attempting to describe this totality abound and yet there is no model of space that one can speak of. In its place is a tradition to discourage discussion of the structure of space for fear of invoking old ether theories.

When lost, the right thing to do is to retrace the steps to where the wrong turn was made. In retrospect, physics' dismissal of the old paradox of space was that wrong turn. Replaced with the wave-particle duality that became the cornerstone on which quantum physics was built, it paled in comparison to the array of paradoxes that logically followed and soon became physics' new norm. But the original problem did not go away and so it still stands unresolved. Worse yet, with the passing years it has been denigrated to the role of the proverbial elephant in the room. The unacknowledged question in plain view is: how can space be both empty and solid?

3. Structure and properties of space

Space has structure. The observed 3 dimensions speak plainly of it. That empty space is flat and that presence of matter curves it also implies the underlying structure. What forces define it?

According to the big bang theory, the universe sprang up out of a singularity, which is a place devoid of dimensions yet capable of having them all. This implies that the perceived spatial dimensions are not given but constitute a particular form of energy. Thus when a universe is born, the first expression of its energy is the geometry of space.

Without making any assumptions about the specifics, if we imagine *empty space* as some perfectly even and regular *structure* then any imperfection in this structure, any bend or wrinkle in it, will appear as *something in space*. This view on things, from the position of the structure of space alone, differs from the prevailing view where the 3 dimensions are treated as given, shaped up by the presence of matter in it.

A striking property of empty space is that light propagates through it at a constant speed, regardless of the speed of the emitting source, while matter moves in the same direction and speed determined by the initial force, until acted upon by an external force.

Assuming nothing about this structure and only following the ancient notion of *Cosmos*, which means O*rder*, we can propose that the main organizing principle of the structure is its propensity to keep as even and regular as possible. Applied to each contiguous segment of space, this principle becomes the source of all movement: any deformation introduced into the structure locally is immediately expelled; the same happens in the next segment, then the next, and so on, like in a game of hot potato. A deformation is expelled into the direction that gives, which in a perfectly balanced structure is opposite from where it comes. Thus in empty space light and matter move in straight lines. Another implication of this principle is that *nothing is ever stationary* with respect to the structure.

The propensity of the structure to keep as even and orderly as possible would also imply that it may absorb some of the energy of light propagating through it, to even out its minute tensions. This would contribute to the light's red shift. This would also mean that light has a stabilizing effect on the structure.

As to what this structure is made of, we could go with Poincaré idea of a *fictitious fluid*[5] or take the Faraday vision of vibrating *lines of force*[6], which happened to nicely resonate with the leading theories of the day. Or we could combine the two in an image of incompressible, perfect fluid, consisting of vibrating strings. Details don't matter. The important thing is that we get a dynamic, vibrating structure that *defines* space.

The natural question arises, how many dimensions this structure should have? The hypothesis I would like to advance is that the most natural configuration for a dynamic vibrating structure that *wants* to be as even as possible is 4D. I base it on the fact that, among many things that make 4D different from all other *n*-spaces, it houses the most regular polytopes[7] and, unlike the next runner up, 3D, its number of dimensions is even, which permits a greater number of symmetries. The idea is that, given an *n*-dimensional dynamic structure that seeks to harmonize the vibrations of its components, the 4D configuration corresponds to its lowest energy state; and, while it can be forced into configurations of higher dimensions, as soon as local pressures let go, it will naturally precipitate into 4D. Thus in line with big bang theory, we could assume that initially the structure had infinite number of dimensions, but once it cooled off, it settled into 4D. (The question of why don't we see the 4^{th} dimension is coming up.)

4. Old paradox revisited

In retrospect and armed with knowledge gained in the past century, the solution to the old problem is straightforward. Leaving time out of it for now, we set out in 4D.

We deconstruct space into its 2 paradoxical constituents. This gives us a 4D *structure* suspended in *empty* 4D space. Supposing the Universe is finite, the simplest shape for a dynamic structure that *wants* to assume its lowest energy state is a hypersphere (4D version of a sphere). In line with the current theories, we can call its 3D surface a *membrane*, or *brane* for short, which is appropriate, considering that it separates the bulk of the structure from emptiness that engulfs it. The membrane is where light waves propagate and where Maxwell equations live. The matter, i.e. the stuff with intrinsic mass, is confined to the outer, empty side of this membrane. The nuclei *glide over it* supported by their electron clouds, which also serve as the interface between nuclei and space. (The question of the size of the engulfing emptiness as well as the nature of nuclei confinement we address later.)

This is the basic conceptual setup. Please note that not only does it resolve the old paradox but also lets us say with certainty "where matter stops and space beings": what we call space is split into the bulk of the structure and surrounding emptiness. Bounding the structure is the membrane, which is the *event plane*, the stage where all interactions between matter and space take place. As an interface between the two contrasting constituents of space, it allows for 2 basic interactions, normal and tangent, exemplified in the following two images, which, however, come with one caveat. The first image is a pebble falling into a pond sending forth concentric circles of ripples. The second is a view of a speedboat cutting through the mirror of a lake leaving a fan wave behind. The caveat is that, unlike the surface of the lake, the brane is 3-dimensional and is viewed from 4^{th} dimension.

Overall, this setup is in line with the models with large extra dimension[8], such as ADD[9] that allows 1mm for the headroom. However, because the EM radiation is *entirely confined to the brane*, it does not matter how big this empty dimension is, except that it should certainly be larger than the largest nucleus, the size of which in 4D is uncertain (a nucleus appears so small from our 3D POV, because it lives outside our 3D, only touching it). This goes in contrast to all other extra-dimensional models preoccupied with various methods of compactification, driven by the assumption that, if extra dimensions exists, they should be equally accessible to both matter and radiation, as the observed 3. The other assumption of the current extra-dimensional models is that all extra *spatial* dimensions, *beyond the first 4*, exist a priori in each and every point in space, rather than springing up into existence in response to the critical energy density in space.

The important feature of this setup is that an interaction, or *an event*, results in 3 distinct *timelines*: one goes into matter in *empty* 4D and the opposing force is split in 2 between the 4D bulk of the *structure* and its 3D *surface* (Fig.1). Each timeline is measured in its own *ticks*, or quanta, which are the cycles, or full rotations, in each adjacent subspace.

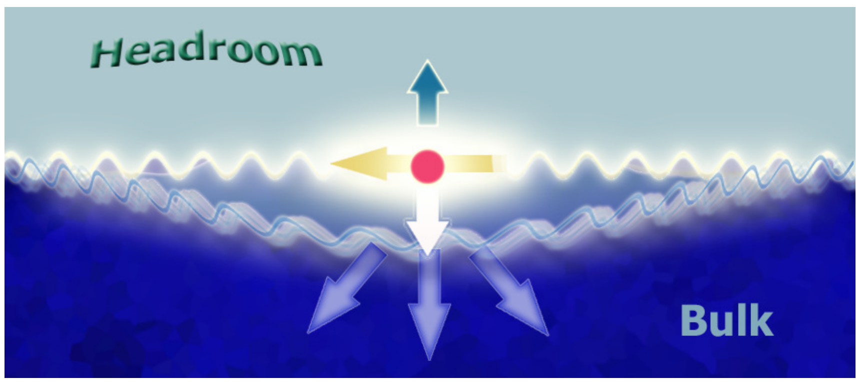

Fig.1 Conceptual diagram of the event plane. The white wavy line is the edge-on view of the brane that separates the bulk of the structure from emptiness. The red dot is an event. The yellow arrow is the timeline tangent to the brane, interacting with EM forces in it. The white arrow down is the timeline of the normal going into the bulk, displacing the brane. The radian length of the displacement arc corresponds to mass (3 arrows down are the reminder that the normal is perpendicular to all 3 planes in our 3D). The arrow up is the timeline that goes into 'matter' (not shown) in the 4D of the headroom.

Essentially, what we perceive are *projections* from 4D on each side of the membrane onto its 3D. This accounts for quantum uncertainties and explains the half-integer spin of fermions: a full rotation in 4D is seen from a 3D POV as 720° or 2 turns necessary to return to the original state. Charge too is defined by projection onto the membrane, while mass is the displacement of a 4D volume beneath it. The particle zoo of the Standard Model makes up various aspects of the underlying geometry.

As to where we fit into this setup and why don't we see it, here a reminder is in order: All information about our world, starting with light and warmth and ending with phone and satellite signals, we get via EM radiation *and it is confined entirely to the surface of the hypersphere, which is our world,* which is the membrane.

In a way this setup resembles a 3D display, a sort of a touch screen that shows to some collections of atoms what other collections of atoms are doing. Imagine, each atom touches the display sending forth disturbances throughout. These disturbances reach collections of atoms attached to it elsewhere, and some of those collections may say, *wow I see things!* What they can't see is the emptiness that surrounds them in which they are free to glide just over the display on their electron clouds.

4D is notoriously difficult to visualize, but the model is easily grasped with one dimension removed. This warrants a trip to Flatland[10].

5. Flatland cosmology

Flatland world is the surface of a sphere (with some headroom). The sphere's 2D surface is called the Plane and it is continuous and invariant in both directions, each point of it being equidistant from the center of the sphere. The Plane is where the Flatland version of radiation lives.

A typical flatlander, Mr. Square lives on the Plane (Fig.2). All the information about his world he gets via the radiation and it is *confined entirely to the Plane*. It allows Mr. Square to see his surroundings and enjoy the warmth of a summer day or be chilled by night. To Mr. Square, the 2D space of his world appears perfectly flat and empty and he believes that he moves in it with ease. Unbeknownst to him, he and all Flatland matter actually exist in 3rd dimension. Oblivious of this fact, Mr. Square only perceives what is attached to the Plane and he is riveted to it in more than a figurative sense.

If we take a cross section through the Plane with Mr. Square in it and look at it from our 3D POV edge-on, we will see that instead of living *within* the Plane as befits a plane figure proper, Mr. Square actually sits on top of it, in the 3rd dimension. If we zoom all the way to the Flatland nanoscale, we will see that nuclei comprising his body exist *outside* the Plane (Fig.3). Our privileged perspective also allows us to see invisible to Mr. Square dynamic *lines of force* that crisscross the Plane throughout, making it impermeable to matter. The nuclei move along these lines as if on rails with their electron clouds serving as rollers. Interestingly, Flatland photons also move along the same lines. Viewed from this perspective, the Flatland version of the double-slit experiments involving both light and matter would not seem mysterious at all!

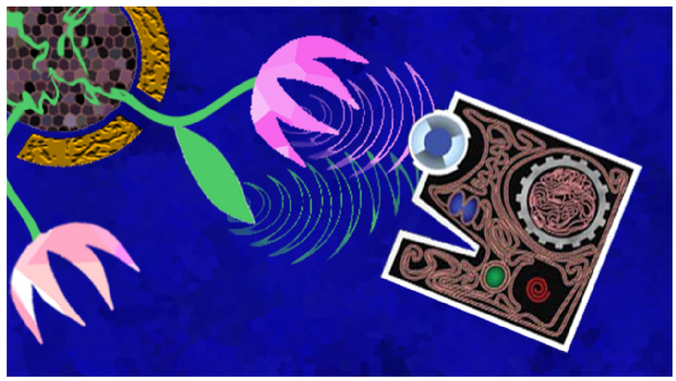

Fig.2. Mr. Square looking at a flower. Light waves bounce off the electrons of the flower and propagate through the Plane reaching the electrons of his eye.

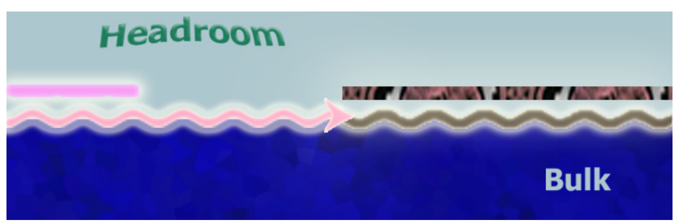

Fig.3. Edge-on view of the cross section of the Plane (wavy line). On the left is the pink flower and Mr. Square on the right. Both glide just over the Plane in the 3rd dimension.

6. Mass and gravity

While Flatland analogy is helpful in visualizing the 4D model, it is worth remembering that the Plane is analogous to the brane only in the sense that both are surfaces of a sphere and hypersphere respectively. Being 2-dimensional, the Plane allows for only one of the membrane's 2 inseparable components. But this comes with one advantage: it makes the Plane very much like the surface of water. In fact, Flatland is best modeled as a surface of perfect fluid. The Plane's water-like surface tension results in it bending so smoothly that Mr. Square cannot tell whether his 2D world is curved or not. To him it appears perfectly flat throughout and he can only infer the curvature from how light bends around massive objects, and he can feel it in the pull of gravity. It is only from a 3D POV that the curvature of the Plane is readily seen.

Regarding mass and gravity, it is natural to assume that in Flatland, just like in our world, it is the intrinsic mass that confines matter to the Plane; i.e. mass is the force that pushes on matter, pressing it into the surface toward the center of the (hyper)sphere, displacing a volume, increasing the surface area and causing it to curve. The resulting curvature is gravity. Some may object that I use gravity to explain gravity. But in this model there is no gravity per se. What we call gravity emerges entirely from the interaction of mass (which is displacement of volume) with the surface tension of the structure *wanting* to minimize its surface area.

Thus no gravitational attraction exists between an electron and a nucleus. The electron cloud integrates the nucleus into the Plane, filling in the dent it makes and leveling it out with the surface. In a 4D world, from a 3D POV, it looks like a swarm of electrons are busy mending the hole that the presence of a nucleus makes in the 3D fabric of space. Because all motion is dictated by the structure *wanting* to be even locally, a naked nucleus is not tolerated and gets kicked around until it finds some electrons.

In this model mass corresponds to the displaced volume described by its bounding surface. The shape of this volume reflects the interplay of the tangent (tension) and normal (mass) (Fig.1). At leisurely speeds the angle of momentum to the tangent and normal is balanced between them. At higher speeds this angle shifts toward the normal, which corresponds to the increase in mass.

Gravity is the curvature and 'attraction' is the radian length of the arc between objects. Due to the surface tension, at micro scales, the curvature in radian length is practically 0. A single atom does not make much of an indentation in the surface locally, but on large scales all atoms combined contribute to the radian length of the arc of the displacement of the surface from where it would be if it were empty (Fig .4).

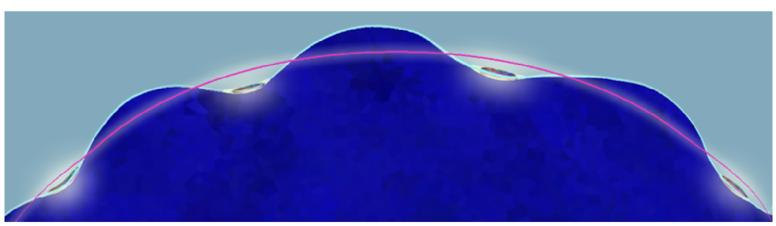

Fig. 4. Segment of a cross section of Flatland's sphere, macro scale, showing the curvature of the Plane (greatly exaggerated for clarity). Matter aggregates in the troughs while the crests are void. The red line shows where the Plane would be if matter did not displace it. If we sum up this curve's troughs and crests throughout, the resulting 2-space would be practically flat, in line with Mr. Square's assessment of his world.

The incompressible-fluid-like tension of the Plane has an interesting consequence: the depressions in the Plane locally cause it to bulge outward elsewhere. If negative curvature corresponds to gravitational attraction, then positive curvature should imply repulsion. This would mean that, at least in Flatland, gravity can be both attractive and repulsive. The attractive aspect of gravity would live in the troughs of the curve and the repulsive, in its crests.

The crests in the curve of the Plane make up the Flatland version of intergalactic voids, the vast stretches of emptiness that also characterize our world. In our world, enormous empty voids predominate, while matter appears *squeezed* in between[11]. What looks like a hillcrest in Flatland would look like a bubble in our world, which is in line with the observed foam-like structure of the cosmos: galactic filaments are squeezed between vast empty bubbles. This implies that at large scales the major players are not aggregations of matter but voids that surround them, *pushing* on them. Thus one should experience a deceleration when approaching a void, as if climbing uphill. Should a galaxy be hurled into a void by some force, the repulsive aspect of gravity there would first deform and then take apart its structure.

It so happens that in our world, very few galaxies are seen within voids.[12] This is in line with this model which predicts 2 outcomes for those galaxies: either they will turn out to be a mirage, i.e. actually located on the perimeter of the void and only appearing inside (possibly due to some calculations quirks or overlooked lensing), or we should first see the deformation and then, if a galaxy lingers in a void, disintegration of its structure[13].

The other interesting implication of this model is that the visible universe should appear much larger than it actually is. This is because space is mostly empty and so not much is there preventing light from going around more than once. Thus, in principle, looking in 2 directions of the sky, one should be able to see the same object twice (the trick would be to determine what *directions in our 3D* correspond to the *opposite directions on the hypersphere)*.

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7. Expansion

The current assumption that Universe is expanding is largely based on the redshift. In this model *empty space* is synonymous with a perfectly regular, balanced structure. A balanced structure will remain balanced after light passed through it. But if it had some tensions, it will absorb some of the light's energy to even them out. Thus light has a supporting influence on the structure of space, smoothing it out like an iron cutting through a creased cloth. (And if the cloth was smooth to begin with, no harm is done!) This alone could account for a good portion of the observed redshift.

The other contributor is the trivial Doppler effect due to receding from each other bodies. First, the differences in these two causes should be apparent in signatures of the waves themselves. Second, to the question of *why the bodies are receding*, the simplest answer is in line with this model of space that conjures up an image of a 4D ocean, on the 3D surface of which we live. Imagine, waves propagate, the brane wobbles, a swell may be coming in… When a swell is coming, to the creatures living on the planets in the affected intergalactic filaments it would appear that space around them is expanding. And that is exactly what goes on *locally*. But it hardly means that the whole ocean is expanding.

In this model, the expansion of the universe is equivalent to the increase in the surface area of the hypersphere, which implies increase in its volume, or bulk. Could it be that something, or someone, is pouring tremendous energy into our world? Maybe it is draining from another world? These questions force us to consider if Universe is an opened or closed system, with all the implications to energy conservation laws. Another reason for expansion could be, in line with the prevailing idea, that the structure is still cooling off after the initial bang. Out of these 3 possibilities, i.e. a *swell*, an open-system universe, or the bang still in process, the first appears the simplest.

8. + time

In this model space, time and energy are intimately linked. This stems from the simple idea that the structure of space is made of energy. A unit of it can be visualized as a luminous *point* whose energy is expressed in intrinsic oscillations, or cyclic changes in its energy state. Each oscillation defines some unit of spacetime as if carving out a bit of order out of disorder, even if temporarily.

It is natural to assume that a luminous point is imbued with some *total energy* and that it can spend this energy either in many small increments or in a few spectacular outbursts. However, because we measure the time of a process in terms of another process (ex. life is measured in cycles of Earth around Sun), having nothing to compare our *point* with, we can't tell how long an oscillation is, either in the time it takes nor distance is traces. We can't even tell whether one cycle differs from another! We can only assume that a *point* will last only as long as its total energy permits. It is in this sense that time becomes equivalent to energy. With the *length* of oscillations undefined, there is only one way to measure time with certainty and it is to count the oscillations themselves. Thus time of a *point*, as well as its energy, can be expressed in the number of its oscillations, or ticks, whose lengths, in either temporal or spatial units, are undefined.

The picture changes when we consider not a single luminous point but a myriad of them mingling in tight constraints of a singularity. If we propose that they can preserve some of their energy when they bump into each other (provided that subspaces they define happened to coincide), then this could serve as the organizing principle that would quickly lead to the emergence of a vibrating structure that defines the geometry of some spacetime. And so the ontological question of why the structure emerged can be answered simply: because those luminous points that did not organize with others eventually spent their energy and thus ceased to exist, while the organized points still persist in exchanging most of their energy between themselves back and forth. Tick-tack.

This brings us back to the organizing principle of the structure: all space *wants* is to last as long as possible, to maximize the number of its ticks, because that is how time is ultimately measured. For this it must harmonize the vibrations of the components of its structure, which is accomplished by keeping it as orderly as possible.

9. Conclusion

In retrospect, it should have happened 100 years ago. The 19th century was the time when the geometry of higher dimensions was vigorously developed. The ideas spilled from universities into popular culture and captured people's imagination. In the period spanning the last decades of the 19th century through the first of the 20th, the theme of Fourth Dimension permeated all strata of society. References to it were made in literature and philosophy; it inspired great and works of music and art[14]; and the marketplace babbled with wild speculations. It seems, the stage was set and the time was ripe to accept the reality of a 4th spatial dimension, just as before there was a time to accept that the Earth was round. And yet it did not happen.

Those who experienced firsthand the frustration with the immobile ether are no longer with us. Thus the tradition to discourage discussion of the structure of space is hardly justified. This essay attempts to open such a discussion and gives an example of how the topic can be approached with very few assumptions in mind. It shows that, consistently applied, a simple organizing principle can lead to a solution to the old paradox of space and reveals a coherent and visualizable in detail scheme of the world which also allows for what is sorely missing in physics today, and that is a little sign with an arrow, *you are here*.

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