<u>Unbaked Layer Cake</u>

I feel the need to call this what it is: a play in speculation. It has nothing rigorous to back up the claims, it simply tries to be an exploration of thought for its own sake, not making any claims of Truth. On top of that, it's likely a fair bit naive, considering my level of knowledge, but hopefully someone finds value or at the very least entertainment within.

Let's start with the premise that the universe is mathematical in nature, like the Mathematical Universe Hypothesis (Tegmark). If you are unwilling to take on this assumption, just think of this whole paper as an analogy. Now what would this mean? It would mean there exists (an) isomorphism(s) between mathematical structure and the structure of reality. Perhaps mathematical structures that have yet to be discovered even. It would mean that the universe has no other properties besides mathematical properties.

How might this look? It could potentially be a formal encoding of the axioms and rules of physical theories. This was one of Hilbert's goals for his program to formalize all of mathematics. He wanted "to treat those physical sciences, in which mathematics play an important role, by means of axioms like geometry.", at least according to (Tambakis). This would simply be a mathematization of the rules we have come up with. Some view our models as simply approximations of reality, so this may not wholly satisfy the masses, but would be a good start. Could we get any further than this, and create a bijective mapping of mathematical structure onto reality? How could we go beyond our models? There may be inherent limitations on doing this based on the fact that we can only talk about reality through symbolism of one form or another. To create a mapping onto reality beyond a simple model would require something else. Not to mention the 'true' nature of reality is up for debate, which is why I take as an assumption in this paper that "reality is mathematics".

Is there any evidence for this? This is no rigorous proof, but if and only if an example of Gödel's undecidability is found within nature, can we claim that there is something more fundamentally mathematical about reality, than the math simply being a useful tool. This is because a truly undecidable result would only be possible if there existed a true mathematical structure underlying reality. In fact there are a few instances of undecidable results being found within quantum mechanics (Cubitt, Moore). Now the question pops up, are these results due to the fact that we are using mathematics to describe quantum mechanics, or are they because nature is fundamentally based on mathematical structure? The first instance suggests the former and the other the latter. It is much easier to entertain the idea that there are at least some structures and properties of our universe that behave isomorphically to a mathematical structure even if the universe in its entirety does not. So this may be all that (Moore) is suggesting, but nevertheless it is interesting all the same.

Why would we want to do this? It leaves open the possibility that we could essentially understand the whole of the cosmos, if we could just find a way to formalize it all. The obvious drawback is that Gödel's incompleteness theorems bite this dream in the butt. Even if we could formalize reality, there would exist statements that could not be proven nor disproven, paths that could not be followed, theorems that must be taken on faith. But with this in mind, in mathematics, we always assume the axioms, so it may not be too much more to ask that these undecidable theorems themselves be taken on faith.

If this were true, what would be the consequences of it? Here is where I would like to start building some speculation upon this speculative claim, for fun. There is a common theme that goes hand in hand with a reductionist view of the world, amusingly portrayed in an xkcd comic ("Purity"), that physics is just applied math, chemistry is applied physics, biology is applied chemistry, psychology is applied biology and so on and so forth. The structure in this view looks like a layer cake where each layer below is "more fundamental" than the one on top of it. Taken from the point of view that "everything is mathematics", mathematics is the most fundamental layer of the cake, from which everything is built. In other words subsequent layers of cake, physics, then chemistry, then biology etc., could be seen as systems of theorems, or derivations from more base principles, all the way down to "the ultimate axioms of nature", if such things exist. For example, the rules of, say, chemistry would be considered theorems, rather than axioms, of the underlying mathematical structure, built from the axioms of fundamental physics. Now, some of these theorems would be emergent properties. Because emergent properties do not exist at the lowest layer of the cake, emergent properties must be viewed as theorems of the original axioms.

The Cambridge Dictionary of Philosophy (Dictionary) describes two forms of emergence, descriptive and explanatory. Descriptive emergence means "there are properties of 'wholes' (or more complex situations) that cannot be defined through the properties of the 'parts' (or simpler situations)". Explanatory emergence means "the laws of the more complex situations in the system are not deducible by way of any composition laws or laws of coexistence from the laws of the simpler or simplest situations". There are many different types of emergence as characterized in (Fromm), but that is not exactly what we are to discuss here. The ones I wish to discuss are the ones that are not deducible from simpler pieces. Because certain emergent properties cannot be explained from their constituent parts, from the point of view I am taking in this paper, they must be examples of Gödel truths. In other words, truths of reality that can not be derived from more base principles. They could be seen as fundamental in their own right, though not necessarily universally pervasive. These would be equivalent to Sabine Hossenfelder's definition of strong emergence, though with the additional mathematical basis (Hossenfelder).

To remedy the situation, that, of course, may not even exist, I borrow an idea from last year's essay contest winner (Adlam). The idea that we need to change our ideas of what fundamental means, but I take it one step further. I am suggesting more of a loop of fundamentality, taking the strongly emergent properties as fundamental pieces themselves, adding them to the assumptions or axioms of the overall picture. No layer of the cake would then be necessarily any more fundamental than another, because each layer could have a strongly emergent property that would also be considered fundamental. Each layer would then be reliant on pieces both above and below. A layer cake looping back on itself, folding, and intermingling almost like the cake batter before it's been baked. Creating a chaotic image of the way in which the different layers are connected. The original fundamental pieces would then still be fundamental, but layering them from low level to high level no longer makes as much sense, because in this view, certain fundamental properties would also be high level properties. The idea of stacking the layers from the bottom up could still be done, but it may not provide the clearest picture of how everything is connected. It may never be that we find all the connections between everything, because this could easily become chaotic. What I mean to say here is that these strongly emergent properties could be affecting lower and higher level phenomena, and to find all the ways in which everything is affected may become difficult. Not to mention, taking these new strongly emergent properties as fundamental, builds new structures that are potentially just as susceptible to Gödel's incompleteness theorems themselves, creating a never ending line of new strong emergent properties. The only thing that could end the line in any practical manner would be a lack of any 'interesting' new strong emergent properties.

Again, I wish to reiterate, Gödel's theorem states there exists truths that can not be proven. If the only properties that exist are mathematical in nature these unprovable truths are synonymous with strongly emergent properties, so strongly emergent properties must exist. I suggest these properties should then be taken as fundamental in their own right and investigations could be done to see how they may be affecting phenomena at different scales.

Even if one is unwilling to take on the assumption that 'everything is mathematics', the argument presented here could still prove useful in thinking about the different scales of reality, and how certain emergent properties seem to be unexplainable. If they are taken to be fundamental properties even though they are emergent on a higher level, perhaps some progress could be made in new directions. It may not lead anywhere, but it could at least be interesting to pursue, in the spirit of the pursuit of mathematics, where certain assumptions are made simply to see where they lead. A consequence of this idea, is that there likely exist strong emergent properties in nature even if mathematical properties are not all that exist. If only certain properties are mathematical in nature, but are sufficiently complex, there would still exist properties which are undecidable, and could be taken as fundamental. That isn't to say these properties would be interesting or novel. Perhaps some of these are already known and already taken to be fundamental, since science makes discoveries from non-axiomatic approaches.

How would a strongly emergent property be identified if it were found? Similar to how many of our scientific theorems are never proven, but are simply given more evidence to support them always looking for something to disprove them, the strongly emergent properties would never be able to be explained by lower level properties. We could continue to search for an explanation, but if none are found, then they would have to partially be taken on faith. Additionally, using these emergent properties in new models to explain certain other phenomena would help solidify them as strongly emergent, especially if higher level emergent properties could be used to explain lower phenomena. Is this possible? The world may never know.

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Thank you Keaven!!