Physics, mathematics: using information theory

Christophe Tournayre

Abstract

In this short essay, I propose to explore the relationship between physics and mathematics using components of the information theory.

About the author

I am passionate about insights. In my professional life, I am designing supply chain solutions. I graduated in European Logistics Management.

Introduction

As the subject of this contest highlights, the relationship between mathematics and physics is somewhat not clear.

- On one side, mathematics principles are used and proved to be very effective in improving our knowledge of the physical world.
- On the other side, it is not a one rule fits all. Using probability to calculate the speed of a car is not exactly the right tool.

Why some mathematical models are useful to understand physical objects while others are not?

Kolmogorov work

To answer this question, I propose the generalization of Kolmogorov complexity principle.

In information theory, Kolmogorov complexity is a measure of the computability resources needed to specify the object. Further information regarding Kolmogorov complexity can be found on the internet [1]. I also want to introduce Kolmogorov-Smirnov Test which tries to determine if two datasets differ significantly [2].

What I found interesting in Kolmogorov work is that it measures information variance. I believe mathematical models are applicable to physical objects when the information variance between the two is null.

The challenge is to reduce the information variance between the mathematical model and physical objects towards 0. Here again, Kolmogorov gives us clues on how to achieve it. Reducing information variance is not the same thing as switching arbitrary any variables in a model. Kolmogorov shows that some information can be reduce to lower complexity while others cannot. It is this information that cannot be reduces to lower complexity on which the model should be based upon.

Examples

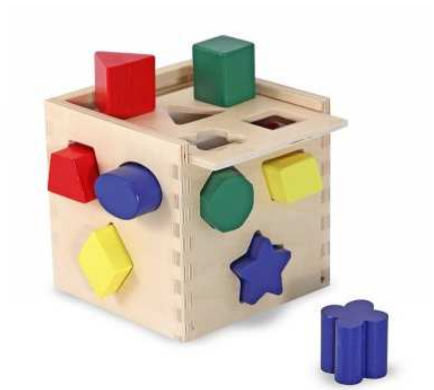
Here are a couple of examples to highlight these ideas:

1. The first one is standard example used to introduce Kolmogorov complexity applied to strings. It is presented by Michal Koucky [3]

33333333333 (1)
31415926535 (2)
84354279521 (3)

"The notion of randomness is connected to patterns in strings and to a way how we can describe them. The first two strings in our example have very short descriptions (few words) whereas the last string has very long description as it lacks any regularity. The longer the necessary description of a string the more randomness is in the string. This intuition leads to the following definition of Kolmogorov complexity of a string $x \in \{0, 1\}^*$: the Kolmogorov complexity of x is the length of the shortest description of x. Of course the length of the description depends on the language we use for our description—we can use Czech or French or English. . . ."

2. A second example: Here, we have a game for children.



In this game, what I find interesting is that the child is not overwhelmed by the information in front of him. After all, this box is made of wooden plate and most of its area is pale. Instead his mind focuses on the objects trying to fit the triangle into the triangle slot (the bright colors are here to help him).

The child mind has reduced the complexity of the pale wooden area to “nothing” although its covers most of the area. The problem for him now is trying to figure out a model for the color and the shapes because he cannot reduce their complexity.

Reducing complexity beyond Kolmogorov definition

Reducing complexity is a subject in itself. It is not straightforward because objects are highly contextual.

Contextual objects

I want to highlight that objects that we perceive are contextual.

Let's take the example of an apple falling from a tree. It can be seen as food, as an apple, as a group of atoms, as energy … the definition of this object depends from what angle we look at it.

The brighter the perception, the more angles can be seen.

Out-of-context mathematics

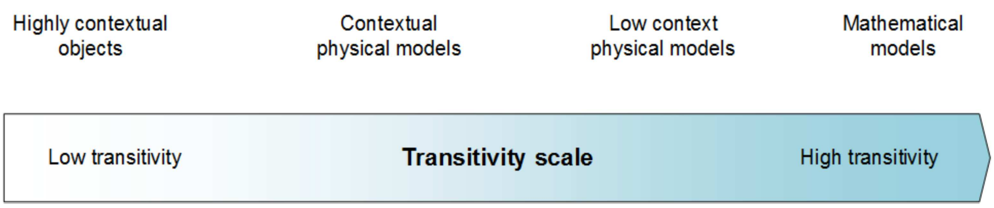
A property of mathematical models is that they are independent of any context.

$$e^{i\pi} + 1 = 0$$

Transitivity and context

I propose that a driver to establish a relation between contextual physical objects and mathematical models is transitivity.

"In mathematics, a binary relation R over a set X is transitive if whenever an element a is related to an element b, and b is in turn related to an element c, then a is also related to c." – Wikipedia [4]



On one side of the scale, one can find very transitive information: for example, 1+1=2 is applicable independently of where or when (with limits to our known world).

$$1 + 1 = 2$$

On the other side of the scale, one can find very intransitive information: for example, I bought milk this morning in my Paris local store.

Along the scale, one can find various models that are applicable to other situation (transitivity) but have limited application. For example, Newton’s laws of motion are applicable in daily life context. Yet, they come short of explaining reality when the context differs (very high speed or very small scales).

Switching costs

To help understanding transitivity, it can be useful to introduce economic concepts such as path dependency and cost to switch. Transitivity is the ability to switch from a to b.

“Switching barriers or switching costs are terms used to describe any impediment to a customer's changing of suppliers. In many markets, consumers are forced to incur costs when switching from one supplier to another. These costs are called switching costs and can come in many forms.” – Wikipedia [5]

- In economics, like highlighted in this extract, they are a costs associated with change. The origin of this cost is contextual to the object analyzed.

“Examples of switching costs include the effort needed to inform friends and relatives about a new telephone number after an operator switch; costs related to learning how to use the interface of a new mobile phone from a different brand; and costs in terms of time lost due to the paperwork necessary when switching to a new electricity provider.” – Wikipedia [5]

- On the other side, when a = b, switching a to b costs nothing. The relation between a and b is fully transitive.

Is it a coincidence that a core element of our mathematics “ **=** ” is based on 0 switching cost property?

I want to point out that I am not sure that humanity has an interest in improving transitivity without careful planning. Transitivity is useful but stability (intransitivity) of our environment is important for long term future.

Kolmogorov and transitivity

I want to come back to Kolmogorov complexity and how to reduce it. I believe when an information is fully transitive, its complexity is 0. I also believe that the missing link to quantify transitivity is derivative.

- The action of deriving is identifying the transitivity in a relation.
- Any information is a derivative

Conclusion

In this essay, I tried to introduce up a couple of insights that could be interesting for further research. First, Kolmogorov complexity, then how transitivity can be used to establish a relation between mathematics and physics. Yet, the content of this essay remains highly speculative.

References

[1] http://en.wikipedia.org/wiki/Kolmogorov_complexity.
[2] http://en.wikipedia.org/wiki/Kolmogorov%E2%80%93Smirnov_test
[3] http://iuuk.mff.cuni.cz/~koucky/vyuka/ZS2013/kolmcomp.pdf
[4] http://en.wikipedia.org/wiki/Transitive_relation
[5] http://en.wikipedia.org/wiki/Switching_barriers