The easiest way of putting it -- what is fundamental -- is how we define the imaginary unit. It's symbol is i its role in maths is to close the algebra on the geometry. "The Fundamental Theorem of Algebra" states: the field of complex numbers is algebraically *closed* if i equals the square root of minus one. The imaginary unit is defined by solving uniquely the equation $x^2+1= 0$. That is, i is a one unique distinguishable number defined as the square root of minus one, i.e., $i \equiv +\sqrt{-1}$.

This "constant of closure" or "$i \equiv +\sqrt{-1}$" is used in General Relativity via Minkowski's space-time continuum equation:- the speed of light equals the imaginary unit – c=i – to produce a mathematical model of Einstein's Special Relativity axioms;"the speed of light is constant" and "the laws of physics (or the equations) are the same in all reference frames".

Clearly by making the speed of light by definition the imaginary unit, we imbue "the speed of light" with all the "properties of the imaginary unit" which are the properties that are *necessary and sufficient* to close all equations. That is, what the imaginary unit can do, the speed of light can do to. Clearly the imaginary unit via The Fundamental Theory of Algebra forces "c=i" to behave as a universal constant always timelessly available for all observers. That is, the imaginary unit is the "timeless" number that closes algebra on a geometric number field, all numbers are "forced" by the power of mathematical certainty (obtained by deductive proof) to obey the terms and conditions of the Fundamental Theory of Algebra which states that every non-constant single-variable polynomial with complex coefficients has at least one complex root. That is, there are no "places" without the constant of closure for General Relativity of "c=i" that is, this "constant of closure" is universal and acts as a timeless initial condition for all polynomials that describe any interactions via single variable equations that are non-constant.

Clearly using c=i we can explain why the speed of light is a "constant of nature" or "a constant of closure". The Fundamental Theorem of Algebra forces "the speed of light" because it has the "necessary and sufficient properties" of the imaginary unit to close the equations of physics algebraically on the stage of Minkowski's space-time continuum $ds^2 = (cdt)^2 – (dx^2+dy^2+dz^2)$ with the condition of c=i.

On one hand we must have c as a physical restriction (or boundary condition) on motion because then $c \neq i$ and since the theorem can also be stated as follows: every non-zero, single-variable, degree n polynomial with complex coefficients has, counted with multiplicity, exactly n complex roots. There cannot be any other roots than the space-time routes (or the interactions) taken by the equations that obey c=i. The speed of light is both 1) a constant of nature (or a universal timeless initial condition available to all observers) which allows us the freedom of movement everywhere and everywhen constrained by 2) mathematical equations that appear as timeless yet explain motion within constant change.

Simply we have motion because c=i (the central idea of Minkowski's paper) is "the intrinsic number" that is necessary and sufficient to describe an invariant "geometry" using s as the *necessary and sufficient length* that gives the geometry $ds^2 = c^2dt^2 – (dx^2+dy^2+dz^2)$ called space-time coherence.

What is fundamental – is how we define the imaginary unit in maths. Recall the imaginary unit is defined by solving uniquely the equation $x^2+1= 0$. That is, i is a unique (i.e. distinguishable) number defined as the <u>square root of minus one</u>, i.e., $i \equiv +\sqrt{-1}$. Since there are two possible square roots for any number $+\sqrt{}$ and $-\sqrt{}$, clearly the square roots of a negative number cannot be distinguished until one of the two is defined as the imaginary unit, at which point +i and -i can then be distinguished. Since either choice is possible, there is no ambiguity in defining i as "the" square root of minus one. What if instead of solving for $x^2+1= 0$ to obtain " $+\sqrt{-1}$", "the" "imaginary unit" – we solve for $x^2+1^2= i^20^2$ or the <u>square root of the area of the imaginary unit.</u> That is, the square root of i^2 to obtain the indistinguishable “imaginary units” $+\sqrt{-1}$ and $-\sqrt{-1}$, that is we can define two “imaginary units” consistently on the one geometry of the bracket since we have now have “two constants of closure” For ease of notation let us define the “bracket” of the area of the imaginary unit as <| |>=i^2 therefore the indistinguishable square roots of the bracket area are:- the bra imaginary unit <| and, |> the ket imaginary unit. The bra <| can be used by General Relativity GR as c=i while the ket |> can be used for Quantum Mechanics QM which seems to close it's equations using i=h. That is, one geometry (the area of the imaginary unit) with two sorts of closure for the equations that describe reality; one set of equations GM that uses c=i for closure while the other incompatible set of equations QM uses h=i for closure.

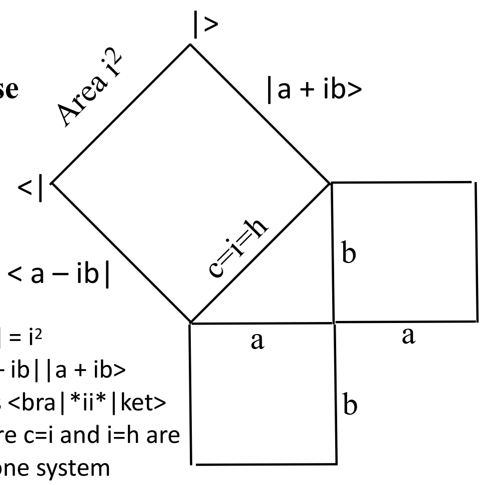
That is, by having two indistinguishable “imaginary units”, the bra imaginary unit <| and the |> ket imaginary unit, we can have both the speed of light and Plank's constant as universal timeless conditions for all mathematical equations that close algebra on the geometry.

So for GR the “constant of closure” is “the bra <|” which is used in Minkowski's space-time continuum equation:- the speed of light equals the imaginary unit – c=i – to produce a mathematical model of Einstein's Special Relativity axioms;"the speed of light is constant" and "all observers are equal". As $ds^2 = (cdt)^2 - (dx^2+dy^2+dz^2)$ with the condition of c=i. Technically Minkowski made c(metres)=i(seconds) denoted as c=i in this essay.

So for QM the “constant of closure” is “the ket |>” which is used in Quantum Mechanics via “Heisenberg's-time” equation:- Plank's constant equals the imaginary unit – h=i – to produce a mathematical model of Schrȍdinger's equation; "Plank's quanta is constant" and "all time is equal for all observers". As $i\hbar\partial\Psi/\partial t = H\Psi$ with condition of h=i since in QM the wave function Ψ is a just a complex number. Plank's constant is also known as the quanta of energy or the quanta of action. Time evolution is the exponential of the Hamiltonian, since the Hamiltonian is the generator of time-translation (equivalently: Energy is the charge of time translation). Hence why h(Joules)=i(seconds) or h=i in the essay.

Clearly by making the quanta of action by definition the imaginary unit, we imbue "the quanta of action" with all the "properties of the imaginary unit" which are the properties that are necessary and sufficient to close all quantum equations. That is, what the imaginary unit can do, the quanta of action can do to.

The Hypotenuse Box



ab – ba = [a,b] = i^2
ab + ba = < a – ib| |a + ib>
Hypotenuse is <bra|*ii*|ket>
or c=i=h, where c=i and i=h are combined in one system

That is, for the bracket of the area of the imaginary unit (<| |>=i^2) we have two closure methods on the one geometric number field. Since the “Fundamental Theorem of Algebra” can use the incompatible bra <| and ket |> imaginary units for the equations.

What is fundamental?
When we use the Born Rule: In Quantum mechanics we postulate a wave function Ψ then the Born Rule states that the wave-function's moduli squared $|\Psi|^2$ or ($\Psi^*\Psi$) obtains probabilities. What if reality is more like this – we have a “bra” < | and “ket” | > that gives us a “bracket” <| |> which equals the area of “the imaginary unit” i.e. <| |>=i^2. That is the (imaginary unit)^2 bifurcates into a bra of <|=i and into a ket of |>=i. Yes we define two sorts of “complex numbers” each with their own complex conjugation. If bra complex numbers are <|=i then the complex conjugation is bra*i=|a + ib> and so for ket complex numbers |>=i which are ket i* = < a – ib|. That is we have two sorts of complex numbers

using the two different roots <|&|> for the bracket area i^2 so we have two sorts of closure for the hypotenuse.

Clearly by the Fundamental Theorem of Algebra if an “imaginary unit” defines “complex numbers” as in our case z=a + ib & zero=0 + i0 it guarantees *closure,* that is, there is at least one complex root in the complex number field and that a real non-constant polynomial function of power n has n complex roots. Also if z=a + ib is a root then the complex conjugate z*=a – ib is also a root, that is, they come in pairs.

Looking at the definition of <||>=i^2 clearly z=a +ib are only the bra*i states. That is, in analogue to complex numbers defined with the bra <|=i which give (z=a + ib) & zero= 0 + i0 we can have the ket i=|> which gives (z=a – ib) & zero= 0 – i0. Yes two sorts of complex numbers since we have an area of i^2 that bifurcates into an equal pair of bra and ket complex numbers. That is using two ways of defining the “imaginary unit” implies (via the Fundamental Theorem of Algebra) we now can close any general point (a,b) in two ways on the geometry of the area of the imaginary unit.

Why do this? Two solutions to the area of the imaginary unit hence two ways that equations can work (that is the closure of algebra) on the geometry. That is two incompatible complex numbers systems can work together seamlessly as one whole. Of example we can let the ket |> solutions be used for Quantum Mechanics where i=h (basic postulate of QM), and we can use the bra <| solutions for Special Relativity where c=i (Minkowski). Table shows summary of ideas.

	<||>=i^2	
	Ψ*Ψ =Born rule *equals* c=i=h	
	<Ψ*| |Ψ>	
Let <|=i		Let |>=i
i=h		c=i
i(second)=h(Joule)	Two sorts of complex numbers	c(metre) = i(second)
$i\hbar\ \partial/\partial t \stackrel{\text{def}}{=} H$ H|>=E|>		$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$ $ds^2 = g^{\mu\nu}\, dx^{\mu}\, dx^{\nu}\ \ g^{\mu\nu} = (1,-1,-1,-1)$
Schrödinger's wave-equation		Minkowski space-time & Einstein's Tensor Equations

So by reinterpreting the Born Rule, as probabilities $|\Psi|^2$ or ($\Psi^*\Psi$) then the wave-functions <Ψ*| and | Ψ> of the ket *i and bra i* states respectfully, give us enough mathematical elbow room to accommodate both Relativity and Quantum mechanics in one scheme.

The two sets of complex numbers, as pictured in the hypotenuse box, share the same geometry of the bracket but are"interwoven" seamlessly. Much as our complex numbers formed from one distinguishable imaginary unit i.e., z=a+ib has "parts" that are "mathematically dealt with separately" such that if we add two complex numbers: 1+i4 plus 2+i5 we add the "real parts" and the "imaginary parts" to get 3+i9, it is only when make an area of (that is multiply the) complex numbers can we "mix these parts" together, (1+i4) times (2+i5) equals (1x2) +(1xi5) +(i4x2) -$i^2$4x5 =22+i13. So using the bracket to obtain the bra and ket imaginary units, gives us two sets of complex numbers or "operators" <a+ib| and |a-ib> that form the two sets of "complex numbers" that are dealt with separately, so the bra complex numbers are the constant of closure for i=c numbers and are used by GR and the ket complex numbers are the constant of closure for i=h numbers and are used by QM. So these "two complex numbers sets" are dealt with separately. Even if their "parts" have the same form, imagine if we use <| and form a+ib, but if we use |> and have a-ib then form the complex conjugate of |a-ib> to get a+ib which "looks" exactly like the a +ib formed from the <|. But as for our complex numbers and their parts which are dealt with separately, we have two constants of closure, so two sets of complex numbers that must be dealt with separately. That is why and how we can have "the same set of a and b" being in superposition, in the triangle above. We made the hypotenuse have a set of x,y that can use matched a and b. We force the hypotenuse to have all the equations from the two closure constants that match a and b. That is even if they have exactly the same form: a+ib derived by different methods from the <| and |> imaginary units. They are not part of the same physical closure system, they will appear to be "separate" yet are "together". So QM seems to work on the micro scale and GR seems to work on the macro scale and they both seem to work together as a whole combined to give us "one" reality based on the bracket of the area of the imaginary unit.

The hypotenuse diagram is a "relative state" diagram that is, all states (or hypotenuses) relative to the area of the imaginary unit. In modern physics – the status of Hilbert space and it rays that project "values" to the "eigen"-entities – it never was clear if these purely mathematical entities were a part or apart from physical reality. They were needed but not "in measurable space". In the above diagram – it is the whole diagram at once that is the theory, not a 'small" subsection. That is, the hypotenuse box is holistic – all parts are of the same whole. There are no “separate” areas from the area of the imaginary unit, the idealisation of the area of the imaginary unit is as much part of the diagram as a and b. Clearly the hypotenuse box is the geometric structure which represents “all relative states” from the area of i^2. That is, the hypotenuse box is an “all entangled diagram where we have both complex numbers systems at once“. That is, the hypotenuse box has the quantum property of monogamy, since only entangled states can have monogamy. Monogamy is a purely quantum behaviour of systems that interact, **monogamy of entanglement** means that an **entangled** state cannot be shared with other parties.

That is, strictly it is the property of monogamy <| |> for the geometry of the area of the imaginary unit i^2 that makes complex numbers act they way they -- why the real parts act together and why the imaginary parts act together, and how it is the area that can mix them up via the area of the imaginary unit. Such as (a+ib) times (c+id)=ac + ibc + ibs $+i^2bd$. Basically quantum monogamy is the disjoint of quantum systems (or simply the conjugates) don't interact once they have been measured or become part of the entanglement. That is, a and b are forced by monogamy to act as "two sets of complex numbers" with an area of the imaginary unit. When we consider the relative states diagram (which is a total entanglement diagram) we can piece out "areas" (formed by length times length) that are geometric with respect to the parameter that is acting as the area of "the imaginary unit". Why is this "area" so important -- it forms a dual space, the area of the imaginary unit isn't a dual but the bracket is a dual space from which all other states are entangled. And dual spaces can form areas that can be used to describe properties (such as entanglement entropy and mutual information for example) proportional to the minimal surface separating the subsystem.

So the state relative diagram which is a totality or the total entanglement of all states relative to the area of the imaginary unit. The diagram ***is*** the property of monogamy as a geometric system as a whole. It describes how monogamy the global property of entanglements forces a and b (in the legs of the diagram) to behaviour as "complex" numbers. The sides labelled a and b **are** in superposition, we can show that certain areas of the diagram force a and b to behave as in a space-time continuum while being in a quantum system. Also monogamy explains why a and b can act as "operators" from a complex plane and why there is only one ray <| |> from the area that is necessary and sufficient for coherence for the diagram.

The diagram is maximally entangled it only shows the minimal surfaces (that is, the entangled hypotenuses of "all states" are shown i.e. the length times length equals area) on one geometry that duals for the area of the imaginary unit. The diagram has many active hypotenuses working all at once. From the bracket hypotenuse we can form <| and |>, from the <a-ib| hypotenuse we can

form the area <a-ib||a+ib>, from the |a-ib> hypotenuse we can form the area <a+ib||a-ib>, and from the c=i=h hypotenuse we can form the area of the imaginary unit. And of course the a and b in the legs can form they own hypotenuses with areas attached. In the diagram we have all lines are active hypotenuses with their areas. But the one geometric object shows the minimal surfaces for each area, the total entanglement is shown as the property of wholism, I.e. drawn as one object.

From monogamy we get why and how complex numbers act the way they do and why we can have "separate" or "dis-jointed" paths for the same a and b. Think like this, it is the difference starting at a and b and going towards the area bracket <||>, or starting at area bracket and going towards a and b. It is totally entangled. So "imagine if we use <| and form a+ib, but if we use |> and have a-ib then form the complex conjugate of |a-ib> to get a+ib which "looks" exactly like the a +ib formed from the <|." This is we why we have dualities in physics, the most famous is the <wave|particle> duality, from above, the wave uses one set of complex numbers while the particles uses the other set, such that both the wave and particle use the same a and b. That is same a and b but they behave so differently we measured.

The hypotenuse box is a geometric structure which is “all entangled” and we have "all relative states', from the area of i^2. That is
(1) Entanglement is geometrical
(2) Entanglement measure is geometrical.
(3) Monogamy of entanglement is geometrical and valid for all systems

Since we have dualities we have shown, in general, that any measure of correlations (or areas) that is monogamous for all states of the hypotenuse box must vanish for all separable states (since they are no separate areas from the hypotenuse box): that is, only entanglement measures (that is areas) can *only* be strictly monogamous. Monogamy of other than entanglement measures can still be satisfied for special, restricted cases |a-ib> and <a+ib|: we show that the geometric measure of discord [a,b] can be pictured as the pure state of the hypotenuse of the diagram. That is, “laws of physics", the “constants of nature”, are in their own pure states (on their own hypotenuse) entangled such that "physical actions” can be "described by them" with no correlations except equations on the same geometry. Clearly monogamy shows the “laws of nature” are the bracket area, and the “constants of nature” are the “constants of closure”, and that any a and b and action of a and b i.e. [a,b], are restricted to *only* the equations of the closure constant used.

Summarising from Minkowski's c=i and Heisenberg's h=i being constants of closure we can draw a hypotenuse box diagram that shows that monogamy (the property) is the area of the imaginary unit projected as a single ray (a hypotenuse) that obtains the bracket area <||>=i^2. Clearly the Born rule *rules* is the idea, that is <||> is the born rule and total entanglement means wave functions become wave equations because of monogamy. The duality of the diagram is total as we can have duals of <wave|particle> we can have a dual of <wave function|wave equation>. After all wave functions are just complex numbers. Clearly this is why timeless “equations” can “track” a and b that form any physical (within time) hypotenuse of an measurement. See Appendix.

That is, the relative state entanglement diagram shows the minimal surface of the hypotenuses of the dual, it is like we have "literally drawn" the "two upright lines" in the bracket <||> as a hypotenuse, and like-wise we have literally drawn the "line" from the <| as a hypotenuse, and again we have literally drawn the line from the |> as a hypotenuse, and then we have literally drawn where a and b are a hypotenuse that has [a,b]=ic and [a,b]=ih. Then we combine all of these hypotenuses in one diagram. Then, within the areas of a and b we can have both dualities expressed as having different appearances, before this the "physical differences" we observe simply aren't part of the entanglement. Clearly the speed of light isn't the same as the quanta of action or Plank's constant for us. The c=i=h hypotenuse "mergers" both of these constants by using the same a and b for two sets of complex numbers. The entanglement diagram looks simple but contains many surprises when looked on as "one block" or as "separate monogamous" systems. The total entanglement diagram as one block is the property of monogamy itself, the 'separate areas attached to each hypotenuse" are monogamous. There is a difference between the property of monogamy which is imbued by all the hypotenuses when considered as one geometry against the systems that are monogamous (any of the areas attached to the hypotenuses). The Born rule comes first then the wave-function then the wave-equations if we are starting from the area bracket, but from within we have the wave-function, then the wave-equation and then the Born rule. Complete entanglement. This explains why in QM probabilities are additive probability amplitudes (areas) and the classical behaviour is additive probabilities (or just using lengths).

In more detail the Born rule <||> then, the wave-functions <| and |> then, the wave equations <a bra complex number| for GR and |a ket complex complex number> for QM then, the constants of nature c=i=h as indistinguishable then, within each box named a and b – the constants of nature become distinguishable. And the unlabelled hypotenuses in the diagram are used when we make an actual "measurement" or make a "hypotenuse" or do an experiment. Those are where we attach the observables we use onto the total entanglement diagram. There are left blank to show all, any and, each & every combination of a and b used to do an experiment (or form a new hypotenuse in the entanglement) can be connected to the area of i^2. See Appendix for details.

When we do an experiment the hypotenuse box diagram expresses why we have laws of nature (the bracket area <||>) and why we have constants of nature i=c and i=h (constants of closure) and why they can both use the same geometry c=i=h (a common area), there are all connected to the area of the imaginary unit. That is, we have intrinsic equations available timelessly for observables a and b. Or from within we have the wave-function (the "laws of nature" where all information comes from), then the wave-equation (the "constants of nature" as the two indistinguishable imaginary units c=i=h) and then the Born rule (the actual hypotenuse). Complete entanglement. .

Clearly the measure (or area) of monogamy for any total entanglement diagram is i^2.

Since "there are no individual systems or objects in quantum mechanics", this is why we have to draw everything at once. The diagram shows the relative states of the total quantum system. There are no isolated systems in QM. However how much the bracket is an "idealisation" <||>=i^2 it is actually part of the "whole" if we start at the bracket <||> then we go to bra imaginary unit <| and the |> ket imaginary unit from which we can have two constants of closure c&h that is, we can form two complex numbers sets <| --> <a -ib| and |> --> |a +ib> one for c=i and the other for i=h, then we get to the hypotenuse which has legs ab and ba that can now follow two set of rules of commutation, ab-ba=[a,b]=ih for QM, and ab-ba=[a,b]=ic for GR. If we ask what is the area of the combined area that is relative to the bracket, we have c=i=h as the hypotenuse so we have [a,b]=i^2 it appears to us users of the equations of physics there is a "mysterious area of the imaginary unit" that is *necessary and sufficient* to describe everything yet we can never experience it. It is saying that the idealisations are needed just as much as the actual physical case we can see that one is the limit of the other (or the dual of each other).

The Conclusion The measure (or area) of monogamy for any total entanglement diagram is i^2.

From the bracket area <||>, the complex number operators <a+ib| & |a-ib>, seem to form an area of "$a^2 + b^2$" which is the same as the area of the hypotenuse of the a leg and b leg of the triangle. That is, the area "$a^2 + b^2$" gives the necessary and sufficient "numbers" timelessly for the legs, and these "numbers" or what we call the "laws of nature" (laws=numbers such as i=c) are literally the timeless numbers that can describe all, any and, each and every outcome when we do a physical experiment (or label the hypotenuse box with a and b with our observables). When we apply our a and b to the hypotenuse box we can see how we can form an area that is "$a^2 + b^2$", this is how the total entanglement diagram works, it shows where we have "areas" formed that have the same form or appearance. This "$a^2 + b^2$" area shows why we have "timeless laws" describing temporal actions.

The common area "$a^2 + b^2$" is formed from the
1.<a+ib| & |a-ib> hypotenuses, and
2, a and b hypotenuses.

The “hypotenuse operator” <a+ib| forms the bra complex numbers "a+ib" and the hypotenuse operator |a-ib> forms the ket complex numbers "a-ib" in the common area of "$a^2 + b^2$". So the bra <| & |> ket hypotenuses act as (better *are* the) operators obtaining areas where complex numbers "a+ib" & "a-ib " are indistinguishable, and these literally are the operators (because they can form areas as well) which obtain, relative to our a and b hypotenuses, for the "areas" we call "numbers" we use in physics.

Since the measure of monogamy for any total entanglement diagram is i^2 we have an "area" where the "laws of physics", the "constants of nature", are in their own pure states (i.e. their own hypotenuse's in the diagram) entangled (the areas formed by length times length) such that "physical actions" (a and b) can be "described by timeless laws with constants" ***with no correlations except for equations on the same geometry.*** Literally the "mathematical equations" and "constants" are "built-in" “timelessly, everywhere and every-when) intrinsically into the area, that is, shared by our a and b observables that we place into the total entanglement diagram because there are no other correlation paths since there are no other hypotenuses within the area. So it appears to us that our actions "a and b" can be described by "timeless numbers" (obtained from the bra and ket operator) and "timeless equations" (obtained from the area of the bracket) which are "mathematical in form" (complex numbers obtained from <a+ib| & |a-ib>) that describe all, any and, each & every interaction a and b. The entanglement is complete. Laws become timeless numbers which can describe how a and b behave. That is, the idealisations we use such as "laws", "constant of nature", "numbers", "equations" etc are as much part of the diagram as are the observable and actionable a and b when we form a physical hypotenuse. When we measure a and b these "numbers" are timelessly available for you to complete the hypotenuse box using actions that aren't numbers. Clearly where literal numbers are and where we are, aren't the same place. In dual math there is an area (define as length times length) that has the "exact" same appearance as "numbers" in the common area of the hypotenuse box. Or we can use areas as numbers which is the simple definition for dual geometry. The entanglement is total and complete. Equations describe actions, actions that can be described by timeless numbers in timeless laws that seem to be always acting everywhere and every-when.

By introducing a new maths (based on the invariant i^2) we have show how quantum monogamy pictured as the minimal surfaces of a hypotenuse box can derive a new combined interpretation for QM and GR equations relative to the bracket area.

Now we can answer with mathematical exactitude Wigner's 1960 paper on *The Unreasonable Effectiveness of Mathematics in the Natural Sciences.* Wigner opines "the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it" and "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.”

What is fundamental – to answer Wigner's question mathematically and physically – is the measure of monogamy for any total entanglement diagram is i^2. Using the concept of the bracket area we can devise a new interpretation of mathematics that is entirely based on duals. This new interpretation uses the bracket area to prove for the hypotenuse box that the total relative state is the property of monogamy for the geometry, and that the areas formed from each hypotenuses are monogamous. From this we can show how the timeless numbers (i=c & i=h) and laws (<| & |>) can describe actions a and b.

This new interpretation of mathematics should be compared with Pythagoras's theorem where a and b the legs of a right angled triangle equal $a^2 + b^2$ the area of the hypotenuse, using the assumption that "numbers" are "lengths". In dual mathematics we show that in the total entanglement that there are "areas" that have the same exact form as "numbers" that can be used to describe observables in physical measurements that can label the hypotenuse box. Our current mathematical models are based on the idea that numbers are lengths -- that is fundamental. But consider if what is fundamental is that numbers are areas. Then we automatically get dual, since an area can be obtained from length times length. Dual mathematics shows that "our current maths thinking" is a "subset" of <||>, precisely the bra <| which allows for an area with a side of <a+ib| which can project what we call the complex numbers z=a+ib with zero=(0+i0), which then can define "real numbered areas", using the equation $x^2 + 1 = 0$ using length as numbers. Simply dual mathematics starts at a different point with the area of the imaginary unit, so we use the equation $x^2 + 1^2 = i^2 0^2$ all based on areas. You see $x^2 + 1 = 0$ mixes lengths with areas so cannot form a hypotenuse on the hypotenuse box, while $x^2 + 1^2 = i^2 0^2$ is all based on areas where we can derive "lengths" that aren't numbers but operators. At its most simple and fundamental, non-dual maths has lengths as numbers while dual maths has areas as numbers. And using the imaginary unit as an area we can devise a geometry that has a measure (of area) of monogamy of i^2.

Conclusion Winger's question on the nature of physics and mathematics can be addressed rigorously using the ***fundamental*** concept -- a number is an area.

What is fundamental? A number is an area not a length. And the imaginary unit is a number which can form monogamous areas using the non-hypotenuse sides, that is, the two sides that are not defining the area. Clearly we only need two sides to get an area, so the other two sides can <u>*encode*</u> operators about the area of the imaginary unit to other monogamous areas of the hypotenuse box. Current maths thinking only uses "one" encoding side -- the complex conjugate of the <a+ib| side -- to obtain areas. Basically in current maths thinking there is only z=a+ib, with zero=0+i0. We can devise a different set of complex numbers z=a-ib with zero=0-i0. And both can be related to the area of the imaginary unit, to obtain a new dual mathematics.

The *purpose* of this essay is to delight, surprise and, amuse any person interested in the current problems in physics, mathematics and, philosophy of science.

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Appendix - There is a well-known problem in quantum mechanics that reflects the "lengths and areas encoding" idea. The puzzle is known as the *watched pot paradox*, which is reminiscent of Zeno's paradoxes. The problem has to do with *continuous observation*.When we continuously observe a quantum system, for example a boiling kettle, the rules of quantum mechanics stipulate that the evolution of the Schrödinger wave-equation evolves only *when in superposition* so, it should, according to this logic, *never evolve*, and so, just as folk wisdom has it, a watched pot never boils. This is actually a general problem in quantum mechanics, not one unique to the MWI. The problem is symptomatic of how discreteness (jumping from measurement to measurement) and continuity (no gaps in measurements) are fundamentally incommensurable, and why reconciling them in quantum mechanics is so difficult.

That is, a continuous measurement of duration T (always with the *same* observer) (the singular collapse of the wave function) or a series of measurements (without the T observer in the gaps dT and elsewhere there are other observers i.e. there many worlds). You see, it's a matter of 1)the universal collapse of the wave-function that is controlled by one unique observer (or area) by the Schrödinger wave equation or 2) the collective collapse by many different observers of the wave-function that is controlled by the Schrödinger wave equation (or the lengths of the non-collapse hypotenuses are used to encode the behaviours of the collective collapse).

Or in other words we have 1)the universal collapse by **Schrödinger himself** of the pot that is being watched by **Wigner** or 2)collective solipsism i.e. collective collapse by many different observers (**Wigner's friends**)

The wave-function isn't the same as the Schrödinger wave equation. This of course is what the *watched pot paradox* actually is showing us. When we mix up one observable with many measurements – what can the wave-function do when constrained by Schrödinger wave equation – we go from 1)solipsism without the solipsist to 2) collective solipsism. These are not dual but implications of the "lengths and area encoding" that is, 1) acts as the area while 2) are the encoded properties of the area i.e. what comes from the length times length of the non-hypotenuse sides . The *watched pot paradox* is a telling story about *intersubjectivity* in quantum mechanics.

In the Many Words Interpretation MWI of QM, there is only one "monolithic time for all the worlds" and this *time* is used *in* the Schrödinger wave equation. In the MWI we use the time-dependent Schrödinger Wave Equation for the "monolithic time that is used by the many words. And these many worlds use the time-independent Schrödinger Wave Equation. We have confused the wave function with the wave equation saying that "*time* is a term *in* the Schrödinger wave equation". Again, it is a matter of how time works, or how we use the time-independent and time-dependent Schrödinger wave equations respectively. Clearly the two views 1) solipsism without the solipsist and 2)collective solipsism, both suggest that there cannot be only one solipsist as an explanation of reality. Since this would be inconsistent with fully unitary time evolution of quantum states. It is the difference between unitary time evolution of the many worlds and unitary-ness the property itself obtained from the one time-dependent Schrödinger wave equation that uses 'monolithic time". This is how can we avoid the wave function becoming (i.e. completely entangled with) the wave equation itself since in the MWI all worlds are unitary obtained from one source of unitary-ism – we must have – *the wave function* is the encoded area of the *wave equation*. These equations are the very heart of quantum mechanics.

The crux is that The Universal Wave Function <||> consists of 1) the Schrödinger time-dependent wave equation <u>*and*</u> 2) the Schrödinger time-independent wave equation – entangled. In case 1) time is monolithic and parameterised by the partial derivative $\partial\Psi/\partial time$, while in case 2) time is parameterised by the Hamiltonian H. The total entanglement of the quantum system allows of wave functions equalling wave equations. Yes, these are the wrong way around (the wave equations have become wave functions) to the standard definitions of the time-dependent and time-independent Schrödinger equations. This allows of the unitary-ism property to be associated with case 1) and the unitary states evolving as case 2) hence the "back-to-front" nature of the The Universal Wave Function definition. Case 1) allows of one monolithic time and Case 2) allows the many unitary worlds to evolve. Whither the wave function hither the wave equation.

That is, using The Universal Wave Function (UWF) as $<||>=i^2$ as both Schrödinger equations we can define **probability** consistently as a totality entanglement. In quantum mechanics probability is defined by the Born rule $\Psi^*\Psi$ where the wave function is the UFW, that is we have a totality to judge the number of samples of "a series of outcomes" or the "sampling of the Hamiltonian" in an absolute way.

Everett's "Relative-state formulation of quantum mechanics " is simple to state but has complicated implications, including parallel universes. The theory can be summed up by saying that the Schrödinger equation applies at all times; in other words, that the wave function never collapse.

Using our new understanding of the nature of time: dependent and independent of the UWF we can restate a new "totality worlds" theory as the UWF implies at 1) that we have all cases and that at 2) we have relative samples of cases (i.e. Everett's Relative-state formulation of quantum mechanics). Yes the Born rule comes first then the wave functions! A case example of this affect follows.

Another paradox in QM goes by the name of the *quantum theory of immortality*. Consider a version of the Schrödinger cat thought experiment that is endless repeated (i.e. indeterminately). Whether or not we accept the MWI, it is possible that the cat in the box to survive without end. This case is the same as an indefinite series of tosses of a fair coin that results in all heads (where "heads" corresponds to the cat surviving and a single "tail" corresponds to the cat going to kitty heaven). Just as we don't observe suitably heated kettles that refuse to boil, we don't observe immortal cats or humans in the real world, or even any that are 10,000 years old. Clearly the endless series of heads goes on and on but it only needs one tail to stop the parade of events. This is a problem in probability how do we get "low frequency events" not to happen often in a sample. **Probability** is really hard – how do you show that low frequencies give low observations in general but not specifically in any one time period. Also why it is deduction and not induction that rules. Clearly you are using "deduction" In the "heads" case since we have a mathematical series of "one alive cat"=heads, "the second sighting of one alive cat"=heads, "the third sighting of one alive cat"=heads, ……., but this reasoning ends when tails appears. That is, in the paradox mathematical induction (confusingly named) which is a form of deductive reasoning which cannot end, ends with one dead cat=tails. Hence giving a measure of almost zero probability (except for at least one case) for all heads in the case of the indefinite sampling of an infinite series with a "tails". Or in quantum language – let there be a superposition of all heads that is pure, allow one impure state for the UWF Universal Wave Function itself, which can act via entanglement as the wave equation for the pure superposition.

That is, quantum immortality in quantum mechanics has a "measure zero" solution. This answer to the problem allows for a vanishingly small but non-zero (or "measure zero") proportion of encoded areas consisting of immortal cats, ageless humans and other bizarre phenomena, as counter-intuitive as it might seem.

The "lengths and areas" encoding has irreversibly converted quantum behaviour (additive probability amplitudes) to classical behaviour (additive probabilities)

Name-calling			
	Turning point		
	Area		i^2 area with two roots that is +i=|> and –i=<| on equal footing until the choice
	<| |>=i^2		Both choices in the bracket are available equally i.e. <||> is not a duality
+i=|> ket	The two choices for the hypotenuse	–i=<| bra	Symmetry seems to been "broken" but both choices are taken at once
length	Legs of the triangle	length	Different laws on the same geometry
[a,b]=ab-ba= 0	Equations obeyed	[a,b]=ab-ba $\neq$ zero	Two ways to close algebraically the legs
Area is square	Area of hypotenuse	Area is circle	Two different areas for Pythagoras hypotenuse
Imaginary unit=c	Constant of closure	Imaginary unit=h	Two constants of nature due to closure
Sorites forward series	How actual objects behave	Sorites backward series	The mathematical counting series used by the objects
Deduction "Why"	Type of inference statements defined	Induction "How"	The actual constants we use in measurements
c is an deductive "constant"	The available how & why statements	h is an inductive "constant"	It is possible to have two sets of statements (i.e. physical axioms) to describe the one reality