Quantum strangeness from the uncomputability of nature

In this essay, ideas from computability theory are used to devise a local mechanism that explains how quantum value indefiniteness and quantum nonlocality may arise. Then we explain why we should use the algorithmic information theory to explore the quantum world and cosmology, in addition to the already successful Shannon information theory.

1. Quantum correlations and value indefiniteness

In a series of papers starting from 1964, John Bell [1] and others have probed into the problem of hidden variable theories (HVT) of quantum mechanics. He and others showed that local HVT leads to inequalities that are violated by quantum measurement statistics. This has been usually taken to mean that local realism is not true. But a more careful analysis of the assumptions of Bell's reasonings tells us that the violation actually means that either nonlocality is true, or the quantum correlations are not explicable (in the sense of Reichenbach's common cause principle).[1] Realism was never an independent assumption to begin with, so the violation does not refute realism, as some may argue.

Kochen-Specker theorem (KS theorem) [2] shows that quantum observables cannot possess non-contextual value assignments. How should we understand this? If one insists that quantum observables are to possess definite values prior to measurements, they can only do so if the measurement contexts are specified. In other words, it is the (observable + context) duo that can possess a definite value. This is called a contextual HVT. Or, one can adopt the view that quantum observables simply have no definite values prior to measurements (except for the observables of which the state is an eigenstate). Asher Peres summarized this view nicely: *Unperformed measurements have no results*. This is the view adopted in this paper.

In the past decade many toy models of quantum theory have been explored, with the goal of recovering mysterious quantum properties in more familiar settings. The most famous one is the Spekkens toy model [3], where many phenomena

[1] Here, freedom of measurement choices is assumed, i.e. superdeterminism is false.

thought to be exclusively quantum are reproduced. However, there are two major problems with that model - it is local, i.e. unable to reproduce quantum entanglements, and it fails to avoid the KS theorem, i.e. its observables have predetermined values independent of the measurement contexts.

Now, combining quantum correlations and KS theorem, we encounter a dilemma: consider the measurements of entangled pair of qubits, let's say a singlet state. Obtaining spin up in particle 1 measurement necessarily entails a spin down in particle 2 measurement in the same direction (and vice versa). The spin measurements are perfectly anti-correlated. Yet, KS theorem tells us that prior to the spin measurements the spin observables for both particles do not have definite values. So, how is it possible that the non-preexisting values be correlated at all?

The common reply to this is nonlocality. Obtaining an outcome on one particle changes the state of its entangled counterpart instantly, allowing for correlation of outcomes. But exactly how this instant effect works is beyond anyone. Entanglement is nonlocal but it does not allow for superluminal signaling, and no violation of special relativity. Some prefer to view entanglement as alocal, i.e. a phenomena that is outside of space. Therefore, spatial distance is not a problem for quantum correlations. This view is popular among researchers who goes further to reverse the logic, claiming that space itself somehow arises from entanglement.

However, I think these accounts are unsatisfactory as they avoids (or deflates) the attempt to explain nonlocality. In this paper, we will describe an attempt to explain nonlocality with a local model, in which the quantum observables are not predetermined.

2. Computability – computable and non-computable numbers

A computable number is a real number that can be computed to any precision by some algorithm within finite steps. All real algebraic numbers are computable. Some transcendental numbers are computable too, such as π, e. Interestingly, computable numbers have measure zero, which means if you cut the real number line with a knife, you will always hit an uncomputable number. This is a consequence of the fact that computable numbers are countably enumerably infinite, while there are uncountably infinite uncomputable numbers.

3. The model

In this model, the state of a spin-half particle is represented by an uncomputable binary number[2]. Without loss of generality, let this number be in the interval [0,1].
Since this is an uncomputable number, its decimal has an infinite number of digits, and there is no program or algorithm to determine each of its digits.

The model is summarized as follows:

1) Each digit in the decimal corresponds to a different direction of spin measurement, for all directions of measurements. Therefore, it is assumed that there are only countably infinite directions. This is a reasonable assumption in a finite universe[3];

2) The pure state $|+\rangle_{\vec{n}}$ is represented by an uncomputable number with value 1 for the digit that corresponds to direction $\vec{n}$.

$$|+\rangle_{\vec{n}} : 0.{*}{*}{*}{*}{*}\ 1\ {*}{*}{*}{*}{*}{*}{*}{*}{*}{*} \ldots$$

Similarly,

$$|-\rangle_{\vec{n}} : 0.{*}{*}{*}{*}{*}\ 0\ {*}{*}{*}{*}{*}{*}{*}{*}{*}{*} \ldots$$

3) The outcome of a measurement in a different direction $\vec{m}$, $M_{\vec{m}}$, does not exist before the measurement. This is due to the fact that the

[2] For spin-n, it is represented by a base-n number.

[3] There are many arguments for a finite universe, such as the Bekenstein bound, which says the information contained in a finite region can only contain a finite amount of information.

uncomputable number (its digits) is not predetermined by any algorithm. If there can be no preexisting algorithm that is able to determine the value, the values simply do not exist. This allows the model to avoid the KS theorem, thereby realizing the slogan “Unperformed experiments have no results”.
Note that in this model, it is mathematically true that the observables have no predetermined values, this is stronger than being true in a physical sense.

4) A measurement is represented by the calculation of one single digit of the uncomputable number.
Now, we make an important assumption on this model:
No amount of measurements would change the uncomputability of the number.
That is, no matter how many trials of measurements are performed on the particle, the information gain from the outcomes does not allow anyone to build an algorithm to output the number in finite steps.

An important and direct consequence of this assumption is the impossibility of performing all the quantum measurements simultaneously, i.e. quantum non-commutativity. For if it is possible to do so, all the digits will be known and the uncomputable number become computable, violating the above assumption.

Another consequence of the assumption is quantum measurements invariably disturbs the system if the system is not in the eigenstate of the measurement. The proof is simple:
Assuming there are only two different quantum measurements. By different I mean they cannot be performed together. If none of the measurements disturbs the system, we will be able to find out all the digits of the uncomputable number by simply performing one of the measurements after another. This violates the above assumption.

Actually, we can say more than this: the disturbance is unknowable - the originally known digit becomes unknown after the disturbance. If the disturbance is computable, all the digits will be computable.

So, in general, $M_{\vec{m}}$ changes the state $|+\rangle_{\vec{n}}$:

$$0.{*}{*}{*}{*}{*}\ 1\ {*}{*}{*}{*}{*}{*}{*}{*}{*}{*}\ \ldots\ \rightarrow 0.{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}\ 0\ {*}{*}{*}\ \ldots$$

Or

$$0.{*}{*}{*}{*}{*}\ 1\ {*}{*}{*}{*}{*}{*}{*}{*}{*}{*}\ \ldots \rightarrow 0.{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}\ 1\ {*}{*}{*}\ \ldots$$

$(m \neq n)$

After the measurement $M_{\vec{m}}$, the digit corresponding to the direction $\vec{m}$ becomes 0 or 1. The digit at direction $\vec{n}$ becomes undetermined.

5) Explaining quantum correlation.
Now, we show how in this model perfect correlations can arise from non-preexisting values, in a local way.
To illustrate this, let's take two identical uncomputable numbers:

$$a = b$$

where a and b are binary uncomputable numbers within $[0,1]$.

$$a = 0.{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}\ \ldots$$
$$b = 0.{*}{*}{*}{*}{*}{*}{*}{*}{*}{*}\ \ldots$$

So, if the ith digit of a is found to be 1 through measurement, we immediately know the ith digit of b to be 1.
In this example, no physical nonlocal effects is required, and yet the correlation can happen as a pure mathematical consequence.

It is also easy to model perfect anti-correlations. Take two uncomputable numbers that add up to 1:

$$a + b = 1$$

If the ith digit of a is found to be 1 through measurement, we immediately know the ith digit of b to be 0.[4]

In sum, we show that if quantum states are represented as uncomputable numbers, important quantum phenomena such as KS theorem and nonlocality could be recovered.

[4] Upcoming papers will include and explain the quantum probabilities in the model.

4. Algorithmic information theory and the Cosmos

In this paper I would like to urge for the use of algorithmic information theory in physics, particularly cosmology. The standard Shannon information theory has been extensively applied in many branches of Physics, from thermodynamics, quantum information, complex systems to astronomy, black holes and even to the formation of spacetime itself. A lot of great insights have been obtained in this quest and I believe more will continue to come. However, I think as the subject of the study gets bigger and bigger, its usage and the interpretation of analyses based on the theory will become more questionable. The reason is as follows.

The entire Shannon information theory is based on the concept of entropy,

$$S = -\sum p \ln p$$

where the entropy is a function of probabilities.

As we know, the standard interpretation of probability is the frequency interpretation. To verify probabilistic results we have to compare it with measurements on an ensemble of subjects. But as the subject of our study becomes bigger, there are less ensemble of the subject that we can measure or experiment on. This makes the use of probability problematic. The extreme case is when the subject is the entire universe. The use of probability and its interpretation loses its foundation, because there is only one universe.[5] So, when someone says the universe had a very low entropy in its early age, be skeptical!

Some would say we still can use the probability theory on the entire universe, but with the epistemic interpretation. This is correct, but for the degrees of belief to have any operational meaning, there must be an ensemble of subjects.

So, should we just give up and refrain to talk about information at the level of the universe? I think not. There is a way to talk about the information content of a single object, it is called the Algorithmic Information Theory (AIT). The idea is very simple. In short, the algorithmic information content of an object is the shortest

[5] In this essay we do not entertain the idea of many universes. We assume that there is only one universe.

length of the algorithms or programs that describe or output the object. For example, the information content of a sequence is the length of the shortest programs that output the sequence in finite time, and halts.[6] This shortest length is called the Kolmogorov complexity of the sequence.

If a sequence x has a pattern, its Kolmogorov complexity $K(x)$ [4] will be much shorter than the length $l(x)$ of the sequence:

$$K(x) \ll l(x)$$

But if the sequence is complex and has no patterns at all, $K(x)$ will be almost the same as $l(x)$, differing up to a constant. In this case, the sequence is random and has less information content.

This is also why in throwing a fair dice, we instinctively think the sequence of outcomes 5,5,5,5,5,5 is quite unlikely, even though it has the same chance of occurring as any other sequence of six outcomes. The sequence 5,5,5,5,5,5 has a pattern, and therefore a smaller $K(x)$.

Now, since AIT describes the information content and complexity of a single object, we can use it to talk about the information of the entire universe. The interesting thing about the Kolmogorov complexity is that it is an uncomputable number (that can be approximated from above)!

[6] Strictly speaking, the length depends on the language used, but they differ only up to a constant. The programs are running on a universal Turing machine.

[1] Bell, J.S., 1964, “On the Einstein-Podolsky-Rosen paradox,” Physics, 1: 195–200; reprinted in Bell 1987b [2004], 14–21.

[2] Kochen, S. and Specker, E., 1967, “The Problem of Hidden Variables in Quantum Mechanics”, Journal of Mathematics and Mechanics, 17: 59–87; reprinted in Hooker 1975, 293–328 (page references to original and reprint).

[3] Spekkens, R., 2004, https://arxiv.org/abs/quant-ph/0401052

[4] Li, M., and Vitányi, P., 1997, An Introduction to Kolmogorov Complexity and Its Applications (Second Ed.), New York: Springer.