Is Kinematics compatible with field symmetries?

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Abstract

An investigation is undertaken of the basis and implications of space-time kinematics. An understanding of the drawbacks and limitations of their use is sought. The possibility of addressing the same problems using the classical methodology of dynamics in rigorous terms is evaluated and the necessary means to make the Maxwell equations invariant in situations of moving bodies or across moving frames is explored. Further exploration leads to a derivation of the relativistic energy-momentum relations within purely 3 dimensional space and independent time using only the Maxwell equations and the Lorentz force equation.

1 Introduction

Relativistic kinematics is centered around the use of a mathematical tool called the Lorentz transformation. The Lorentz transformation is a means of adjusting the initial conditions and variables of the general wave equation so that its form is invariant regardless of the kinematic situation it is employed. According to the prescription of the theory of Special Relativity, it effectively does that not by modifying the length or period of the wave itself, but by compressing the length and time scales of the material objects and processes apart from the wave. It is applied to situations where one particle is emitting energy and a second particle is receiving energy.

But what are the assumptions and postulates one has made to render validity to that process? The plainly stated postulate of Special Relativity is that the speed of light is invariably c in a vacuum outside of the influence of gravitational masses. The rationale for that postulate comes from both the theoretical side and from experimental evidence. The value c is the only obvious velocity parameter embedded in the Maxwell equations. It is derived from the combination of the vacuum permittivity ϵ_0 and vacuum permeability constants μ_0, the former being effectively a conductance facilitating parameter while the later is a resistance to the passage of energy through space, i.e., $c = \dfrac{1}{\sqrt{\epsilon_0\, \mu_0}}$.

What are the latent or unexpressed assumptions? Since no structure or process has been identified that may sponsor the linkage of time to space, the default explanation has become that the linkage ***is*** the structure. After all, symmetries are to be expected, aren't they? They represent simplicity. But perhaps the more important hidden assumptions lie in how the Maxwell equations are expected to function.

We know from experience that the Maxwell equations hide certain potential complexities from us. For example, the spatial extent and shape of a charge, whatever it may happen to be, plays no significant role in nearly all practical problems outside of atomic or sub-atomic cases. Neither do we need to worry about deviations from the principle of superposition. So perhaps we may have become a little bit complacent in thinking that all of those types of potential inaccuracies will not affect us.

A particular point is most interesting. With the Lorentz transformation, one assumes that one is determining the field values for a point in the vacuum. But experimentally one has no means at all to make any measurement. A physical measurement requires a charge to be moved in some manner to register some type of physical indication of change. But the moment a charge is added to allow a measurement to be made, the applicable equations are no longer source free. Is this an unsolvable paradox or merely a complication for calculations? The Maxwell equations seem to work in such a way that issue is normalized away when dealing with stationary bodies or charges. However, it may be a very critical factor when dealing with moving bodies. We will investigate that issue in this paper.

What are the experimental conditions under which space-time kinematics are tested? Most importantly: one does not measure velocity. One may only infer the velocity of electromagnetic waves. Measurements are taken with an interferometer of some type. What is actually being measured are phase differences.

We begin now with a very brief survey of some points of history.

2 Development of the Lorentz transformation

In 1887, Woldemar Voigt published a paper[1] containing, among other things, a comparatively clear and concise derivation of the essentials of the formula for what was later called by Henry Poincare "The Lorentz Transformation"[2]. Voigt's rationale was to quantify the Doppler effect of an emitting body on a moving body. He found a solution to the wave equation for the moving particle's radiation using the technique of variable substitution into solutions for the simpler form of the wave equation, in coordinates that travel along with the particle. The initial conditions and form of the wave can then be transformed into the form required for a moving particle by substitution of the variables that perform the translation.

Voigt applied the accepted procedure of variable substitution[3] to solve a complicated partial differential equation on the basis of solving a more simple one, in this case the wave equation. This wave equation applies only within a medium that is isotropic and non-dispersive, that is to say, a vacuum. Voigt's procedure not only predates that of Henrik A. Lorentz, but it is more general. With the use of his transformation, Voigt studied situations of moving electrodynamics and moving observers wherein no abrogation of the universality of a time delta was invoked.

Similarly, Lorentz' technique for studying moving electrodynamics was to assume the invariance of the form of the wave equation when alternate variables for time and position are inserted. The wave equation, untransformed (on the left) and transformed (on the right using primed coordinates), takes a particularly simple form when the direction of motion is in the positive to negative x direction:

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{\Psi} \quad = \quad \left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\right)\mathbf{\Psi'} \quad = \quad 0$$

Here the wave parameter Ψ may be considered a metavariable or place holder for any field value such as **E, B, D** or **H**.

Expressed in matrix form for the time dimension plus 3 spatial dimensions, the transformation for the above equation is

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\mathbf{v} & 0 & 0 \\ -\gamma\mathbf{v} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \qquad \text{where} \quad \gamma = \frac{1}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}}$$

This has the effect of compressing spatial lengths and time periods and is interpreted in Special Relativity to apply to the material objects and processes, not the electrical objects or electrodynamic processes, for the observer or measurement device that lies in the inertial frame for which the transformation is calculated. In the Lorentz group, rotations and spatial reflections are supported, respectively, by the matrices

$$\begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(\theta) & sin(\theta) & 0 \\ 0 & -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} \qquad \begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

The successive application of a series of these transformations is equivalent to a linear combination of the matrix values.

3 Incompatibilities

One may be led to believe that a linear combination of the 3 matrices above will account for any physically valid symmetry expression involving time and space for a solitary particle reacting to the field fluctuations originating from a body in relative motion to the solitary particle. However, due to the fact that the determinant of the matrix that is a linear combination of the matrices is either 1 or -1, and no continuous transition between those values is possible, the use of the Lorentz transformation locks one into one of the 4 time-space parity classes[4] as follows:

$$(1)\ \ L_+^{\uparrow} \quad det\ a = +1 \quad a_{00} \geq 1 \quad \text{proper orthochronous} \quad (1)$$

$$(2)\ \ L_+^{\downarrow} \quad det\ a = +1 \quad a_{00} \leq -1 \quad \text{proper nonorthochronous} \quad (\text{TP})$$

$$(3)\ \ L_-^{\uparrow} \quad det\ a = -1 \quad a_{00} \geq 1 \quad \text{improper orthochronous} \quad (\text{P})$$

$$(4)\ \ L_-^{\downarrow} \quad det\ a = -1 \quad a_{00} \leq -1 \quad \text{improper nonorthochronous} \quad (\text{T})$$

Where a is the 4 x 4 matrix for each of the 3 transformation equations above, T represents time inversion parity and P represents space inversion parity.

This is problematic because violations of parity are in fact seen experimentally, especially with regard to the Weak Force. ***We therefore observe that Lorentz covariance applied to time and spatial relationships is too restrictive as a constraint to be generally applied to field expressions for elementary particles or to serve as an underlying basis for their determination.***

The situation is this: something outside of Special Relativity or the symmetry of relativistic kinematics is steering the parameters that determine how parity is manifest. Time-space symmetries emerge from a deeper physical basis and cannot of themselves be regarded as the underlying primitives of field manifestation.

The converse situation also exists in association with the Dirac equation (which is considered to comply with Lorentz covariance). Upon reaching a large electrostatic potential step, a wave packet for an electron conforming to the Dirac equation has a certain probability of becoming a positron. This violates the principle of charge conservation. This is called the Klein paradox[5] and it remains unresolved. Another difficulty is that although the free-particle Dirac operator satisfies the commutation operators of the Poincaré group, the operator for an external field does not do so[5].

The algebraic groups that support the mathematics of the Lorentz transformation are the Special Linear or $SL(4)$ and $SL(4, \mathbb{C})$ groups. These are projective groups. We may note that while each separate projection is linear, relationships between separate projected and non-projected elements are not linear. The finite-dimensional representations of the Lorentz transformation cannot be unitary within the non-compact part of the Lie group[5].

Yet another incompatibility is that the metric signature of Minkowski space-time, that is required by a strict interpretation of the Lorentz transformation, prevents the existence of self-dual gauge fields in SU(n) gauge groups[6]. For that reason, the metric signature of Minkowski space (+ - - -) is replaced with the signature from Euclidean 4 space (+ + + +) when much work is done involving elementary particles. Elementary particles are modeled as a rule within Special Unimodular Unitary SU(n) groups.

Additionally, Minkowski space requires a redefinition of the inner product (sometimes called the scalar product). This means, for example, that a true Fourier transform cannot be performed in Minkowski space. The replacement "pseudo-Fourier transform" has very different algebraic rules for determining its values.

4 Explicit determination of the Doppler effect

If the kinematics of Lorentz covariance is too restrictive, what are the alternatives using dynamical procedures? Voigt's transformation is

$$\begin{bmatrix} \tau \\ \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{bmatrix} 1 & -a_0 & -b_0 & -c_0 \\ -\alpha & m_1 & n_1 & p_1 \\ -\beta & m_2 & n_2 & p_2 \\ -\gamma & m_3 & n_3 & p_3 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

Here the variables τ, ξ, η and ζ take the same structural role as those of t', x', y' and z' in the Lorentz transformation, respectively. We observe that the primary coefficient for the transformed time parameter τ, in the upper left hand corner of the matrix above, is 1 rather than γ as it is in the Lorentz boost transformation matrix. This means that time is not subject to the same type of dilation as it is with the Lorentz transformation.

We may also observe that it is distinctly not a physical situation that the wave equation is invariant across different inertial frames. Rather, the wave form must expand and contract to account for the effects of Doppler shift. In Special Relativity, Doppler effects are calculated algebraically and are not based on the form of the wave equation. A physically realistic theory would allow the wave form to conform to the Doppler principle.

From this standpoint, the wave equation that is obtained from the Galilean transformation is compatible with the Doppler principle. To obtain the Doppler shifted wave we generate the differential operators $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t}$ by the use of the chain rule from equations $x' = x - \mathbf{v}t$ and $t' = t$.

$$\frac{\partial}{\partial x} = \frac{\partial t'}{\partial x}\frac{\partial}{\partial t'} + \frac{\partial x'}{\partial x}\frac{\partial}{\partial x'} = \frac{\partial}{\partial x}(t)\frac{\partial}{\partial t'} + \frac{\partial}{\partial x}(x - \mathbf{v}t)\frac{\partial}{\partial x'} = \frac{\partial}{\partial x'}$$

$$\frac{\partial}{\partial t} = \frac{\partial t'}{\partial t}\frac{\partial}{\partial t'} + \frac{\partial x'}{\partial t}\frac{\partial}{\partial x'} = \frac{\partial}{\partial t}(t)\frac{\partial}{\partial t'} + \frac{\partial}{\partial t}(x - \mathbf{v}t)\frac{\partial}{\partial x'} = \frac{\partial}{\partial t'} - \mathbf{v}\frac{\partial}{\partial x'}$$

We may now solve for $\frac{\partial}{\partial x'}$ and $\frac{\partial}{\partial t'}$ to obtain

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x} \qquad \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial x}$$

$$\frac{\partial^2}{\partial x'^2} = \frac{\partial^2}{\partial x^2} \qquad \frac{\partial^2}{\partial t'^2} = (\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial x})^2$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x'^2} \qquad \frac{\partial^2}{\partial t^2} = (\frac{\partial}{\partial t'} - \mathbf{v}\frac{\partial}{\partial x'})^2$$

Then

$$(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{\Psi} \quad = \quad (\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}(\frac{\partial}{\partial t'} - \mathbf{v}\frac{\partial}{\partial x'})^2)\mathbf{\Psi'} \quad = \quad 0$$

and

$$(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2})\mathbf{\Psi'} \quad = \quad (\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial x})^2)\mathbf{\Psi} \quad = \quad 0$$

We may recognize the term $\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial x}$ as the full derivative operator for time $\frac{d}{dt}$ which is also called the convective derivative, material derivative and substantial derivative, and is often given the designation $\frac{D}{Dt}$. Thomas Phipps has reinvigorated the use of this derivative in Electrodynamics to make field values invariant within the Euclidean independent time and space dimensions[7]. This was apparently nearly a lost art that was understood and practiced by, for example, Helmholtz, Maxwell, Heaviside, Hertz, Cohn and Ritz[8].

We therefore define (for motion parallel to the x axis)

$$\frac{\partial}{\partial t'} \quad = \quad \frac{D_+}{Dt} \quad = \quad \frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial x} \qquad\qquad \frac{\partial}{\partial t} \quad = \quad \frac{D_-}{Dt} \quad = \quad \frac{\partial}{\partial t'} - \mathbf{v}\frac{\partial}{\partial x'}$$

The forward and inverse Galilean transformations now give the wave equation pairs a distinctly covariant flavor as follows

$$(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{\Psi} \quad = \quad (\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{(D_-)^2}{Dt^2})\mathbf{\Psi'}$$

and

$$(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t'^2})\mathbf{\Psi'} \quad = \quad (\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{(D_+)^2}{Dt^2})\mathbf{\Psi}$$

One may verify that Maxwell's equations are indeed invariant for moving bodies with this transformation of the field parameters via the convective time derivative. Yet this methodology directly shows Doppler effects. To determine the Doppler shift for a moving body that is emitting radiation we stay within a frame of reference and apply the convective derivative to the wave equation

$$(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{\Psi} \quad \Rightarrow \quad (\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{{D_+}^2}{Dt^2})\mathbf{\Psi'} \quad = \quad f(x,t')\mathbf{\Psi'} \quad = \quad 0$$

Then take the Fourier transform

$$f(x,t')\mathbf{\Psi'} \quad \Leftrightarrow \quad \mathfrak{F}(k,\omega')\widetilde{\mathbf{\Psi}}' \quad = \quad (k^2 + \frac{1}{c^2}(\omega' - \mathbf{v}\mathbf{k})^2)\widetilde{\mathbf{\Psi}}' \quad = \quad 0$$

Here ω, a scalar, is the angular frequency of any sinusoidal component of the wave and $\mathbf{k}$, a vector, is the wave number of any sinusoidal component of the wave. The dispersion equation for the transformed wave equation is then $\frac{\omega'}{\mathbf{k}} = c - \mathbf{v}$. The Doppler shift is therefore $\omega' = \omega\,\frac{c}{c - \mathbf{v}} = \omega\,\frac{1}{1 - \frac{\mathbf{v}}{c}}$ or simply $f' = f\,\frac{1}{1 - \frac{\mathbf{v}}{c}}$.

However, this is merely the Doppler shift for a non-interacting wave. Interactions between the field and the particle, in which force is applied to the particle and from which effects some type of measurement may be made, are to be accounted for also.

5 Field interactions on a moving charged particle

To determine the effects of field interactions on moving particles we employ the Lorentz force equation to find the dielectric tensor using constituent relations[9]. For a simple case we choose a free electron in an otherwise empty vacuum encountering radiation traveling along the positive to negative x axis. The **E** and **B** fields may be determined starting from the Fourier transform of the Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{and} \qquad \nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial}{\partial t}(\ddot{\mathbf{K}} \cdot \mathbf{E})$$

Which is

$$\mathbf{k} \times \widetilde{\mathbf{E}} = \omega \widetilde{\mathbf{B}} \qquad \text{and} \qquad k \times \widetilde{\mathbf{B}} = -\frac{\omega}{c^2}(\ddot{\mathbf{K}} \cdot \widetilde{\mathbf{E}})$$

$\ddot{\mathbf{K}}$ is the dielectric tensor which is determined by equating the right hand sides of the so-called microscopic and macroscopic forms of Ampere's law. It is equal to the identity tensor $\ddot{\mathbf{I}}$ plus the conductivity tensor $\ddot{\sigma}$ divided by the displacement current as such

$$\ddot{\mathbf{K}} = \ddot{\mathbf{I}} - \frac{\ddot{\sigma}}{i\,\omega\,\epsilon_0}$$

Taking the curl of $\mathbf{k} \times \widetilde{\mathbf{E}}$ we get

$$\mathbf{k} \times \mathbf{k} \times \widetilde{\mathbf{E}} = \mathbf{k} \times \omega \widetilde{\mathbf{B}} = -\frac{\omega^2}{c^2}(\ddot{\mathbf{K}} \cdot \widetilde{\mathbf{E}})$$

A non-trivial solution of the above equation is possible if the determinant of the $\ddot{\mathbf{K}}$ matrix can be set to zero. The current density is $\mathbf{J} = e\mathbf{v}$. The force on the electron is $\mathbf{F} = m_e \frac{d\mathbf{v}}{dt} = e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ where m_e and e are the mass and charge of the electron and v is the relative velocity between the emitting or disturbing particle particle and that of the particle involved in a measurement. The force provided by the magnetic field, $\mathbf{v} \times \mathbf{B}$, does not cause displacement in the x axis so that, for our purposes, $\frac{d\mathbf{v}}{dt} = \frac{e\mathbf{E}}{m_e}$. We will assume that the measurement particle is initially at rest. We now have

$$m_e \frac{d\mathbf{v}}{dt} = e\mathbf{E} \qquad \text{and} \qquad \mathbf{J} = e\mathbf{v}$$

or

$$m_e(-i\omega)\,\mathbf{v} = e\widetilde{\mathbf{E}} \qquad \text{so that} \qquad \mathbf{J} = \frac{e^2}{(-i\omega)m_e}\widetilde{\mathbf{E}}$$

The conductivity tensor is defined by the equation $\mathbf{J} = \ddot{\sigma} \cdot \widetilde{\mathbf{E}}$ from which we find

$$\ddot{\sigma} = \ddot{\mathbf{I}}\,\left(\frac{e^2}{(-i\,\omega)\,m_e}\right) \qquad \text{and} \qquad \ddot{\mathbf{K}} = \ddot{\mathbf{I}}\,\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

Where $\omega_p^2 = \frac{e^2}{\epsilon_0 m_e}$ is commonly known as the plasma frequency. It is the frequency at which a free charge particle resonates if it is not moving substantially with reference to a background field. Substituting the dielectric tensor into $\mathbf{k} \times \mathbf{k} \times \widetilde{\mathbf{E}} + \frac{\omega^2}{c^2}(\ddot{\mathbf{K}} \cdot \widetilde{\mathbf{E}})$ we obtain

$$c^2\mathbf{k} \times \mathbf{k} \times \widetilde{\mathbf{E}} + (\omega^2 - \omega_p^2)\,\widetilde{\mathbf{E}} = 0$$

This equation can be written in matrix form as

$$\begin{bmatrix} \omega^2 - \omega_p^2 & 0 & 0 \\ 0 & -c^2k^2 + \omega^2 - \omega_p^2 & 0 \\ 0 & 0 & -c^2k^2 + \omega^2 - \omega_p^2 \end{bmatrix} \begin{bmatrix} \widetilde{E}_x \\ \widetilde{E}_y \\ \widetilde{E}_z \end{bmatrix} = 0$$

Setting the determinant to zero gives the dispersion relation

$$(-c^2k^2 + \omega^2 - \omega_p^2)(\omega^2 - \omega_p^2) \ = \ 0$$

The roots of which are

$$\omega^2 \ = \ \omega_p^2 \ + \ c^2k^2 \qquad \text{(for the transverse mode)}$$

$$\omega^2 \ = \ \omega_p^2 \qquad \text{(for the longitudinal mode)}$$

The transverse mode is of primary concern because energy and field fluctuations are not propagated in the longitudinal mode. The phase and group velocities are

$$\mathbf{v_{phase}} \ = \ \frac{\omega}{k} \ = \ \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} \qquad \text{and} \qquad \mathbf{v_{group}} \ = \ \frac{\partial\omega}{\partial k} \ = \ c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

We see immediately the well known relationship $(\mathbf{v_{phase}})(\mathbf{v_{group}}) \ = \ c^2$. This expresses a description of the dispersion of any energy involved in interactions with the moving charged particle. Group velocity is equivalent to the velocity of the particle $\mathbf{v_{group}} = \mathbf{v}$, while phase velocity describes the speed at which the plane or surface tangent to the wavefront having constant phase travels. Energy relationships are clarified by solving for $\frac{\omega_p^2}{\omega^2}$ below.

$$(\mathbf{v_{group}})^2 \ = \ c^2\,(1 - \frac{\omega_p^2}{\omega^2}) \qquad \text{gives us} \qquad \frac{\omega_p^2}{\omega^2} \ = \ \frac{c^2 \ - \ (\mathbf{v_{group}})^2}{c^2} \ = 1 - \frac{\mathbf{v}^2}{c^2}$$

From which we have apparently discovered the origin of the Lorentz γ factor

$$\frac{\omega}{\omega_p} \ = \ \frac{1}{\sqrt{1 \ - \ \frac{\mathbf{v}^2}{c^2}}} \ = \gamma$$

Maxwell's successor, Sir J. J. Thomson[10], in conjunction with his son's (G. P. Thomson) thorough experimental investigation of electron waves which Louis de Broglie had predicted, knew of these dispersion relations and believed they reflected a constant in quantum relationships in addition to the Planck constant. In a sense he was certainly right in that they apparently underlie de Broglie's Wave Mechanics. However, cast in the form above, it becomes quite apparent that effects associated with relativity are of origin in the dispersion principle - that being that electromagnetic fields impinging on a charged particle cause the particle to move and that movement generates a counter field which recursively mixes with the impinging field which further effects the motion of the particle.

De Broglie[11] has expressed the core of Wave Mechanics as consisting of this simple relationship

$$energy \ = \ h \ \times \ frequency$$

It is known[12], in conjunction with de Broglie theory, that a change in momentum of a particle is related to a change of energy through the relations

$$\mathbf{v} \ = \ \frac{\partial(E)}{\partial(m\,\mathbf{v})} \ = \frac{\partial(E)}{\partial(\mathbf{p})} \ = \ \frac{\partial(\hbar\,\omega)}{\partial(\hbar\,\mathbf{k})}$$

Where E is the total energy of the particle, $\mathbf{p}$ is the particle's momentum and $\hbar$ is the reduced Planck constant. From the de Broglie relations $E \ = \ \hbar\,\omega$ and $\mathbf{p} \ = \ \hbar\,\mathbf{k}$. In harmony with this view, applying $\hbar^2$ to both sides of the dispersion equation for the solitary electron we get

$$(\hbar\,\omega)^2 \ = \ (\hbar\,\omega_p)^2 \ + \ c^2(\hbar\,\mathbf{k})^2 \qquad \text{or} \qquad E^2 \ = \ (\hbar\,\omega_p)^2 \ + \ c^2\mathbf{p}^2$$

By algebraic elimination we may determine that $\hbar\,\omega_p \,=\, m\,c^2$ and thereby complete a derivation of the relativistic energy-momentum relations

$$E^2 \,=\, m^2c^4 \,+\, c^2\mathbf{p}^2$$

Upon eliminating $\mathbf{p}$ above using the relation $\mathbf{p} = \frac{E}{c}$ we obtain

$$E \,=\, \gamma m c^2$$

Which applies for a particle having mass. Here γ expresses the additional amount of energy that is transferred from the incident field fluctuations to the particle because of a greater velocity difference between the two. Eliminating m above using the relation $E = m\,c^2$ we obtain

$$E \,=\, \mathbf{p}\,c$$

Which applies for a particle having no mass.

Thus, invoking only the Maxwell equations and the Lorentz force equation, we have been led to very compelling theoretical evidence for the origin of the critical equations that give rise to the observable physical behaviors normally associated with relativistic kinematics, space-time symmetries, Lorentz covariance and Special Relativity. Our approach here, however, does not suffer from the crippling difficulties relativistic kinematics has in attempting to deal with accelerated or non-inertial motions and many-body physics.

Beeching[12] has presented a detailed review and analysis of very many confirmations of this type of wave mechanics by various experimental physicists in addition to its use in various industrial applications.

6 Conclusions

We have shown that the alternative to space-time linkage in addressing problems of wave energy transfer is to effectively recast the paradigm to a (wave velocity)-energy linkage. Stated mathematically, space-time kinematics requires a redesignation of time, called proper time τ, in response to velocity differences (between emitting and reacting particles) as such

$$\mathbf{p} \,=\, \gamma\, m\,\frac{d\,\mathbf{x}}{d\,t} \,=\, m\frac{d\,\mathbf{x}}{d\,t}\frac{d\,t}{d\,\tau} \qquad\qquad (\frac{d\,t}{d\,\tau} \,=\, \gamma)$$

Whereas the proposal of this paper is

$$\mathbf{p} \,=\, \gamma\, m\,\frac{d\,\mathbf{x}}{d\,t} \,=\, m\frac{d\,\mathbf{x}}{d\,t}(\omega\,\frac{\epsilon_0\, m}{q^2}) \qquad\qquad (\omega\,\frac{\epsilon_0\, m}{q^2} \,=\, \gamma)$$

The interpretational difference between the two approaches is that the first attributes the γ factor to a compression of material objects and processes apart from the wave. Our investigations here indicate that the γ factor is rather a compression of energy within the space of an electromagnetic wave. If this is so then clocks whose timing mechanisms are sensitive to the speed of electromagnetic energy transport may run faster or slower as the situation dictates but they are not otherwise mandated to follow the rules of relativistic kinematics.

A very important observation in this paradigm, would be that c^2 is the invariant (which may be decomposed), not c as it is in Special Relativity and Lorentz theory. A paradigm such as this may lead to deeper, richer and more detailed modeling of particles, fields and interaction processes as well as elicit still new and exciting discoveries.

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