On Constrained Perception

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The argument is made the “Bit” in “It from Bit” is the result of averaging and thus is not definite, and fundamental uncertainty is rooted in set theory. The cosmological constant is also argued to be completely natural. An expansion of the Ricci-tensor over a finite number of terms is also provided.

I. TIME AND INFORMATION

In order to discuss the topic of information, we must first discuss our notion of time and flow. In the most general understanding, time is a monotonically increasing parameter, and a flow is a change of a variable with respect to time. According to ergodic theory [1], we can understand a flow as the group action, φ, which maps the transformation group G acting on set X, back to set X [2]:

$$\varphi: G \times X \to X$$

The group G is the set of real numbers, $\mathbb{R}$, and it is the well-ordered index of the iterated acts of G that is monotonically increasing. This well-ordering of the index is typically assumed to extend to the real numbers as well, making it possible to identify and define the time averages of the past for a general function $g(t)$ as:

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T}^{0} g(t)dt$$

Where t is an index of iterations of G.

However, the well ordering of the reals is not provable from Zermelo-Fraenkel axioms plus the axiom of choice and the general continuum hypothesis (ZFC+GCH) alone [3], and would require the assumption of the axiom of constructability to prove, which is an assumption that has not been shown to be always true [4]. Given also that the simultaneity of events is not absolute in the physical world as shown by Einstein [5], we are forced to accept a certain level of imprecision, or error, or uncertainty, in the ordering of the transformations occurring on set X.

Given this imprecision in the ordering, if A and B are transformations, and Y is the ordering of a set after two successive transformations, it might not be provable that B followed A or that A followed B in which case:

$$ABX = BAX = Y$$

$$AB - BA = 0$$

This means the transformations A and B are permutable, or Abelian, when the uncertainty of the associated ordering is sufficiently high [6]. It is this type of imprecision that von Neumann was identifying in his proof of the ergodic theorem in quantum mechanics. If we recognize position and momentum operators Q and P as being transformations which can never be performed simultaneously in quantum mechanics, then there is an order to those operations, and this corresponds to the situation where:

$$QP - PQ = \frac{\hbar}{i}$$

This is not something that we intuitively understand from our everyday experience, where we are able to measure position and momentum simultaneously and:

$$QP - PQ = 0$$

John von Neumann showed that no contradiction existed between the non-abelian relation of position and momentum in quantum physics and the abelian relationship that exists in classical physics simply because there is an imprecision one must live with when working in the classical world, which at best is only an approximation to what is actually occurring [7].

It is imprecision which leads us to our notion of information as well. Within any interval of time, there is an uncertainty associated with the precise sequence of transformations that occurred during that interval. It is the uncertainty in a sequence which we typically associate with information, and the discovery of that sequence which we associate with knowledge.

If we follow the approach outlined by Wiener [6], we can identify the knowledge gained by comparing *a priori* and *a posteriori* information of a sequence after examination. In that case, we can argue that if we know *a priori* a variable lies between *0* and *1*, an *a posteriori* it lies in an interval between *a* and *b* inside interval 0 and 1, then the knowledge gained can be quantified as:

$$-\log\left(\frac{measure(a,b)+\epsilon}{measure(0,1)+\epsilon}\right)$$

Here an error term has been added to the relationship. As a result, this value will approach zero as the measures of *(a,b)* and *(0,1)* approach error ϵ. In other words, in the limit of error, the possible amount of knowledge to be gained is zero. More interestingly, as the measure of *(a,b)* approaches zero, the amount of knowledge gained is not infinite, and is limited by the error. We can interpret this to mean there is a fundamental amount of information associated with the error that cannot be removed from our calculation of knowledge to be gained.

This leads to the realization there is a limit to the amount of classical information that can be transmitted over an energy limited channel. Bremermann, discusses this topic in some detail, and he derives the limit over any physical channel as [8]:

$$C=\frac{E_{max}}{\hbar}\ln(1+4\pi)=\frac{mc^2}{\hbar}\ln(1+4\pi)$$

This result is slightly different than the one reached by Beckenstein [9]. This is because Bremermann's is directly derived from quantum mechanical principles, while Beckenstein's is derived through geometric arguments, but both show the knowledge to be gained from some transmission is bounded directly by the energy of the system.

The channel capacity associated with the energy is also closely related to the frequency with which states can change from one orthogonal state to another as derived by Margolus and Levitin for a classical system:

$$v_\perp \le \frac{2E}{h}$$

Where *E* is the zero of energy at the ground state of the system [10].

This is of concern when *E* is equivalent to the mass energy associated with the cosmological constant, since it implies that an object the size of the universe is changing from state to state at a rate of 10^{105} times per second given a mass of 10^{54} kg.

II. QUANTUM CONSTRAINT

Ever since the development of quantum mechanics we know that this there is limit to our precision, and thus there is a certain amount of error that we must accept in the statements we can make. It is of fundamental importance that the information associated with this error is not lost as the universe evolves. This error is associated with the existence of multiple potential states, and the elimination of this error would be equivalent to the collapse of the universe into one final state. This, in principle, would lead to a perfectly deterministic classical world without any notion of "free-will".

However, if it is truly undecidable that the reals can be well-ordered, then it can be argued the fundamental error is not loosely rooted in the axioms of quantum theory, but strongly rooted in set theory as well. This would imply that our universe, and the information in it, is a fundamental consequence of the limits of knowledge in general, and will continue to evolve indefinitely, eventually passing through an infinite number of classically orthogonal ground states.

III. OUR PERCEPTION

An observer's internal model is developed based on knowledge gained through observations. In the most general sense, an observer is not a conscious being, but can be any point in space. Therefore, each point in space must have some local model as to how it relates to every other point in space. This is the point's (e.g. observer's) perceived reality.

Real information received at each point must be accompanied by a transfer of energy, and the transfer is mediated by the fundamental particles. To within acceptable limits, the models must agree upon the fundamental values associated with those particles. This is analogous to our notion of a communications channel, where each fundamental particle is associated with a channel for information flow.

Quantum correlations serve to place constraints on possible relationships between data, not the state of the data. After an appropriate amount of time, sufficient mixing can occur allowing systems to decohere into classical states. However, as discussed in the first section, these states are not entirely stable themselves, and will in fact devolve over time. This is possible because of the approximate nature of classical states.

If one were to poll observers, the best answer one could get to the question of what the value of a classical state is would be an average value. Given sufficient communication between observers, the effects of the law of large numbers, and the central limit theorem, it is reasonable that all the observers will arrive at the same average value and distribution for a classical state. Given uncertainty though, the agreed upon average will change over time. This causes our perception of reality to have a flavor of the moment quality. Effectively, averages determine our perception of reality, and those averages change over time.

IV. NATURALNESS OF THE COSMOLOGICAL CONSTANT

One of the most common debates in physics surrounds the cosmological constant and how it appears to be "fine-tuned", e.g. the smallness of the cosmological constant relative to the Planck mass is too great and thus viewed as "unnatural". From an information viewpoint, the value appears too specific to be generated by a random process. However, supporters of the "anthropic principle" will argue that the number is in fact random, and that it is extremely likely to be small, and even if that were not the case, if we require a small value in order to exist, then we will only ever see a small number. Neither of the two camps offer very persuasive arguments, yet both are actually making statements about information. Effectively, the former arguing there is very little information in the universe, the latter there is an overabundance.

It is essential to argue the value of the cosmological constant has some very natural guides and is not "unnatural" at all, and yet still permit for randomness in its value. As pointed out by the articles by Hawking [11] the conventional argument is the cosmological constant is significantly lower than its natural value by over 120 orders of magnitude:

$$\frac{|\Lambda|}{m_p^2} < 10^{-120}$$

It has also been measured to have a slight positive value which has proven difficult to understand. Supersymmetric models offer some explanation, as they allow bosons to make positive contributions to the constant, and fermions to make negative contributions. Since supersymmetric particles come as a bosonic and fermionic pair, when the masses of the particles are the same, the cosmological constant takes on a value of zero. However, supersymmetry is a broken symmetry, and the masses of bosonic and fermionic components are not the same.

Ordinarily, to keep the cosmological constant near zero, we must be concerned with the values of all the decimal digits for the mass of each particle all the way to the scale of the cosmological constant itself. So there is a question of how these values sum to a slightly positive value. When the particle masses in natural units are plotted (Figure 1), a logarithmic relationship of the masses becomes apparent. What this means is that the level of precision needed to validate the most massive particle does indeed cancel the least massive particle is the difference between the scales of the particles. For instance, if the lightest particle is on the order of 10^{-65} and the most massive on the order of 10^{-21} the then level of precision needed to reconcile is: $65 - 21 = 44$. This seemingly unobtainable level of precision is why the cosmological constant problem is viewed as a "fine tuning" problem, the precision demanded is simply greater than what one would normally expect.

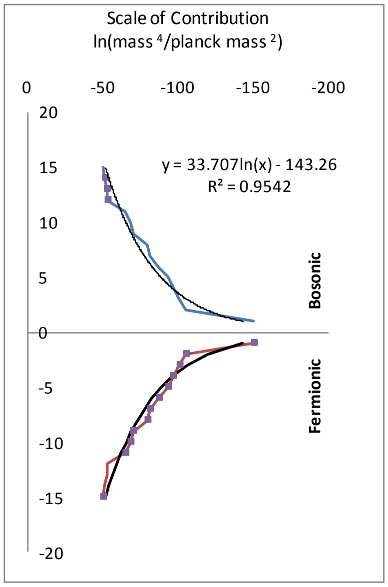

Figure 1. Plot of known fundamental particle masses in natural logarithmic units. The x-axis is vertical and indicates the position of the particle from lightest to smallest, with fermionic contributions taking negative values, bosonic taking positive. Taking the absolute value gives the actual ordinal position. The scale of contribution is the greatest contribution of the particle when summed.

However, this logarithmic relationship is also a clue as to how to address the issue at hand. The explicit expansion of particle contributions to the cosmological constant can be written as:

$$\frac{c_\Lambda \sqrt[4]{\Lambda}^4}{m_p^2} = \sum_i^N sgn(i)\frac{c_i \mu_i^4}{m_p^2} < 10^{-120}$$

Here, the cosmological constant given notation similar to the naturalization process required of all masses. The constants in the expansion should be relatively close to one.

From this we can also observe the log value of the first term of the expansion is proportional to the total number of fundamental particles, *N*:

$$\ln\left(\frac{c_1 \mu_1^4}{m_p^2}\right) \propto N$$

Further, the approximate ordinal position, *n*, of a particle with respect to the others can be crudely approximated as:

$$\left(\frac{c_i \mu_i^4}{m_p^2}\right)^{\frac{1}{b}} e^{\frac{kN}{b}} \cong n$$

Converting this to relationship and inserting it into our expansion, we get:

$$\frac{c_\Lambda \sqrt[4]{\Lambda}^4}{m_p^2} = \sum_{n=1}^N sgn(n) n^b e^{-kN} < 10^{-120}$$

We should see the above equation as a simple linear series which satisfies a constraint. The sum should equal some small number near the value of one (or even zero), with the exponent of the cosmological constant determined by the number of terms in the series. This implies the non-zero value of the cosmological constant is the result of error in the actual mass values due to uncertainty (or, perhaps an appropriate error function). If we consider the comparative case of the function where one subtracts the sum of lower values from the highest values:

$$f(N) = N^b - \sum_{n=1}^{N-1} n^b$$

For any value of the exponent *b*, the function will eventually shift from a positive value to a negative value and will do so quite suddenly. Figure 2 is an example plot of the phenomenon, using the value for *b* determined in the best fit line shown in Figure 1.

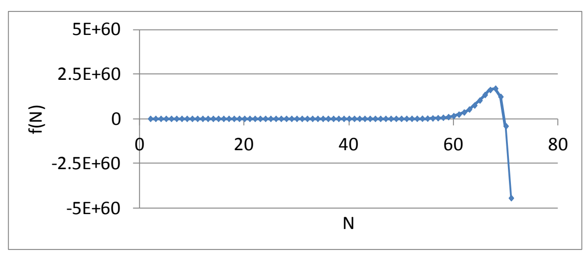

Figure 2. Graph when b = 33.707

The function in Figure 2 cross zero somewhere between the values of *N = 69, 70.*

Setting the function to zero gives:

$$N^b = \sum_{n=1}^{N-1} n^b$$

For each value of *N*, there is a value of *b* for which this expression holds. In case where *N =15*, the value of *b = 9.3573221003829…*; which was arrived at with a numerical calculation program.

The data used is in Figure 1 is empirical, and the sudden shift in value seen in Figure 2 is purely mathematical in nature. There is an interesting relationship with Bernoulli's formula and Bernoulli polynomials, but as of this writing, the relationship is still being explored. In any case, the above evidence shows it is not unreasonable or unnatural to expect a series with the addition of changing sign values to suddenly cancel even when the number of terms in the series are finite. In the case of *N =15*, with *n = 12*, *13*, *14* set as positive and all others as negative, there are two solutions at *b = 4.5398288172557…*; *b = 6.47982306289307…*; again solved with a numerical calculator. The two numbers result from two crossings of the function at zero. The first is going from positive to negative, the second from negative to positive. The suddenness of the changes and the excursion into negative values is reminiscent of the rapid expansion seen in inflation, where one can imagine there being a transition from one zero value to another, causing an inflationary epoch.

If we insert the derived expansion into the vacuum field equation and set the Planck mass and the remaining constant to one, we get:

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

$$R_{\mu\nu} = \frac{m_p^2}{c_\Lambda} \sum_{n=1}^{N} sgn(n) g_{\mu\nu}(n)\, n^b e^{-kN}$$

$$R_{\mu\nu} = \frac{1}{c_\Lambda} \sum_{n=1}^{N} sgn(n) g_{\mu\nu}(n)\, n^b e^{-kN}$$

$$R_{\mu\nu} = \frac{1}{e^{kN}} \sum_{n=1}^{N} sgn(n) g_{\mu\nu}(n)\, n^b$$

If the metric is the same for all values of n, then:

$$R_{\mu\nu} = \frac{g_{\mu\nu}}{e^{kN}} \sum_{n=1}^{N} sgn(n)\, n^b$$

The complex version can be written as:

$$R_{\mu\nu} = \frac{g_{\mu\nu}}{e^{kN}} \sum_{n=1}^{N} {z_n}^{2b}$$

As discussed above, the values of *b* associated with the value of *N* are solvable, allowing one to zero out the series, thereby arriving at the equation for a Ricci-flat manifold:

$$R_{\mu\nu} = 0$$

If the value of *N* is strictly associated with the underlying gauge group of a quantum theory, then the above expansion is a simple form of quantum gravity. Whether this approach can be of use in determining a candidate theory from the many that have been proposed has yet to be explored. Specific relationships, if any, to other Ricci-flat manifolds, such as Calabi-Yau manifolds are yet to be explored as well.

V. CONCLUSION: IT FROM WHICH BIT?

The ultimate question is not whether it arises from bit, but which bit does it arise from? Anthropic proponents are just as guilty as the fine-tuning proponents in that they impose one version of the universe on all its occupants. Clearly, a single observer must think the world is perfectly suitable for itself even if its existence is a random occurrence, and yet it seems terribly unfair for all the other observers to be pure flukes of the imagination.

It has been argued here it’s the averages that determine perceived reality, the classical realm is an approximation, classical states are not stable, there is a limit to our knowledge of the past, and the cosmological constant is perfectly natural. The result is our perceived shared reality is merely the product of averaging over knowledge gained through observations. This means that the “bit” is an average bit, thereby denying it a definite existence.

APPENDIX A. OVERVIEW OF FLOWS AND MEASURES

This concept of flow is important to the discussion of information in several ways. Principally, each component in our definition of flow (*G*, *X*, *φ)* has a certain amount of information associated with it. Furthermore, it is of principle interest to determine the measured value *x* that *X* takes after each iteration of group action is complete. Since a measurement often follows a process, it is reasonable to think of measurement as an act.

If we want to take a measurement of *X*, we must first define a sigma algebra, *Σ*, which is the collection of subsets of *X* that is closed and contains the complements of each subset of *X* and each union of subsets in *X* as well as *X* itself. The pairing of set *X* and algebra *Σ*, is called a measurable space, or sometimes a field of sets and is written as (*X*, *Σ*).

Since we now understand the sigma algebra, *Σ*, as being a collection of subsets of set *X* and their combinations, we can define the measure, μ, as the entity that takes each subset, σ, in *Σ* to some positive real number:

$$\mu : \Sigma \rightarrow [0, \infty)$$

Since *X* and *Σ*, are paired, this tells us that when a measurement is performed and *X* takes on value *x*, there is a definite subset, σ, composed of subsets of *X*, that the value *x* is representing. The combination of a measurable space and a measure creates a measure space represented by the triple (*X*, *Σ*, μ) [12].

This definition of measure space, while useful, is not dynamic and thus not entirely satisfactory. The process of measurement itself must be as dynamic as the definition of flow, and each iteration of flow must consider the measured values of the previous iteration as being part of its initial conditions. If group *G* serves to permute the subsets of *X*, then one must be careful in the definition of measurement to ensure that the information contained in set *X* is not lost by mistakenly thinking the information in the measured values of *x* is less than the information in set *X* itself.

It is desirable to think that each iteration of set *X* has some *Σ* associated with it, however, it does not seem reasonable to expect that the measure μ is the same with each iteration. This brings us into conflict with the concept of measure preserving transformations, ergodicity (e.g. the condition where a systems spatial and time averages are the same) and the ergodic hypothesis. However, the quasi-ergodic hypothesis tells us [7]:

> *A system's point in phase space will, in the course of its motion (determined by the differential equations of mechanics), come arbitrarily close to every point of its energy surface—indeed, the time it spends in any region of the latter in the long time average is proportional to the measure of that region.*

It is Von Neumann's proof of the quasi-ergodic hypothesis in quantum mechanics that showed that the theory was fundamentally correct, and the information was preserved as part of the fundamental imprecision of the theory.

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