The Reasonable Effectiveness of Math Gets Us From Non-being to Consciousness

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Abstract

The effectiveness of math is reasonable only to the extent that the assumption that math is taken to be a discovery is reasonable. Starting with the metaphysical assumption that the Abstract Realm of Mathematics (ARM) 'predates' the beginning of the physical universe, this leads to a logical progression from abstract logical supervenience and emergence through physical supervenience and emergence. Starting from the premise that space and time are as discrete as mass and energy, it would naturally follow that these building blocks of nature would follow the mathematical logic of all that is countable. The universe did not spring from nothing. It had an infinite number of steps to develop its potential configurations. Teleological bias enters as the conjugate attribute of existential threat experienced by any critically self-organized complex adaptive system with skin in the game.

Is mathematics ante rem (before objects) or does it only exist in-rebus (in objects)?[1] Is math a discovery or an invention?

If Wittgenstein insists that nothing should be said about things that cannot be known for sure and Kierkegaard insists that uncertainty is the highest rational truth then, with our embodied minds, having skin in the game, what are we to do?

Evolution seems to have equipped us with Bayesian brains to get around that impasse, to act in our own best interests by constructing a phenomenal model of the reality we are embedded in, predicting best-guess outcomes, choosing between them, and formulating an agenda to navigate in that direction.

My guess is that math has an existence ante rem as this seems to be more fruitful than the alternative assumption: that only physical objects have existence and the whole of mathematics only exists in the mind as concepts.

The abstract logic and structure of mathematics gives all physical things their commanding form, all the more so the more closely things are examined. These mathematical relationships are extracted as discoveries, not inventions imposed post-hoc as explanations.

Perception

Our explanations begin with the phenomenology of perception: the only information we have to go on is the information we take in with our senses. With as many attributes as we can discern, our mind reconstructs an image which purports to explain what is going on. By collecting and crosschecking this information into a paradigm -- a Standard Model of accumulated scientific knowledge --, we make it as objective and accurate as we can. We must then turn around and take it back in as more perceptions.

We think by analogy. We compare the features of a thing we are trying to understand with something else we understand better. We refine our understanding of things by refining the comparisons. Our best comparisons of things and our deepest understanding of them come from comparing their features with mathematical relationships.

Our perceptions get us asymptotically close to a sensible picture of reality in so many situations but so far away in others. It is all a matter of what assumptions we jump to with the axioms we accept as being obviously true. For instance, relax Euclid's fifth postulate and all of the richness of Riemannian space springs from the flat and stable space of solid geometry.

The Abstract Realm of Mathematics

I take as my first axiom the abstract existence of being and nonbeing. This is a restatement of the axiom of the empty set. Following on that by logic is the relationship between the two: the first duality. Set theory takes the empty set to be what we label as zero and the set that contains that to be what we label as the natural number one.

I take the axiom of infinity as the second. By the axiom of infinity, each new element in the set of abstract objects is self-generated from its preceding element. Set theory generates the natural numbers out to the first infinity which is countable, $\aleph_0$. All of the possible combinations of relationships between these natural numbers, and thus all of the possible mathematical structures, exist within the power set of $\aleph_0$. This takes us out to the first uncountable infinity, $2^{\aleph_0} = \aleph_1$. Intermingled within this power set is the subset of mathematical rules that constitute the eigenstates of physical being; relationships that will inevitably lead to Schrödinger's equation, complex numbers, Bose Einstein and Fermi Dirac statistics and the Heisenberg uncertainty principle.

Along with the axioms of set theory, which I take to exist ante rem, I take as an axiom the duality [supervenience/ emergence] as a restatement of the axiom of infinity. Each new element which emerges is supervenient upon its predecessor for any abstract, logical or structural relationships that might accrue. Apart from nonbeing, every emergent relationship follows from a duality.

It is not unreasonable to assert that infinity exists ante-rem in the Abstract Realm of Mathematics and that any in-rebus implementation of the physical universe is not infinite, given its beginning as a quantum fluctuation at the Planck scale and the finite length of time the physical universe has been in existence. Given that infinity consists of the totality of all mathematical relationships, it contains within it not only all that is physically possible but also all that is not.

We sort subsets of these relationships into branches of mathematics to organize our thinking, somewhat arbitrarily dividing them into sets which have application to specific real-world situations. And we assign names to concepts we form around these relationships. But the relationships are simply there irrespective of any grouping or categories to which we might associate them. All of the numbers and relationships are there simultaneously but they do have a first-order structure based on sequence.

Why does nature behave in lockstep with math? Once the effect of discrete granular structured space and time has been considered and assumed for the sake of argument, and a differential geometry has not been arbitrarily applied to a flat structureless space-time background, it simply follows that a universe composed of numbered and countable elements would naturally follow the laws and logic of mathematics. Logical entailment in the ARM leads to physical entailment in the in-rebus implementation (IRI). An in-rebus implementation is simply a physical system in which a mathematical structure is instantiated.

The Pre-geometric state

The study of pre-geometric states was first formulated by Wheeler when he conceived of an underlying structure to the universe independent from a background metric of space-time as a bucket of dust.[2] With Boral sets as one likely candidate for its mathematical structure, the field of study has become a highly developed field of inquiry. The pre-geometric state is a precursor state to the Big Bang.

Just as sets precede fields, which precede topology, which precedes geometry in a supervenient/emergent hierarchy of mathematical relationships. "Before" there was space or time, there was pre-geometry in the ARM. This accrues in just the same way that the egg for the first chicken was laid by a "pre-chicken". In the set of emergent relationships, some are more basic than others. Pre-geometry embodies that set of relations that contain proximity without distance and sequence without duration. The equality of all possible juxtapositions of points held in superposition is the pre-geometric condition.

Contained within the power set in the ARM, is the Boral set and a sigma-algebra that bestows a probability function on it. Boral sets have the same cardinality as the real numbers. A Boral set is of the continuum but it is a subset of $2^{\aleph}$. The Boral set is the collection of sets that can be formed from open or closed sets by union, intersection and complement.[3] Think of it as most of the relationships between all the sets that can comprise it. The Cantor set is a Boral set which can be used to visualize the co-creation of space and energy, irrespective of its actual connection with the pre-geometric to geometric phase change that led to physical reality.

Zeno's paradox would seem to render it impossible to combine an infinite number of dimensionless points into the single step needed to start any journey. But what of an infinity of discrete infinities given one of them is composed of Cantor dust: the infinity of points on the continuous line between zero and one broken and re-broken and re-broken into thirds and thirds again until there are an infinite set of discrete infinities between zero and one. Each of them a Boral set equipped with a sigma-algebra and all of the mathematical structure that that bestows.[3] This is how Zeno can proceed.

The First Symmetry Breaking

Now consider a symmetry breaking, the collapse of the part of the universal wavefunction that led to the crystallization of one geometry out of the potential set of all possible eigenstates held in superposition. Symmetry breakings are generally associated with phase changes in which a system loses degrees of freedom. Fewer choices remain in potential.
I envision at least three modes of wave function collapse: quantum probability (indeterminism), classic determinism (decoherence) and conscious choice (statuo ergo sum). The quantum leap must rely upon probability alone. We can see this in the leap from nonbeing into being of a virtual particle by virtue of the vacuum expectation value.

Usually this symmetry breaking is thought of as an equal amount of matter and anti-matter spontaneously created, ex nihilo, out of the vacuum. This is not the ersatz nada of the false vacuum which is said to have structure and which is really what we are looking for in terms of space. The void has no structure. It cannot be said to be smooth or continuous or flat. It has no physical attributes of any kind.[4]

Symmetry breakings can only occur along discontinuities in the mathematical structures that their in-rebus implementations embody.

If indeed the physical universe is quantized into physical building blocks, discrete with respect to space and time as well as mass and energy, then our smooth continuous differentiable emergent abstract mathematical logic that extends past the integers into the real and complex numbers are only approximations in the limit as the finitesimal approaches the infinitesimal with its discrete structure intact. The suchness of a curved line is not diminished by division into finitesimal parts. The character of physical law would disappear if each point in space was truly dimensionless and structureless as in the abstract Euclidian limit. Rather the character of physical law accrues as a result of the structure, dimensionality and discrete nature inherent in its fundamental building blocks.

The separate elements of Space, Time, Energy and Mass (STEM) come down to us as the separate building blocks of broken symmetries. We know that Space and Time (S) and (T) can be combined into a unity by way of special and general relativity. We know that Mass and Energy (M) and (E) symmetrically fit together as a unity through $E= MC^2$. We also know that the pair MS (in the form of momentum and position) and the pair ET form conjugate attributes in Heisenberg's uncertainty principle.

Physical objects do not follow equations, but instead follow the mathematical relationships to which equations point. They obey all the logical relationships that count. Equations are conceptualizations of the underlying relationships between numbers which form the basis and the heart of our understanding of math.

Our background then is a pre-geometric one -- not composed of a void or absence of structure but rather of the infinite richness of all possible geometric structures, existing in potential superposition, from which the one physical reality we now inhabit precipitated through a symmetry breaking phase change, the collapse of the wave function of the universe.

Only the infinite fecundity and structural richness of the abstract realm of mathematics can provide a substantial enough background which will allow the two primary entities of physical existence, grains of space and quanta of energy to come into co-contingent being.

The Grand Opening

Three of the dimensions opened up and unfurled to produce what we now perceive to be our 3-D world. The other closed dimensions compactified down near the Planck scale and can stretch out luxuriously with nodes, bumps, crevices and crannies stretching up from 10^{-35} meters to 10^{-17} meters or so to the size of a quark in the nucleus of an atom. But no one really knows for sure how much geometry or physics lurks in this eighteen-orders-of-magnitude-wide band of size difference. That same 10^{18} jump in magnitude of scales gets us from the size of an atom to the size of the orbit of Venus around the sun.

The Planck scale is thus the first quasi-constant. The exponential variation with time of the first grains of space forming is just another way of thinking of inflation; both expanding and contracting at the same time. With only that first grain, there is no basis of size comparison, but the vacuum expectation value, which can be thought of as the space-energy co-creation potential, adds more grains of space and quanta of energy at an exponentially decreasing rate. In the regions of space between galaxies where these grains are spaced far enough apart, new

grains pop into being along with a bit of dark energy consistent with the quantum statistical probability we variously call vacuum expectation value or the cosmological constant.

The co-creation of space and energy is more like a grand opening than a big bang.

A re-imagining of the nature of space and time

When we think about particles and their interactions we must conjure a background against which to imagine them. For us it is most natural to see physical processes occurring within a three-dimensional space which classically has been taken to be a flat featureless void. Then we go on to assign a host of geometric attributes to the space -- such as flatness, curvature, and a metric with an associated coordinate system giving a way to calculate distances between points.

We grant these attributes an abstract existence and in keeping with general relativity give the space itself a real physical reality. And we go on to grant a reality, intermediate between abstract and physical, to the fields, and their associated virtual particles, and the imaginary phase space which they inhabit. Indeed this extended space leads into a host of higher dimensional realms in string theory.

Contrasted against this void, the false vacuum of space pops out as a thing with structure. It can have curvature, a frothing uncertainty, a vacuum expectation value, but above all it comes with a metric, an inherent dimensionality; more potential than substance, it limits the number and types of vibrations that are permitted to occur. From our understanding of QM, this structure can be said to have a minimum size: the Planck length, or 1.6×10^{-35} meter.

Grains and particles

We need to have a replacement for the point in space that both possesses a structure and allows us to visualize its properties. In his book, The Shape of Inner Space, Shing-Tung Yau describes a candidate mathematical structure, the Calabi-Yau Manifold.[5] In order to describe a similar structure with additional properties that might aid visualization I will dub these grains of space the Mandelbrot-Yau Orbafolds (MYOs) and speak of them interchangeably.

Now as we begin to see this physical space as quantized on the Planck scale in the form of MYOs, we again find ourselves in need of a background against which to visualize these grains just as we did for the particles of mass energy.

One of the key ideas to reimagining the grid is to realize that nature does not “do continuity.” The grid, and the field interactions that connect the MYOs to their neighbors, is discontinuous. Each MYO is a discrete location of space (not *in* space!). “Between” them is the true void, nothingness. In your head, redraw the grid with dotted lines. Think of each dot as an MYO. Their orientation in a causal lattice constitutes the metric of space.

The “rubber sheet” analogy doesn't explain a thing, instead it just reorients the planets’ vectors (that originally pointed to the sun) in new directions, downward into the depression the sun makes in the rubber sheet -- so now it points in the direction of the thing it is trying to explain. It's not one huge funnel with the planets on a rubber sheet and the sun at the bottom; it's zillions of tiny funnels with their tiny Klein holes twisted around and pointing normal to the direction of the highest concentration of mass-energy. The geodesics of space-time follow the lattice which is differentially twisted and compressed by the mass-energy stored in the MYOs.

It is useful to think of the particles that inhabit these discrete locations in space as also having structure. Here we can imagine the constraints that would be imposed upon them by the structure of the MYOs as string vibrations.

In his E8 simple theory of everything, Garrett Lisi[6] has uncovered the known symmetry groups: the U(1) of electromagnetism, U(2) associated with the weak force and the SU(3) of QCD within the exceptional Lie group E8 and its associated eight-dimensional polytope relating all of the known particles: baryons (quarks, hadrons and mesons), fermions (electrons, mu and tau leptons and neutrinos), and the force-carrying particles, the bosons (photon, gluon, B and Z boson and the Higgs). All these known particles with their respective antiparticle partners almost completely fill out the 248 symmetries within this eight-dimensional phase space which leaves room for another 20 yet-to-be-discovered particles such as the graviton.

The eigenstates of the possible strings are the possible energy vibrations on the orbafolds. You can think of an MYO as a bell that will ring in 248 modes. Each mode would constitute a type of particle.

Starting at the Planck scale, 10^{-35} meter, and extending up into the unexplored region of miniscule dimensions up to the size of quarks and atoms there is room enough to encode all of the physical parameters of masses and interaction strengths. The range in values of the quasiconstants would seem to require a proportional range of scales within the indices of the orbafolds.

At this point it is helpful to have in mind a good understanding of the main difference between fermions and bosons. Fermions obey the Pauli exclusion principal; they cannot be in the same place at the same time. Bosons obey Bose-Einstein statistics and do not obey the Pauli exclusion principal. Before the second symmetry breaking, all the energy can be contained within one MYO because of the finite probability of finding any number of bosons together at zero spacing. This configuration would constitute a metastable symmetry that a quantum perturbation would topple into a cascade of space-energy production.

The grid is minimally-fixed by the Higgs field, in accord with Mach's principle of a universal rest frame. Inertial mass comes from and is conserved by the Higgs field. A change in velocity, either speeding up or slowing down, requires a force. Changing from one geodesic to another requires a change in energy. For Einstein, $G_{\mu\eta}$ was the new ether over a continuum of structured space. Now the Higgs field is the new luminiferous plenum over the discretum of structured space.[7]

There is a perplexing interplay between the raggedy edge of existence at the Planck scale and that mysterious imaginary quantity, i, the square root of minus one that we may loosely associate with our ideas of what these compactified imaginary dimensions might be like. Quaternions[8] and octonions provide a fruitful field of exploration for connections between these compactified dimensions on the MYOs and the complex numbers as a bridge between the ARM and its IRI lurking within the inner product and the interference pattern it might induce. Octonion triplets rotating in 8-D space behave like quaternion doublets. Both sets square to -1.

From Feynman's sum over histories, particles do not take simple straight line paths but all possible paths from A to B, with equal probability amplitudes, and only the phase by virtue of the Huygens principle of constructive and destructive interference determines the evolved

state of the wave function at position B. This is more in keeping with discrete quantized space with a complex path between neighbors. L. F. Abbott showed that the path of quantum particles has a fractal Hausdorff dimension of two.[9]

All things physical come into and go out of being as interference patterns on the causal lattice. Any particle with spin has a component of its existence in one of the complex planes of a compactified dimension.

Time

Emergent from the grains of space and the quanta of energy is the relationship or link between them. All motion in the universe is derived from the movement of the quanta from grain to grain by way of a quantum jump. Time arises from the minimum delay in duration for this jump. The propagation of an electromagnetic field, for example, spreads at the speed of light, from grain to grain, at a mean Plank spacing of 1.616×10^{-35} meters per Planck time of 5.39×10^{-44} sec, or 299792458 meters/sec. The local spacing between each graininstant of spacetime carries with it its own yardstick and its own clock. The appearance of these local yardsticks and clocks appear to be Lorentz-Poincare transformed as viewed from other inertial and non-inertial spatially separate reference frames.[7] The relativistic warping of space and time is a result of the change in spacing or rotation due to the action of a field.

A Self-Tuning Universe

As the various curvatures of the unfurling and compactifying dimensions (and their ratios) diverged, the quasiconstants of nature departed from unity to assume their present values. And in this process they fine-tuned themselves by self-organized criticality.

The quasiconstants **c**, **α**, **ħ**, all the particle masses, and coupling constants all change with the changing indices of the orbafolds that they occupy. The equipartition of the vacuum expectation value throughout the expanding number of grains goes as the inverse of their increasing number.

Complexity and Self-Organized Criticality

Agency resides in any complex adaptive system (CAS) that interacts with its environment through feedback loops. The interaction between the environment and the CAS flows both ways and each in turn is changed. If the CAS is a dissipative critically self-organized system, its state is progressing at the border between order and chaos. [12, 13, 14, 15]

Each interconnected part of an ecosystem can be an agent of change for the rest of the system.[16] This causality echoes back and forth between a system's parts in a cyclical reverberation through the feedback loops that connect them: negative feedback loops damping these cycles into stable configurations and positive feedback loops amplifying the cycles with the potential of pushing them through chaotic regions of the system's phase space into emergent configurations of higher complexity or into new islands of stability[17, 18] found in any chaotic system. Add to this the intentionality of volition, born of the teleological bias found within any organism with skin in the game, and you have the makings of a co-evolving ecosystem complete with a fitness landscape where its agents are driven to preferentially choose pathways, each according to its own best interest. At some point in any scientific account, intentionality is added in to gain explanatory traction: Nature abhors a vacuum.

Emergence of Purpose: Evolution and Teleological Bias

Now we jump from particles to living things. The science between these two ranges of phenomena is well explored territory.

Of course there is a much deeper problem: how can it possibly be argued that any form of intentionality precedes an implementation of life? In lieu of this, what can possibly motivate evolution? The classic Darwinian answer to this is random probability. Those that hold that position have not looked at the odds against it: the random chance of assembling a minimally viable, self-replicating 1000 base pair structure out of the soup of 20 amino acids is 20 to the 1000th power and comparing that with the total number of particles in the universe, 10 to the 81th power, the probability of random assembly is almost infinitesimally small.

A random walk driven across the probability landscape by the dissipative processes of self-organized criticality might very well be nature's first stumbling attempts at a utility function of fitness for service. The mindless default process of self-organized criticality drives nature to a first order approximation of the abundant commanding form we see in the inanimate instances of fractal shapes such as clouds, mountains, lightning, sand dunes, a meander, breaking waves, the dancing fingers of fire, all self-similar but never repeating exactly. All these things owe their structural forms to the variability of their Hausdorff dimension in phase space.[13] The emergent commanding form we see in the physical universe mirrors the emergent commanding form we discover in the ARM.

The teleological turning point seems to come at the point of evolution of eukaryotes. Up to this point, the slow build of self-organized criticality of complex adaptive systems following a mindless auto-catalysis[15] led to a level of complexity in organic chemistry that culminated in self-reproducing single-celled organisms. The advancement to multi-celled organisms with nervous systems, the first primitive sense organs, marked the point at which there could be the beginnings of agency in an individual organism. This led to the Cambrian explosion 543 million years ago, with its rapid proliferation of multi-celled species and the concomitant teleological bias that the perception of existential threat nerve endings bestow upon their suffering owners. Thus begins the mindful (albeit most primitive) self-interest of the individuated agents inhabiting the co-evolutionary fitness landscape.

There now seems to be a paradigm shift back toward the efficacy of Lamarckian inheritance in the evolutionary process. Whatever adaptation to an environmental stress an individual member of a species has been able to effect may somehow be encoded into a modification of its genetic code. John Cairn's work on epigenetics describes lactose enzyme-deficient bacteria grown in a lactose medium. The bacteria would undergo a mutation to subsequently be able to produce this enzyme. An enzyme coded for by a particular gene was feeding back to change that gene itself.

The tree branch is stressed by the wind. The cells that received a threshold of stress, mostly at the outermost transverse fibers, are modified and begin to grow. Cellular damage is the precursor to pain receptors.

Shortly after nature invented the first sense organs, the very first word was spontaneously derived to express the essence of this new sensory input: *ouch!* All critters, from eukaryotes on up, have "skin in the game." It took Nature 3.3 billion years to develop the nerve

end but less than 25 million years after that to develop a grand profusion of viable and competitive versions of the brain.

For the up-and-coming organism there is often a need to resolve conflicting impulses such as whether to remain safe in the cave or venture forth in search of sustenance. If one course of action is compelling or straightforward enough, then the decision to act can be based on instinct. In complex situations there is an overwhelming amount of accumulated associative information surrounding the myriad course of open actions. A heuristic, rule-based system doesn't work so well when dealing with novel situations. It must be supplemented with the creative act of thinking "outside the subconscious box." Consciousness emerges to resolve the puzzle.

Consciousness

The sentient observer, always the clever accountant, finds nature to be accountable. Look at a mountain (or anything produced by a natural process) and you will see self-organized criticality. Look at a city (or anything produced by the hand and mind of a sentient being) and you will see teleological bias.

The future is open to choice to the extent that an observer can predict the future then she can select between eigenstates by an act of free will according to her T-bias.

"I perceive, feel, think and decide -- therefore I am" should be the obvious axiom from which we proceed. We are just beginning to unravel the great mystery of how consciousness came to be. We may never fully know the secrets of its in-rebus implementation. But given the obvious origin of the teleological bias right at our fingertips, there should be no evolutionary mystery as to why. Consciousness evolves around a sense of jeopardy.

All this category symbology and conceptual organization is our reconstructive attempt to organize the underlying relationships that reside in selected subsets of the power set of all relationships, $\aleph_1$. They are consistent and they can be searched through for the ones that seem best to correspond with the physical systems they purport to describe. These mathematical concepts and symbols are all in service of our phenomenal reconstructions. But, at the same time, they represent the logical abstract relationships that are there to guide the evolution of the universe at every step in the way. The more mathematical structures are discovered the more there are to choose from to aid the visualization.

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