Complex Time, Causality and the Anthropic Principal

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Abstract

Complex time, with real (sequential) and imaginary (simultaneous) components facilitates the requirement of an observer to collapse the wave function of the universe. The end state informs the beginning. The infinite configurations of virtual n-dimensional space-time offer a fecundity of choices with consistent and persistent histories of our familiar four-dimensional space-time. The future becoming the past is concurrent with the collapse of the wave function, both in a global sense and within us, a collection of localized and individuated observers.

The logical mind has a strong bias to think in terms of cause and effect. But to search for a cause for the first effect, the formation of the Universe, leads to a chicken-and-egg conundrum, a synthetic a priori hypothesis. The time might be ripe to look at an illogical, or at the very least, acausal explanation.

Here the logical mind has a strong bias to associate "logical" with "true", and "illogical" with "false", but Gödel's incompleteness theorem [1,2] tells us that there are true propositions, which can not be proven within any given logically consistent system. These propositions are logically discontinuous, or not to put too fine a point on it, illogical but true.[3]

Types of time

That time can be cyclical, in some respects, seems to be a feature of nature. Clocks cycle through the hours from midnight through noon and back to midnight seemingly endlessly. The 9,192,631,770 oscillations of a light wave emitted from a cesium atom give us our precisely measured second. But for a life these ticks and cycles mark out a measured pace for a beginning and an end. This sequential directionality is also an undeniable feature of time. Entropy renders time irreversible.[4, 5]

Most familiar to our subjective sense is tensed time: past, present and future. A more objective tenseless time puts events into a sequence of before and after.[6] A highly malleable psychological time allows our perception of events to transcend their normal limits in times of stress.[7] There is block time, which treats space-time as a four-dimensional 'block' and denies the objective flow of time in favor of an eternal now. Einstein said of it "For us devout physicists, the distinction between past, present and future likewise has no significance beyond that of an illusion, albeit a tenacious one."

Any single notion of time seems to cover only half the bases. Block time fits with relativity but not the subjective experience of time. Tensed time covers the subjective experience but not relativity.

As cosmological observations begin to accumulate we see an expanding universe and run the clock backwards to a beginning point, the big bang. This gives us a number of possible outcomes for the endpoint: The single bang that continues to expand outward forever, The single bang that, given enough mass, collapses back upon itself into a Big Crunch. A cycling of bang-crunch-bang-crunch. And there are new cyclical cosmological theories that do not require singularities, inflation or boundary conditions.[4, 8]

Relative time and the death of free will

Newtonian time, absolute, infinite and passive, which is uniform everywhere and the same for all observers fell to a less intuitive but yet more accurate relativistic space-time where the flow of

time and even the sequence of events is dependant upon the position and velocity of the observer. The following is an excerpt from The Labyrinth of Time by Michael Lockwood:[6]

> "Suppose, however that we follow Einstein in regarding the contents of all parts of space-time as being equally real. Then, as far as the free-will debate is concerned, it becomes simply *irrelevant* whether or not the universe is entirely governed by deterministic laws. For regardless of whether our future choices and actions are fixed relative to earlier events or states of affairs, they are, if they are real, fixed absolutely in virtue of their reality alone.
>
> But are we actually obliged to take this line? Strictly speaking, no. Nothing in the physics of special relativity actually forces us to abandon the common-sense picture, according to which there is an objective, albeit constantly shifting, boundary that separates the real, and wholly fixed, past from a currently unreal, and partly open future. We can, if we wish, postulate the existence of a uniquely privileged foliation or 'slicing-up,' of space-time into simultaneity hyperplanes, the contents of which are successively actualized by way of an objective passage of time. It has to be said, though, that, in the absence of any *scientific* reason for doing so, this would strike many people as comparable to embracing an article of religious faith. What we cannot consistently do, however, is simultaneously accept that there is no such privileged foliation –thus regarding all inertial frames of reference as metaphysically on a par with each other—and yet resist regarding the contents of all regions of space-time as equally real."

It turns out, as we will see, that the reports of the death of free will are greatly exaggerated. This argument against free will is based on the exclusive validity of temporal determinism. This notion is valid for classical reality but it is not valid for quantum reality.

The illogical nature of quantum mechanics

At the turn of the twentieth century, the great revolution in physics of the development of quantum mechanics brought to light a great dichotomy in nature: a true paradox. It is a discontinuity that is at the root of being.

Classical Newtonian-Relativistic physics and Quantum Mechanics describe two mutually exclusive principals of nature, which are at work at the same time; both valid; both amenable to scientific analysis and experimental verification. Indeed, QED (Quantum Electro Dynamics) [28] is the most successful theory in science. The predictive power of this theory agrees with experiment to eleven decimal places.

The hallmarks of Newtonian mechanics are its certainty and exactitude; it is totally logical and in conformance with cause and effect. The hallmarks of quantum mechanics (QM) are its acausal and probabilistic nature. It is subject to Heisenberg's uncertainty principle, which limits what can be know about a given system. The quantum state, operant at the scale of quarks, atoms and molecules (and, indeed, larger systems – the only requirement is coherence) and again at the scale of super-massive black holes, is described mathematically in terms of a non-commutative, probabilistic wave function† characterized by a system simultaneously possessing wave-particle duality; capable of interfering with itself; being in a superposition of all possible states (being in two or more places at the same time), taking all paths through space; and being entangled.[9] All parts of itself are in instantaneous communication irrespective of the extent of physical separation. If you leave any of these features out, you get the wrong answer!

These strange aspects of reality are usually described to the lay reader using the two-slit experiment as an example.[10, 11, 12, 13]

The results from the two slit experiment are so goofy that the experiment has been conducted many times in many forms with ever increasing levels of accuracy and sophistication to try to eliminate the discontinuity or to refute the result, but to no avail.

One neglected aspect of the two-slit experiment is what happens to the wave function when it interacts with the screen. In the experiment without external observation it goes to completion with the collapse occurring naturally by decoherence. The results bear the unmistakable signature of

wave coherence and superposition pointing to the complex nature of the information that existed before the collapse. In the observed case the imaginary component of that complex information has been removed by the observation and only the real component remains.

Another interesting aspect of quantum behavior is a phenomenon called wave entanglement where, if two or more particles 'cross paths' or interact in a quantum way, they are said to be entangled. If an observation is made on any one of the particles, all of the others are immediately affected, no matter how far they are separated in space-time. This is also known as action at a distance.

The collapse of the quantum state to a classical state is dependant on the observer. The observer can arbitrarily choose which of the two mutually exclusive attributes, particle or wave, manifests. Archibald Wheeler put forth the notion of a participatory universe. In Wheeler's words, "the weak anthropic principal allows for an observer; the strong anthropic principal requires an observer."

Einstein fully understood that quantum mechanics was illogical and refused to accept its acausal nature and spooky action at a distance. He famously said, "The Old One does not play dice with the universe!" He clearly saw that the concept of block time is incompatible with the concept of probability, which tacitly assumes an open future. Assigning weights to various outcomes assumes the possibility of those outcomes. The outcome of the tossed coin, buffeted by Brownian motion, hovering between heads and tails (the calculation is intractable), can not be predicted. But no matter how sensitive to initial conditions it may be, if those initial conditions are frozen in time, then the outcome is fixed. Einstein contended that there must be some hidden variables that would make the description of reality complete. In answer to these objections, stated in the form of the EPR paradox, Bell's non-locality theorem [15] rules out the possibility of any local hidden variables and reveals this aspect of reality in all of its illogical glory. Bell's theorem, experimentally verified by Aspect and others has shown that this information transfer is simultaneous within the limits of the experiment. Experiments over distances of many kilometers have shown this information transfer to occur at a rate exceeding 10,000 times the speed light. [16]

One of the main arguments that has been deployed to tame quantum weirdness is the contention that the photon used in the measurement process disturbs the quantum system just enough to cause the uncertainty. However, as in the case of decoherence, this does not explain wave-particle duality or superposition, or the temporally acausal features of entanglement as demonstrated by the quantum eraser experiment and the experimental verification of Bell's theorem.

Quantum Mechanics is *illogical*, if only by philosophical predisposition, and *in only one respect: it is not temporally causal*. To steadfastly hold to temporal causality as the single operant principal in the universe is to deny the acausal nature of this most fascinating mode of being.

I should mention in passing, the existence of closed time-like curves, discovered by Gödel, which exist as solutions to Einstein's field equations.[6] Whether or not these CTCs facilitate the transmission of end state information to the initial state of a quantum system is an open question.

Quantum computing

Computers store bits of information in groups. The number of bits in a group determines the number of states that can be represented. Each additional bit in the group doubles this amount. One bit gives two possible states, two give four, eight give 256. In a conventional computer, each of the bits can only be in a single state at any given time, thus a group of 8 bits can only be in one of 256 possible states. Each state must be acted upon in a sequential manner, say for factoring a number. In a Quantum computer the group of 8 bits (qbits) is held in superposition, thus the group is in all 256 states simultaneously.

The field is in its infancy. So far, only small numbers have been factored in this simultaneous manner. One group of materials that shows particular promise for holding these qbits in superposition is organic molecules.

The evolution of life through random chance alone seem highly unlikely. To self assemble a string of the one thousand base pairs necessary for self replication from a subset of four out of a soup containing over twenty amino acids gives one chance in 20^1000; many more orders of magnitude than the total number of particles in the known universe. To oversimplify, a quantum computer with twenty groups of qbits in one thousand superpositions collapsing to the solution that works solves this puzzle more efficiently.[13]

Chaos, dissipative systems and self-organized criticality

Large interactive systems evolve to a state of critical complexity where a single small input can precipitate anything from a small to a catastrophic or universal change. The adage, 'the straw that broke the camel's back' encapsulates this. Self-organized criticality (SOC) is characterized by the following attributes: They are dissipative systems, which operate far from equilibrium, exhibit coherence, non-linearity and sensitivity to initial conditions. Some examples of SOC systems are earthquakes, avalanches, market crashes, populations and most simply, in the angle of repose of a sand-pile.

Criticality is reached when the probability of branching and non-branching reactions is balanced. Both the super-critical (overly complex) and sub-critical are attracted to the critical state. "Fractal structures and flicker noise are the spatial and temporal fingerprints of self-organized criticality."[35] Whereas entropy can start out from any initial condition and evolve irreversibly to the stability of equilibrium, SOC is critically dependant upon initial conditions and evolves to a metastable state. Biological systems exemplify all these attributes.

Looking to higher dimensions

Kaluza, mathematically exploring a space of 5-dimensions, discovered that Maxwell's equations of electromagnetism could be derived from its properties. Klein suggested that this additional dimension was rolled up or compactified. Murray Gell-Mann used the symmetry groups of Lie (pronounced Lee) algebra to describe the transformations of subatomic particles. From this he described the workings of the particles he dubbed quarks and went on to predict additional particles then unknown but subsequently discovered.

String theory, formulated in 10-dimensional space-time, is now being explored as a possibility of unifying the three forces of the standard model from the quantum domain with gravity. The five most consistent versions have been shown to be mappings into an 11-dimensional space-time of M-Theory.

Garrett Lisi has formulated a very elegant theory using the E8 Lie group of 8 dimensions which naturally contains the electromagnetism of (U1) , the electro-weak unification of (SU2), and gravity SO(3,1). This formulation predicts the existence of the Higgs Boson, a particle required by the Standard Model, as well as a number of new particles; possibly those predicted by super-symmetry.[18] The E8 symmetry group also appears in string theory.

Looking to the dimensions in between—Fractals

The self-similarity of nature's forms faithfully follows a fractal function. They are the marriage of real and imaginary numbers. Fractals point to forms in nature that evolve in space such as clouds, mountains, trees, blood vessels, and the nervous system. The formation of solar systems follows the pattern of formation of galaxies.[20] They are, in fact, ubiquitous. The universe bears their imprint on *every* space-time scale. They tie into the chaotic, non-linear processes that evolve in time; the dripping of water and a heartbeat.[21]

The Platonic realm of mathematics

Perhaps the most enigmatic thing with being is the Platonic realm of mathematics (PRM). It is at once the most ephemeral and the most highly and richly structured thing we can imagine. But its existence is independent of our imaginings. All things in nature, from the big bang onward, mirror its structure and follow its rule. It is the embodiment of all information without a medium. Where is it written? It is ubiquitous, it is specific and exact. Identity, equality, symmetry and sets. Sets take in objects and their relations also as objects. Non-being is the unique member of the empty set. That empty set is the first object with being. This, in turn spawns the first relationship, the duality of being and non-being. All the numbers arise from sets: natural, rational, irrational and real.[2] Fibonacci numbers and the golden section blooming forth into sunflowers and pinecones, and ticking away at the heart of the dodecahedron. Most pertinent to this discussion are the imaginary numbers that give birth to Euler's equation,[22] $e^{i\pi} + 1 = 0$ (which Feynman called the most remarkable formula in math), and also the richness of fractals. It embodies infinity. It embodies 3-dimensional and higher Euclidian space, Riemann and Hilbert space, and the infinite forms of Calabi-Yau orbafolds in an infinite number of dimensions. The PRM is a set of abstract relations, which is independent of physical being and independent of the symbols that conscious beings use to encode them. A mathematician discovers the abstract relationship and forms it into a concept then invents a symbol or a word to encode it. Neither the word or the symbol or the concept is the relationship itself.

Along with the Physical world and the Mental world, Roger Penrose considers the Platonic mathematical world to be one of the three mysteries. He illustrates this in Figure 1.3 & 1.4 in The Road to Reality[23]. He writes, "I am allowing that only a small part of the world of mathematics need have relevance to the workings of the physical world."

Just as the abandonment of Euclid's fifth axiom opens up the vastly richer realm of curved space, so nature, by abandoning exclusive temporal causality, has provided herself with a mechanism to be able to choose, from the fecundity of mathematical relationships, the subset that works for physical reality. Temporal causality and consciousness are co-determinants in the universe. This unverifiable and un-falsifiable belief in the efficacy of conscious choice *is* tantamount to a leap of faith, but it is a simple explanation which is in agreement with the experimental evidence.

The transfinite number of possible configurations of the Calabi-Yau orbafolds [11, 31] provides us with an embarrassment of riches. Within this fecundity of infinite possibilities is the possibility of simultaneous time: of all possible universes existing simultaneously in superposition. This brings us to a serious logical problem having to do with temporal causality. The logical intractability of determining the origin of this discontinuous transition – how one possibility is chosen from the superposition—led Hugh Everett III to formulate the many worlds hypothesis which basically denies that the wavefunction collapses. Each possible superposition gives rise to an additional universe. All possible universes are extant, either spread out in space with others identical to ours at an average spacing of $10*10^{118}$ meters [24] or having existence in other, inaccessible dimensions.

Imaginary time

In A Brief History of Time, Stephen Hawking sets out requirements for a unified theory:

> "We are fairly certain of some features that a unified theory should have. One is that it should incorporate Feynman's proposal to formulate quantum theory in terms of a sum over histories. …. One must add up the waves for particle histories that are not in the "real" time that you and I experience but take place in what is called imaginary time…. That is to say, for the purpose of the calculation one must measure time using imaginary numbers, rather than real ones. This has an interesting effect on space time: the distinction between time and space disappears completely."[4]

Attributes of the Observer

Andrie Linde takes note of the fact that:

> "…the essence of the Wheeler-DeWitt equation (DeWitt 1967), which is the Schrodinger equation for the wave function of the universe, is that this wave function *does not depend on time, …* Therefore if one would wish to describe the evolution of the universe with the help of its wave function, one would be in trouble. *The universe as a whole does not change in time…* In order to answer this question one should first divide the universe into two main pieces: (i) the observer with his clock and other measuring devices and (ii) the rest of the universe."[25]

Duality is fundamental to observation.[33] It provides the necessary contrast between object and background. Whatever attribute can be assigned to the universe, or any object in it, the observation will conjure its opposite or conjugate attribute. Both halves of the duality must be present to provide contrast and produce the unified whole. There is the wave-particle duality of mass-energy. The particle aspect of mass (fermions) or energy (bosons), which is localized and individuated in space-time, corresponds with the simple classical description of physical reality. It exists in that state after an observation has collapsed the wave function. The wave aspect of mass-energy, acting as a coherent ensemble and non-localized in space-time corresponds to the complex quantum mechanical description. It exists in that entangled, superposed state, capable of internal observation (watching to see if it is being watched), before an external observation has been made.

We have seen that observation actualizes physical reality from out of a superposition of virtual potentialities. We come into being both by observing and being observed.

From the many examples above, a strong argument can be made for a local-nonlocal duality of space-time. The sequential component of space-time exhibits temporal causality and excludes free will. The simultaneous component of space-time, exhibits acausality and facilitates consciousness and free will. A corollary to the strong anthropic principal is that the essence of consciousness is volitional choice.

The metaphysical extension of this analogy would be that there would be a duality between a classical individuated observer, a scientist say, external to the system, that would be localized in space-time, and a non-individuated internal observer that would not be localized in space-time. It is obvious that the external, individuated observer is a consciousness. What then can we make of the internal unindividuated nonlocal observer? This is the great mystery. We should start by acknowledging the possibility that it is there.

Consciousness

Consciousness is not an afterthought. To be open to the quantum lessons of nature is one step closer to being open to discovering the nature of consciousness as a fundamental building block of reality. Descartes took consciousness as the only given: "I think therefore I am." This notion that consciousness is the ground of all being has been called the perennial philosophy.[12]

The weak anthropic principal wins us a place in our cookbook universe with its finely tuned constants but it fails to explain how it came to be this way. There is an order that arises out of consciousness. Buildings, social structures and all things from the hand of humanity come from volitional choices. Humanity is not the soul custodian of consciousness. There is volitional order in an anthill, in a beehive and in a bird's nest. How far down this consciousness extends is an open question.

The current and most radical biological view is that life is a quantum state[29] and that evolution is a quantum process.[13] Penrose places the seat of consciousness in a small structure in the brain called the microtubule[26] which consists of a grid of molecules in a sheet which is formed into a cylinder, offset at the joining edge. This offset allows for a cyclical pattern of cellular automata to spiral down its length.[17] Figuratively, a microtubular orchestra of a million times a million entangled qbits playing the resonances of the threads and the sub-threads of thought, collecting together in superposition, compatibles reinforcing and cognitive dissonances cancelling, into a

coherent electromagnetic wave of thought, until all possibilities are accounted for and are given their respective amplitudes, then one thread wins out.

We turn to philosophy to supplement our understanding of that part of nature that cannot be put into a Skinner box. The first requirement is that it will not contradict the science. Any philosophy, which does not take the strange, counterintuitive aspects of QM into account will leave a major piece of the puzzle behind. The hard problem of the subject-object split giving rise to the conscious experience can not be completely accounted for with a purely reductionist or materialist approach.[27] There is also the binding problem, what could be called perceptual coherence, or how the separate sensory inputs are combined into a single unified perception and how that perception feeds back and combines with selective memories to produce reflection. The neural correlates of consciousness, such as microtubules or neurons provide a stage but not the actor for this play. Explaining the brain's physical form and function do not cross Levine's explanatory gap[27]. How and why does physical reality differentiate between a particular frequency of an energy quanta in the electromagnetic spectrum, which can be said to have an objective reality and the subjective, inner experience of the color blue, said to be a qualia of consciousness. If consciousness is not reducible to physical law then it must be thought of as a fundamental component.

The collapse of the wavefunction can be thought of as a remapping or transfer of information from the coherent and simultaneous Quantum domain to the Classical domain where information is spread throughout space-time and is only available to the external observer in a temporally sequential form. This irreversible data mapping occurs concurrently with the collapse. At the beginning and the end, the entire system is coherent. When the wave function begins to collapse, it collapses in stages each fixing a part of the system in time, giving it a sequence and a history. There is a spatial and temporal mix of collapsed and un-collapsed states on any scale that will allow coherence. At the point this information is made available to the observer, memories become accumulatively fixed and tangible, each in their own time domains. There are many pathways through time.

From Gödel and Heisenberg, a description of the world will necessarily be incomplete and uncertain to a degree. And seen from within our individual internal, subjective viewpoints (the other half of the equation), in the timeless words of Kierkegaard, "Rational uncertainty is the highest truth."

By Occam's razor, having a single universe in the past chosen from the possible virtual multiverses in the future is the simplest, most logical explanation. It follows the principal of least action. It is the most efficient. Where would the mass-energy come from for an infinite number of branching universes lacking the ability to collapse their wave functions at each bifurcation? In what real space-time would they be stored? We are a world of individual conscious beings who share a consistent and persistent history. We collapse the wave functions of the remaining possible virtual futures in our heads (or wherever else this consciousness resides) then manipulate the bits and pieces resulting from previously collapsed wave functions according to our creative abilities.

Nature is the perfect recycler, disassembling this impermanent assemblage, on both the microscopic and macroscopic scales, by entropy and chaos, and then putting it back into superposition.

Conclusion

The nature of quantum mechanics with its collapse of the wavefunction to a single state from a superposition of mutually exclusive possibilities chosen faster than light and backwards in time would not seem to be reconcilable with temporal causality, yet this has been shown to be how nature works. The abstract relationships of mathematics exist independently of physical reality and independently of a conscious entity to perceive them It self generates an infinity of relationships, a transfinite number of space-time dimensions in a transfinite number of configurations. From this

fecundity of infinity, at least one, if not many, gives rise to the configuration where consciousness can evolve as an endpoint in sequential time. This endpoint informs the choice of one configuration from the many existing in superposition in simultaneous time.

The future becoming the past is concurrent with the collapse of the wave function, both in a global sense and within us, a collection of localized and individuated observers. How this relates to a non-localized, unindividuated observer is the great mystery. All great stories both in science and in life are mystery stories.

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* Wikipedia† See Order out of Chaos, Chapter VII for a clear description of the wave function and its related concepts.