What is more fundamental than a bit?

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Abstract

In a conference talk in 1989, John Archibald Wheeler suggested that 'every physical quantity, every it, derives its ultimate significance from bits, binary yes-or-no indications, a conclusion which we epitomize in the phrase, it from bit.' Several authors, before and after Wheeler, have pursued similar ideas. However, no practical realisation of Wheeler's agenda within a realistic physical framework has emerged. Using the example of Wheeler's agenda, I pursue the question of what is fundamental in physics.

1 Introduction

In 1948, Shannon [1] introduced the term 'information' as a measurable quantity on the basis of discrete units of information, e.g. decimal or binary digits. The fact that thereby complex information can be broken down into a number of bits, made Wheeler ask whether, in a similar way, the complex structures of physics can be broken down into elementary and fundamental units of information. In a conference talk in 1989, Wheeler condensed his vision into the following sentences [2]:

'No element in the description of physics shows itself as closer to primordial than the elementary quantum phenomenon, that is, the elementary device-intermediated act of posing a yes–no physical question and eliciting an answer or, in brief, the elementary act of observer-participancy. Otherwise stated, every physical quantity, every it, derives its ultimate significance from bits, binary yes-or-no indications, a conclusion which we epitomize in the phrase, it from bit.'

Wheeler's statement seems to suggest that 'bits' are something fundamental, 'forming the base, from which everything else develops'[1]; actually, Wheeler's 'it from bit' was occasionally understood in this sense [3].

Wheeler concretised his vision in a practical agenda of 6 issues. I have condensed, rearranged, and, in part, reformulated this agenda in my own words:

- One: Start from an elementary quantum phenomenon that at bottom has the 0-or-1 sharpness of the definition of a bit.

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- Two: Go beyond statistics and determine what has to be added to obtain all of standard quantum theory.
- Three: Survey with an imaginative eye the powerful tools that mathematics offers and determine whether they are suitable for describing the world of bits.
- Four: Examine the composed systems of bits that display the level-upon-level-upon-level structure of physics.
- Five: Translate the quantum version of string theory and gravitation from the language of the continuum to the language of bits.
- Six: Take advantage of the findings and perspectives of the evolution of organisms. Understand time and space and all the other features that mark physics — and existence itself — as similarly self-generated organs of a self-synthesised information system.

Neither Wheeler nor anyone else has ever practically implemented this agenda. The reason may have been that Wheeler suggested an implementation within the framework of string theories or quantum gravity. In my attempt to understand Wheeler's ideas, I have instead tried an implementation within an experimentally accessible physical environment.

2 Realising Wheeler's vision

I will largely follow Wheeler's agenda, which will lead me to a 'physics of information' that describes the flow of information within 'elementary quantum phenomena' and 'derives its ultimate significance from bits.'

2.1 Issue One: Start from an elementary quantum phenomenon that at bottom has the 0-or-1 sharpness of the definition of a bit.

An elementary quantum phenomenon in Wheeler's sense is an 'elementary device-intermediated act of posing a yes–no physical question and eliciting an answer.' A practical example [4] is the act of measuring the spin[2] of an electron. If we measure the direction of a spin, the result will be either 'up' or 'down' relative to the orientation of the measuring device. If in the first measurement the result is 'up', then an immediately following second measurement will again result in 'up'. If we rotate the device before the second measurement by 180 degrees, the result will be 'down' instead. In both cases the result of the second measurement depends deterministically on the result of the first measurement and the angle of rotation. The situation is different if, before the second measurement, the measuring device is rotated by an angle β different from 0 and 180 degrees: then the result is no longer deterministic, but, if we repeat the experiment often enough, we obtain a statistical distribution of 'up' and 'down', which is a function of the angle of rotation.

2.2 Issue Two: Go beyond statistics and determine what has to be added to obtain all of standard quantum theory.

Here, there is something essentially different from classical statistical mechanics: even within this statistical distribution, the measured spin direction is always 'up' or 'down' relative to the orientation of the measuring device, independently of the angle of rotation of the device. Interestingly, even at this elementary level, we encounter a principle that says 'physics is the same in all frames of reference.'

2.3 Issue Three: Survey with an imaginative eye the powerful tools that mathematics offers and determine whether they are suitable for describing the world of bits.

Obviously, we need two tools: one that encodes the relevant information about the experimental preparations and a second one that, on the basis of the encoded information, predicts the results of the analysing measurement. What is the relevant information? It is, firstly, the result of the preparing measurement, 'up' or 'down', and, secondly, the relative angle of rotation β of the device between the preparing and the analysing measurement. A suitable tool for the first step, well known from quantum mechanics, is the two-dimensional complex vector space[3] of a spin representation[4] of the rotation group[5]. This vector space has two basis vectors[6]: $|u\rangle$, representing the measurement result 'up', and $|d\rangle$, representing the result 'down'. A rotation of the measurement device by an angle β is then encoded as a rotation of the result vector, leading to the superposition of state vectors

$$|\psi\rangle = a\,|u\rangle + b\,|d\rangle\,, \tag{1}$$

where a and b are complex numbers depending on β.

The encoded information is objective in the sense that it is independent of an external observer; it is realistic in the sense that it describes the result of a real (and repeatable) measurement in combination with a real manipulation of the measuring device. How then does probability come into play? If there is no preparing measurement, then the result of the analysing measurement would be undetermined, simply because information cannot emerge out of nothing. If, however, we have information from a previous measurement, the randomness is modified to a 'conditional expectation', which, in general, still has a probabilistic character. This is not because we don't have enough information, but simply because continuous information, i.e. the angle of rotation before the second measurement, cannot be mapped one-to-one onto a discrete variable (a bit).

We need a second tool to extract the conditional expectation value from the information encoded in the state vector. A suitable tool, also known from quantum mechanics, is the Born rule, which states that given a normalised state vector $|\psi\rangle$, the probability of obtaining the value n as the result of the measurement is given by $|\langle\phi_n \mid \psi\rangle|^2$, where $\langle\phi_n \mid \psi\rangle$ is the scalar product[7] between $|\psi\rangle$ and the normalised vector $|\phi_n\rangle$ corresponding to the value n. This yields the expectation value for measuring 'up' or 'down', depending on the information encoded in the state vector. The uniqueness of Born's rule follows from a theorem due to Andrew Gleason [5].

Both tools, taken together, exhaustively describe the act of taking consecutive measurements at the most elementary level. It cannot surprise that they constitute nothing other than the well-known quantum mechanics of spin. Note, however, that I did not deliberately draw on the rules of quantum mechanics, but rather followed Wheeler's advice and selected those mathematical tools that adequately describe the act of repeated measurements of a spin.

Most welcome is the self-explanatory property of the first tool, which describes the preparations of the experiment in a very transparent way. This transparency makes the meaning of the state vector — also called the 'wave function' — very clear and supports the 'empiricist interpretation' of quantum mechanics, which holds that 'quantum mechanics does not describe the microscopic objects themselves, but just relations between preparing and measuring procedures, mediated by microscopic objects' [6].

The first tool has the property of being reversible, whereas the second one describes an irreversible process, accompanied by a loss of information.

Measuring devices in 3+1-dimensional space-time can not only be rotated, but also translated. Therefore, to complete the description of the spin experiment, also translations need to be taken into account. This requires that the representation of the rotation group be extended to a representation of the full Poincaré group[8] [7]. The associated vector space has a basis of momentum eigenstates[9] $|\mathbf{p}\rangle$. A general state $|\psi\rangle$ of this vector space can, therefore, be written as

$$|\psi\rangle = \int d^3\mathbf{p}\; c(\mathbf{p})\; |\,\mathbf{p}\rangle\,, \tag{2}$$

where $c(\mathbf{p})$ are (in general) complex coefficients. The momentum eigenvalues p_μ obey the mass shell relation $p^\mu p_\mu = m^2$, where m is a mass that together with the spin characterises the representation.

The momentum parameter derives from differences of the position of the measuring device between the preparing and the analysing measurement; it describes neither intrinsic properties of the measuring device nor of the spin, but rather a property of the measuring process as a whole.

By incorporating a representation of the Poincaré group, spin is given the mathematical structure of a quantum mechanical spin-1/2 particle with momentum and a mass.

Note, however, that here (and in the following) the term 'particle' does not refer to an ontic entity of Nature, but rather to the informational structure of the relation between spin (or bit) and measuring device, more precisely, to the structure of the act of consecutive measurements of a spin.

2.4 Issue Four: Examine the composed systems of bits that display the level-upon-level-upon-level structure of physics.

Representations of the Poincaré group describing single particles can be combined to make product representations[10], providing a mathematical basis for describing multi-particle configurations. In this way, we can move step by step from single particles to two-particle systems, then to many-particle systems, to macroscopic bodies, and systems of macroscopic bodies.

Of special interest is the step from a single particle to a two-particle system, because here something new emerges: the phenomenon of interaction [9].

Whereas single particles have only an intrinsic spin, two particles together can form an orbital angular momentum. Measuring the total momentum $\mathbf{p}$ and the orbital angular momentum m of a two-particle configuration leaves it in an eigenstate $|\mathbf{p}, m\rangle$ of the total and angular momentum. (The existence of simultaneous eigenstates of $\mathbf{p}$ and m is ensured by the structure of the Poincaré group [7].)

Being an eigenstate of the angular momentum, this two-particle state must be a superposition of product states $|\mathbf{p_1}, \mathbf{p_2}\rangle$ such that together with each product state, also those states that are obtained from it by rotations contribute to the superposition. This requires an integration over the the full angle of rotation (360 degrees), i.e. over a one-dimensional closed path in momentum space. The integration gives the two-particle state a momentum entangled[11] structure

$$|\mathbf{p}, m\rangle = \int_\Omega \omega \; d^3\mathbf{p}_1 d^3\mathbf{p}_2 \; c(\mathbf{p}, m, \mathbf{p}_1, \mathbf{p}_2) \mid \mathbf{p}_1, \mathbf{p}_2\rangle \,. \tag{3}$$

The domain of integration Ω is a finite subspace of the two-particle mass shell

$$(p_1^0 + p_2^0)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 = M^2, \tag{4}$$

where $\omega = V(\Omega)^{-\frac{1}{2}}$ is the required normalisation factor of the two-particle state. Due to this construction, the coefficients $c(\mathbf{p}, m, \mathbf{p}_1, \mathbf{p_2})$ are essentially phase factors.

If in a two-particle configuration, described by the state $|\mathbf{p}, m\rangle$, the individual particle momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are measured, this measurement will yield only statistical results. Although the total momentum is well-defined and constant, the individual particle momenta are not: the two-particle system behaves as if there is an exchange of momentum between the particles. This phenomenon can be described by a 'virtual exchange' of quanta of momentum. The Standard Model of particle physics uses the same mechanism to describe interaction — there the virtual quanta are interpreted as 'gauge bosons'.

This entitles us to say that two particles in a state with well-defined total linear and angular momenta interact by the exchange of virtual quanta of momentum. The exchanged momenta are restricted (bounded) by the finite domain of integration. This leads to well-defined two-particle states without the notorious singularities of the Standard Model.

2.5 Issue Five: Translate the quantum version of string theory and gravitation from the language of the continuum to the language of bits.

Wheeler's proposal to translate string theory or gravitation 'into the language of bits' indicates that he had in mind some digitised structures at Planck scales; he obviously did not expect that such a translation would be possible for theories at experimentally accessible scales. This did not prevent me from trying to 'translate' the best known and best verified part of the Standard Model, which is quantum electrodynamics (QED). Actually, there is no need for a translation, because Richard Feynman already carried out this translation in his seminal papers of 1949–50. Although QED is commonly considered as the prototype of a quantum field theory, Feynman, in his own words, formulated it 'as a description of a direct interaction at a distance (albeit delayed in time) between charges' [8]. Again using Feynman's words, this interaction is described by the exchange of 'virtual quanta' of momentum between the particles. As shown in the previous section, the term 'exchange of momentum' has a well-defined counterpart in the language of bits.

What can be said about the strength of this interaction? Consider the following gedanken experiment: Two independent incoming particles with momenta $\mathbf{p_1}$ and $\mathbf{p_2}$ are described by a pure product state $|\mathbf{p_1}, \mathbf{p_2}\rangle$. A measurement of the total momentum and the orbital angular momentum of this two-particle configuration brings it into a state $|\mathbf{p}, m\rangle$ with the same total momentum $\mathbf{p} = \mathbf{p_1} + \mathbf{p_2}$, but now with a well-defined orbital angular momentum m; the associated probability amplitude is $\langle \mathbf{p}, m|\mathbf{p_1}, \mathbf{p_2}\rangle$. Now measure the individual momenta of the particles. This measurement will obtain the momenta $\mathbf{p_1'}$ and $\mathbf{p_2'}$ with a probability amplitude of $\langle \mathbf{p_1'}, \mathbf{p_2'}|\mathbf{p}, m\rangle$. The total transition amplitude is then

$$\langle \mathbf{p_1'}, \mathbf{p_2'}|\mathbf{p}, m\rangle \langle \mathbf{p}, m|\mathbf{p_1}, \mathbf{p_2}\rangle\,. \tag{5}$$

Because in the intermediate state $|\mathbf{p}, m\rangle$ the information about the initial particle momenta is lost, the transition amplitude will in general not vanish; we will observe a scattering pattern, but what determines the scattering amplitude?

The only constant that can have an influence on this amplitude is the normalisation factor ω in Equation (3), which enters into the amplitude (5) as ω^2. The numerical value of ω^2 has been calculated [9]:

$$\omega^2 = \frac{9}{16\pi^3}\left(\frac{\pi}{120}\right)^{1/4} = 1/137.03608245. \tag{6}$$

This matches the value of the fine-structure constant α, the square of the electromagnetic coupling constant, with the empirical CODATA value [10]:

$$\alpha = 1/137.035999139. \tag{7}$$

Therefore, we can say that two particles in a two-particle state with well-defined total quantum numbers are subject to an interaction with the same structure and strength as the empirical electromagnetic interaction.

This result strongly suggest that this interaction and the interaction that is so successfully modelled by QED are one and the same. This identification explains why α has the value it has. It further shows that the electromagnetic interaction is not based on an additional principle (of gauge invariance), but is rather the mathematical consequence of the description of measurements in a Poincaré symmetric space-time environment.

2.6 Issue Six: Take advantage of the findings and perspectives of the evolution of organisms. Understand time and space and all the other features that mark physics — and existence itself — as similarly self-generated organs of a self-synthesised information system.

By superimposing momentum eigenstates $|\mathbf{p}\rangle$, new states localised at the position $\mathbf{x}$ of position space [11] can be constructed:

$$|\mathbf{x}, t\rangle = (2\pi)^{-\frac{3}{2}} \int \frac{d^3\mathbf{p}}{p_0}\, e^{-ipx}\, |\mathbf{p}\rangle\,. \tag{8}$$

Although such states are localised in space, they cannot be localised in time: in other words, time is running, it cannot be stopped. Just as with momentum, the position of a particle is relative to a measuring device: this means that particle positions form a particle- and device-specific parameter space.

In multi-particle configurations, we can embed each particle-specific parameter space into a shared parameter space by referring them to one and the same measuring device. This act introduces 'distances' between the particles in a 3+1-dimensional Poincaré symmetric 'space-time' structure. The expectation values of the distances are determined by the states of the involved particles i, k within the multi-particle state Φ, as I have indicated here

$$\int d^3\mathbf{x}_i\, d^3\mathbf{x}_k\, (\mathbf{x}_i - \mathbf{x}_k)\, |\langle \Phi | \mathbf{x}_i, \mathbf{x}_k, t\rangle|^2\,. \tag{9}$$

A network of mutual distances defines a space-time structure. In a classical limit, this structure can be expected to become a Riemannian space-time manifold; this would be an essential step towards a geometric theory of gravitation, such as General Relativity or Conformal Gravity [12].

In space-time, the laws of momentum space obtain new forms: in a non-relativistic approximation, the law of exchange of momentum is translated into the Coulomb law [9]. The Coulomb potential describes attractive and repulsive forces. The attractive Coulomb force between negatively charged electrons and positive protons lead to the formation of atoms and from there to the 'existence' of macroscopic structures in 3+1-dimensional space-time — some of them may be devices capable of measuring spins and momenta, completing the construction of a 'self-synthesised information system.'

3 Conclusions

From the implementation of Wheeler's agenda, closely tied to the act of measuring the direction of a spin, mathematical structures have emerged that describe the flow of information between elementary acts of measurement. Let me subsume these structures and their relations under the phrase 'physics of information'. The physics of information is based on transparent, well understood mathematical descriptions of elementary physical processes. It provides a serious alternative to well-established, though not fully understood physical theories, such as the axiomatic theory of quantum mechanics and the phenomenological theory of quantum electrodynamics.

Eugene Wigner [13] was surprised at the 'unreasonable effectiveness of mathematics in the natural sciences.' He might have asked how pure mathematics can know about physical objects such as charged particles. The paradox is solved when we understand a 'particle' not as an ontic entity of Nature, but rather as a mathematical structure that describes the flow of information between two consecutive measurements.

This resembles the ideas of 'ontic structural realism' [14], which claims 'that there are no "things" and that structure is all there is.' However, the structures encountered here are not simply 'there': rather, they emerge from the act of acquiring information by measurements, or, in Wheeler's words, by 'the elementary device-intermediated act of posing a yes–no physical question.'

Already Niels Bohr [15] was convinced that 'physics is to be regarded not so much as the study of something a priori given, but rather as the development of methods of ordering and surveying human experience.' This clearly applies to the physics of information, which basically does not describe Nature, but rather the flow of information in our attempts to study Nature by making measurements and trying to describe and explain their results by physical theories. It is a theory about acquiring information, manipulating information, and combining pieces of information, by physical means within a Poincaré symmetric environment.

An outstanding feature of the physics of information is that it does not depend on the validity of abstract axioms (of quantum mechanics), first principles (of gauge invariance), or preconceptions (of local interaction); rather, it derives directly from the mathematical description of the elementary act of measurement, which is the really fundamental element of the theory. In other words, it is based on the very definition of physics, which implies making measurements and describing their results. The absence of additional assumptions makes it a reliable foundation for phenomenological theories such as the Standard Model.

The physics of information has a bottom, which naturally coincides with the most elementary level of information. Beyond this level, there is no information about Nature that could be unveiled by physical means.

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Notes

[1]Definition of ‘fundamental’ in Cambridge Dictionary online.

[2]Spin is an intrinsic form of angular momentum carried by elementary particles and atomic nuclei.

[3]A vector space is a collection of objects called vectors, which may be added together and multiplied by numbers.

[4]Group representations describe abstract groups in terms of linear transformations of vector spaces.

[5]The rotation group is the set of all rotations about the origin of three-dimensional space.

[6]I use Dirac’s bra-ket notation: $|...\rangle$ for vectors and $\langle...|$ for covectors.

[7]A scalar product $\langle ... \mid ... \rangle$ is an algebraic operation that takes two vectors and returns a single number. A vector is normalised if the scalar product with itself equals 1.

[8]The Poincaré group is the set of symmetry operations of space-time. It includes translations, rotations, and boosts.

[9]An eigenstate of a linear operation is a vector that only changes by a scale factor, when that operation is applied to it; the scale factor is known as the eigenvalue associated with the eigenstate. Here, the linear operation is a translation and the eigenvalues have the property of momentum.

[10]A product of representations is a tensor product of vector spaces underlying representations together with the factor-wise group action on the product.

[11]A two-particle state is entangled if it cannot be written as a product of one-particle states.