# It from Bit, or Bit from It? 

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#### Abstract

The basic concepts and numerical relations of theoretical particle physics, including quantum mechanics and Poincaré invariance, the electromagnetic and the gravitational interaction, the leptonic mass spectrum and the mass of the proton, can be derived, without reference to first principles, from intrinsic properties of the simplest elements of information, represented by binary information. What we comprehend as physical reality is, therefore, a reflection of mathematically determined logical structures of information.


## 1 Introduction

"The past century in fundamental physics has shown a steady progression away from thinking about physics, at its deepest level, as a description of material objects and their interactions, and towards physics as a description of the evolution of information about and in the physical world." This statement, formulated by the organizers of the FQXi Essay Contest, explains the theme for 2013: "It from Bit, or Bit from It?"

Niels Bohr was probably the first to clearly articulate that there must be a difference between the physical world (Nature) and the information about it: "It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature." [1]. If Bohr had already been familiar with information theory, which was developed by Shannon [2] only in 1949, he might have replaced "what we can say about Nature" with "our information about Nature."

According to Bohr's dictum, the question of "how Nature is" cannot be answered by the methods of physics. Therefore, for a physicist, the question in the title is reduced to the following: Do the recognizable structures in our information about Nature reflect some of the hidden properties of Nature? Or are these structures inherent in information and the process of acquiring information?

When John Wheeler coined the phrase "It from Bit" 3], he noted "[...] it is not unreasonable to imagine that information sits at the core of physics, just as it sits at the core of a computer" (4). So he obviously had the suspicion that physics may not reflect structures of Nature but rather structures inherent in information.

I will address the question formulated in the title of this essay by examining structures that exist within the most elementary form of information, represented by elements of binary information, or, for short, "bits". I will then compare these structures with empirical structures of elementary particle physics and show to what extent these empirical structures are already inherent in information.

## 2 Properties of binary information

Let us step right into the world of bits and consider a set of binary elements that may take on one of two states. These states are abstract symbols without any meaning, unless we assign a meaning to them by calling them "true" and "false", if they refer to a statement, or "up" and "down", if they refer to a direction, or denote them by the numbers " 0 " and " 1 ", if they refer to the absence or presence of a certain power of 2 ( = "binary digit" or "bit") in a binary number in a computer program.

By assigning a semantics to the states of a binary element, we introduce a reference frame that defines the meaning of the abstract states. The information contained in an element is, therefore, always information relative to a semantic frame of reference.

The state of a binary element has a binary degree of freedom, in the sense that it may point to either one meaning or to the alternative meaning, as defined by the semantic frame of reference. This degree of
freedom can be described as a symmetry with respect to the permutation of the states, or, more generally, as a symmetry with respect to the "orientation" of the state relative to the semantic frame of reference.

Information is not always exact, because it is not always possible to answer a question only with "yes" or "no". Very often the answer is "perhaps". Therefore, when setting up a mathematical description of information, we must be prepared to express inexact information in terms of probabilities.

Let us now see how we can find a mathematical description of a set of binary elements that meets these requirements.

## 3 Mathematical description of binary information

Let us represent the states $d$ ("down") and $u$ ("up") of a binary element by two-component vectors, also called spinors,

$$
\begin{equation*}
|d\rangle=\binom{0}{1} \quad \text { and } \quad|u\rangle=\binom{1}{0} . \tag{1}
\end{equation*}
$$

The degree of freedom with respect to permutation of the states can be described by two operators: the operator $\tau_{+}$converts the state $d$ into $u$ and the operator $\tau_{-}$converts the state $u$ into $d$. There is a third operator, $\tau_{0}$, which is the commutator of $\tau_{+}$and $\tau_{-}$. The commutator of two operators $A$ and $B$ is defined by $[A, B]=A B-B A$, it is also sometimes called the Lie bracket. The operators $\tau_{+}, \tau_{-}$and $\tau_{0}$ satisfy the well-known commutation relations

$$
\begin{equation*}
\left[\tau_{0}, \tau_{+}\right]=\tau_{+}, \quad\left[\tau_{0}, \tau_{-}\right]=-\tau_{-}, \quad\left[\tau_{+}, \tau_{-}\right]=2 \tau_{0} \tag{2}
\end{equation*}
$$

which define the Lie algebra of the group of special orthogonal transformations in three dimensions, $\mathrm{SO}(3)$, or simply the group of rotations in three dimensions.

A more common form of the commutation relations results from replacing $\tau_{+}, \tau_{-}$, and $\tau_{0}$ by the operators

$$
\begin{equation*}
l_{1}=\frac{1}{2}\left(\tau_{+}+\tau_{-}\right), \quad l_{2}=\frac{1}{2 i}\left(\tau_{+}-\tau_{-}\right), \quad l_{3}=\tau_{0} \tag{3}
\end{equation*}
$$

These operators satisfy the more symmetric commutation relations

$$
\begin{equation*}
\left[l_{1}, l_{2}\right]=i l_{3}, \quad\left[l_{2}, l_{3}\right]=i l_{1}, \quad\left[l_{3}, l_{1}\right]=i l_{2} \tag{4}
\end{equation*}
$$

By identifying $l_{k}$ with $\frac{1}{2} \sigma_{k}$, where $\sigma_{k}$ are the Pauli matrices

$$
\sigma_{1}=\left(\begin{array}{ll}
0 & 1  \tag{5}\\
1 & 0
\end{array}\right), \quad \sigma_{2}=\left(\begin{array}{rr}
0 & -i \\
i & 0
\end{array}\right), \quad \sigma_{3}=\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right),
$$

we obtain a representation of the $l_{k}$ on the states (1).
By forming linear combinations of the two basis vectors with complex coefficients $c_{1}$ and $c_{2}$

$$
\begin{equation*}
|\phi\rangle=c_{1}\binom{0}{1}+c_{2}\binom{1}{0} \tag{6}
\end{equation*}
$$

we obtain a complex vector space, spanned by the basis vectors. On this vector space, the operators $l_{k}$ of the Lie algebra generate, by exponentiation

$$
\begin{equation*}
t=e^{i \omega_{k} l_{k}} \tag{7}
\end{equation*}
$$

with three real parameters $\omega_{k}$, a representation of the group $\mathrm{SO}(3)$. Because the $l_{k}$ are Hermitian operators, $t$ is unitary, which means that the inner product

$$
\begin{equation*}
\langle\phi \mid \phi\rangle \equiv c_{1} c_{1}^{*}+c_{2} c_{2}^{*} \tag{8}
\end{equation*}
$$

is left invariant by these transformations. Therefore, without loss of generality, the coefficients $c_{1}$ and $c_{2}$ can be normalized so that they satisfy

$$
\begin{equation*}
c_{1} c_{1}^{*}+c_{2} c_{2}^{*}=1 \tag{9}
\end{equation*}
$$

Together with the inner product (8), the states (6) form a Hilbert space.
This vector space now allows describing inexact information. Inexact information at the level of binary information is expressed by a probability $w_{d}$ that the element is in the state $d$ and a probability $w_{u}$ that it is in the state $u$. Of course, these probabilities must satisfy

$$
\begin{equation*}
w_{d}+w_{u}=1 \tag{10}
\end{equation*}
$$

Comparison with Equation (9) shows that

$$
\begin{equation*}
w_{d}=c_{1} c_{1}^{*} \quad \text { and } \quad w_{u}=c_{2} c_{2}{ }^{*} . \tag{11}
\end{equation*}
$$

A transformation $t$ describes a change of the orientation of the states relative to the semantic frame of reference. In the simplest case this means a permutation of the states. In the general case, the orientation allows expressing inexact information, with probabilities in a continuous range from 0 to 1 .

The transformations (7), although they change the coefficients, do not change the form of the states (6) and, more importantly, do not change the inner product (8). We can say that the binary elements transform covariant with respect to changes of the semantic frame of reference. In other words, the Hilbert space is invariant with respect to the symmetry group $\mathrm{SO}(3)$.

The Hilbert space formalism, outlined here, is nothing other than the quantum mechanical description of angular momentum in three dimensions. The state (6) describes an object with an angular momentum (spin) of $1 / 2$.

## 4 Poincaré invariance

In the last section I introduced the semantic frame of reference of a single binary element. As long as we consider binary elements as uncorrelated, we have to provide each element with an element-specific reference frame. To be able to treat the set of binary elements as a whole, it makes sense to define a set-specific frame of reference.

The individual element's frames can then be oriented to the set's frame, being either parallel or antiparallel to a fixed axis within the set's frame, say, the rotational axis defined by the operator $l_{3}$ of the set's frame. However, as will be explained later, there are situations where the orientation ought to be left open.

The symmetry group generated by the three rotational operators $l_{1}, l_{2}, l_{3}$, referring to the element's frame, and a single rotational operator $l_{3}$, referring to the set's frame, is the product group $\mathrm{SO}(3) \times \mathrm{SO}(2)$.

Let us analyze the properties of a coordinate system that has this group as a symmetry group in more detail. If $x_{1}, x_{2}, x_{3}$ are Cartesian coordinates in the $\mathrm{SO}(3)$ symmetric part of the coordinate system, then the quadratic form

$$
\begin{equation*}
x_{1}^{2}+x_{2}^{2}+x_{3}^{2} \tag{12}
\end{equation*}
$$

is invariant under all transformations that are elements of $\mathrm{SO}(3)$. Similarly, if $t_{0}$ and $t_{1}$ are Cartesian parameters of the $\mathrm{SO}(2)$ symmetric part, then the operations of $\mathrm{SO}(2)$ will leave the quadratic form

$$
\begin{equation*}
t_{0}^{2}+t_{1}^{2} \tag{13}
\end{equation*}
$$

invariant. Although the $t$ and $x$ spaces are independent parameter spaces, we are free to connect them into an $x-t$ space. In the following, we will consider the (indefinite) quadratic form in $x-t$ space

$$
\begin{equation*}
c^{2}\left(t_{0}^{2}+t_{1}^{2}\right)-\left(x_{1}^{2}+x_{2}^{2}+x_{3}^{2}\right) . \tag{14}
\end{equation*}
$$

If we measure $x$ in units of length and $t$ in units of time, then the constant $c$ has the dimension of a velocity.
In addition to the transformations of $\mathrm{SO}(3) \times \mathrm{SO}(2)$, the quadratic form (14) is invariant under the transformations known as boost transformations. They describe the change from a stationary to a moving coordinate system. Boost transformations do not belong to the original symmetry group, but describe dynamic transformations of the coordinate system. They extend the symmetry group $\mathrm{SO}(3) \times \mathrm{SO}(2)$ to a "dynamic" $\mathrm{SO}(3,2)$ group, the de Sitter group.

Consider an $\mathrm{SO}(2)$ symmetric orbit in the plane $t_{0}-t_{1}$ with a very large radius, e.g., the earth's orbit in the ecliptical plane. At a given point of the orbit, the tangent to the orbit approximates the orbit within
a neighborhood $\mathcal{N}$ of this point. Within $\mathcal{N}$, the rotations of $\mathrm{SO}(2)$ can, therefore, be approximated by translations. This approximation is known as group contraction and has been formulated in a mathematically strict sense by Inönü and Wigner [5]. By group contraction, the de Sitter group, in $\mathcal{N}$, is approximated by the Poincaré group $\mathrm{P}(3,1)$, also known as the inhomogeneous Lorentz group. Without the translations, $\mathrm{P}(3,1)$ is identical to the (homogeneous) Lorentz group $\mathrm{SO}(3,1)$, which is a subgroup of $\mathrm{SO}(3,2)$.

The transformations of $\mathrm{PO}(3,1)$ leave invariant the quadratic form

$$
\begin{equation*}
c^{2} t_{0}^{2}-x_{1}^{2}-x_{2}^{2}-x_{3}^{2} \tag{15}
\end{equation*}
$$

They can be described by three rotations in 3-dimensional space, three translations in space, one translation in time, and three boost operations. The Poincaré group is central to Einstein's special theory of relativity.

It is no coincidence that the construction of the Poincaré group presented here is reminiscent of Barbour's construction of time in his award-winning essay "The Nature of Time" 6].

## 5 Binary elements in energy-momentum space

Let us now try to describe a single binary element in a way that is covariant with respect to its orientation with the element-specific and the set-specific semantic frame of reference, and also with respect to boost operations.

Similar to the representation of the group $\mathrm{SO}(3)$ on a two component vector space, we now represent the product group $\mathrm{SO}(3) \times \mathrm{SO}(2)$ in a vector space built from four-component basis vectors,

$$
\left(\begin{array}{l}
1  \tag{16}\\
0 \\
0 \\
0
\end{array}\right), \quad\left(\begin{array}{l}
0 \\
1 \\
0 \\
0
\end{array}\right), \quad\left(\begin{array}{l}
0 \\
0 \\
1 \\
0
\end{array}\right), \quad\left(\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right)
$$

The first two vectors describe a spinor with its reference frame parallel to the set frame. The second two describe the same spinor in an anti-parallel configuration. Linear combinations of these basis vectors form the state space of Dirac spinors.

The matrix

$$
\gamma^{0}=\left(\begin{array}{cc}
I & 0  \tag{17}\\
0 & -I
\end{array}\right)
$$

where $I$ is the $2 \times 2$ unit matrix, then delivers an eigenvalue of +1 if applied to the first group of Dirac spinors, and -1 if applied to the second. This formulation is not yet covariant with respect to the transformations of $\mathrm{SO}(3,2)$. To make it covariant, we have to find $4 \times 4$ matrices that transform together with $\gamma^{0}$ in the same way as the reference frame. In other words, we have to find a representation of $\mathrm{SO}(3,2)$ by $4 \times 4$ matrices.

The representation of the $\mathrm{SO}(3)$ rotations is straightforward. Their generators are obtained, as before, from the Pauli matrices

$$
\sigma_{i j}=\epsilon_{i j k}\left(\begin{array}{cc}
\sigma_{k} & 0  \tag{18}\\
0 & \sigma_{k}
\end{array}\right), i, j, k=1,2,3
$$

The boost operations are generated by the $4 \times 4$-matrix

$$
\sigma^{0 k}=-\sigma^{k 0}=\left(\begin{array}{cc}
0 & i \sigma_{k}  \tag{19}\\
-i \sigma_{k} & 0
\end{array}\right)
$$

When we close the algebra of the matrices that we have defined so far, with respect to the commutator, we find the additional matrices

$$
\gamma^{k}=\left(\begin{array}{cc}
0 & \sigma_{k}  \tag{20}\\
-\sigma_{k} & 0
\end{array}\right)
$$

We can combine the indices 0 and $k$ to an index $\mu=0, \ldots, 3$, and use the metric tensor $g^{\mu \nu}=\operatorname{diag}$ $(+1,-1,-1,-1)$ to raise and lower indices, as in the theory of special relativity.

The matrices (17) and 20) are Dirac's $\gamma$-matrices in the so-called standard or Dirac representation. The $\gamma$-matrices satisfy the anti-commutation relations

$$
\begin{equation*}
\gamma^{\mu} \gamma^{\nu}+\gamma^{\mu} \gamma^{\nu}=2 g^{\mu \nu} \tag{21}
\end{equation*}
$$

and the commutation relations

$$
\begin{equation*}
\frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right]=\sigma^{\mu \nu} \tag{22}
\end{equation*}
$$

It can be shown that $\frac{1}{2} \sigma^{\mu \nu}$ and $\frac{1}{2} \gamma^{\mu}$ together form a representation of the Lie algebra of $\mathrm{SO}(3,2)$, by verifying the commutation rules of the Lie algebra of $\mathrm{SO}(3,2)$.

Since the Dirac matrices form a representation of $\mathrm{SO}(3,2)$, and so also of $\mathrm{SO}(3,1)$, a Lorentz transformation of a Dirac spinor can be constructed from the appropriate Dirac matrices. Thereby the spinor state is changed into a linear combination of all four basic spinor states, with coefficients that are functions of the parameters of the transformations. The parameters $p_{\mu}$ which correspond to transformations generated by $\gamma^{\mu}$, form a parameter space with a metric tensor $g^{\mu \nu}$, the energy-momentum space.

Lorentz transformations leave the product $\gamma^{\mu} p_{\mu}$, applied to a Dirac spinor, invariant (the proof can be found in textbooks, e.g., (7). So we end up with the well known Dirac equation

$$
\begin{equation*}
\left(\gamma^{\mu} p_{\mu}-m\right)|p\rangle=0, \tag{23}
\end{equation*}
$$

which is the quantum mechanical description of an "elementary particle" with spin $1 / 2$, momentum $p$, and mass $m$. The properties of this mass will be examined in more detail in Section 9.

## 6 The Pauli exclusion principle and baryonic structures

The Pauli exclusion principle was formulated by Pauli in 1925. It says, in short, that two fermions cannot exist in the same quantum state, more generally, that they obey Fermi-Dirac statistics. A new proof, presented by O'Hara [8], shows that Pauli's principle applies also to binary elements if they are represented by Dirac spinors.

In addition, O'Hara extended the Pauli principle to $n$ spinors by studying possible spin-correlations, which he defined in the following way: " $n$ particles are isotropically spin-correlated, if a measurement made in an ARBITRARY direction $\theta$ on ONE of the particles allows us to predict with certainty, the spin value of each other of the $n-1$ particles for the same direction $\theta$."

O'Hara then showed that

$$
\begin{equation*}
|\psi\rangle=\frac{1}{\sqrt{2}}(|u\rangle|u\rangle+|d\rangle|d\rangle) \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
|\psi\rangle=\frac{1}{\sqrt{2}}(|u\rangle|d\rangle-|d\rangle|u\rangle) \tag{25}
\end{equation*}
$$

are the only isotropically spin-correlated states permitted for a system of $n$ particles.
This means that when three spinors are coupled, only two of them can be coupled in parallel to a triplet state or anti-parallel to a singlet state: the third spinor must be statistically independent. Therefore, in case of anti-parallel coupling, the extended Pauli principle forces the spinors into the state

$$
\begin{align*}
\psi\left[\lambda_{1}, \lambda_{2}, \lambda_{3}\right]=\frac{1}{\sqrt{3}} & {\left[\psi_{12}\left[\lambda_{1}, \lambda_{2}\right] \psi_{3}\left(\lambda_{3}\right)\right.} \\
& +\psi_{31}\left[\lambda_{1}, \lambda_{2}\right] \psi_{2}\left(\lambda_{3}\right) \\
& \left.+\psi_{23}\left[\lambda_{1}, \lambda_{2}\right] \psi_{1}\left(\lambda_{3}\right)\right] \tag{26}
\end{align*}
$$

where

$$
\begin{equation*}
\psi_{i j}\left[\lambda_{1}, \lambda_{2}\right]=\frac{1}{\sqrt{2}}\left(\psi_{i}\left(\lambda_{1}\right) \psi_{j}\left(\lambda_{2}\right)-\psi_{i}\left(\lambda_{2}\right) \psi_{j}\left(\lambda_{1}\right)\right) \tag{27}
\end{equation*}
$$

and $\lambda=(p, s)$ stands for the momentum and spin of a Dirac spinor. This suggests that the spinors "are in a dynamic equilibrium with each other, with the coupling continuously broken and then reformed among different [spinors]."

In the standard model of particle physics, a similar "dynamic equilibrium" is interpreted as a "dynamic exchange of gluons" between the constituents (quarks) of a proton.

O'Hara concluded: "if it is assumed that only singlet state coupling is stable, then all spin- $3 / 2$ configurations will necessarily decompose." This closely resembles the structures of spin- $1 / 2$ and spin- $3 / 2$ baryons.
"However, [now] their ... structure may be explained in terms of the coupling principle, without any recourse to the concept of color."

Structures derived from the state (26), and their interactions, are certainly an interesting topic for future research.

## 7 Entanglement and interaction

Two Dirac spinors can be combined into a two-particle product state in such a way that they form an irreducible two-particle representation of the Poincaré group. The individual spinor states are then necessarily momentum entangled.

In a recent article [9, I have shown that this entanglement is responsible for an interaction between the Dirac spinors, with the same structure and strength as the electromagnetic interaction. The strength of the electromagnetic interaction is expressed by the electromagnetic coupling constant, also known as the fine structure constant $\alpha$. A theoretical value of $\alpha$, determined from the geometry of the irreducible two-particle state space, agrees with the empirical value up to a factor of 1.0000005 . This means that two Dirac spinors, forming an irreducible two-particle representation of the Poincaré group, interact electromagnetically, in exactly the same way as do electrons and positrons.

The existence of the electromagnetic interaction answers a question about the nature of boost operations, which in Section 4 were introduced in a rather formal way: the electromagnetic interaction allows, in principle, accelerating Dirac spinors, which then are in relative motion to an observer. Therefore, the inclusion of boost operations in the symmetry group of a Dirac spinor is required for reasons of consistency.

## 8 Space-time and gravitation

In another article [10, I have given reasons why irreducibility may also be responsible for the gravitational forces.

Within a quantum mechanical description, we can easily switch between a representation in energymomentum space and in space-time. Both representations are formally connected by a mathematical operation, the so-called Fourier transformation.

A more detailed analysis shows that we have to differentiate between space-time, as a parameter space, and space-time, derived from the structure of probability distributions in parameter space-time. Within irreducible two-particle representations, the latter has a curved structure, which can be described by a non-Euclidean geometry, as known from Einstein's general theory of relativity.

The coupling constant of the resulting gravitational interaction is essentially determined by the quotient of the electromagnetic coupling constant and the square root of the total number of heavy particles (protons, neutrons) in the visible universe, which is $10^{80}$. This coupling constant has the same order of magnitude as the empirical gravitational constant.

## 9 Proton mass and leptonic mass relations

I have to come back to the contraction of the de Sitter group $\mathrm{SO}(3,2)$ to the Poincaré group $\mathrm{PO}(3,1)$, which becomes the symmetry of the parameter space in the neighborhood $\mathcal{N}$ of a given point of the $\mathrm{SO}(2)$ orbit. However, a state of a binary element is not confined to $\mathcal{N}$. It is still a state of the $\mathrm{SO}(3,2)$ symmetric state space. Only within $\mathcal{N}$ does it "look" like a state of an $\mathrm{SO}(3,1)$ symmetric state space. Since the group $\mathrm{SO}(3,2)$ is larger than $\mathrm{SO}(3,1)$, there must be a multiplicity $v$ of $\mathrm{SO}(3,2)$ states that in $\mathcal{N}$ are identified with (approximated by) the same $\mathrm{SO}(3,1)$ state. The multiplicity $v$ is identical to the number of representations of $\mathrm{SO}(3,1)$ that fit into a representation of $\mathrm{SO}(3,2)$, as given by the number of elements of the quotient group $\mathrm{SO}(3,2) / \mathrm{SO}(3,1)$, or, more precisely, by its volume, $v_{e}$. This has been calculated [11]:

$$
\begin{equation*}
v_{e}=V\left(\frac{S O(3,2)}{S O(3,1)}\right)=\frac{16 \pi}{3} . \tag{28}
\end{equation*}
$$

If we prepare a state with momentum $p$ in $\mathcal{N}$, we cannot be sure to which representation of $\mathrm{SO}(3,1)$ it belongs. To express this inexact information, we have to set up the state as a superposition of all states that in $\mathcal{N}$ have the momentum $p$.

Each of the states of the superposition satisfies, within $\mathcal{N}$, a Dirac equation with mass $m_{0}$. When we add up these Dirac equations, with the understanding that the resulting momentum $p$ is the sum of the momenta $p^{(i)}$, each of them referring to one of the representations of $\mathrm{SO}(3,1)$, then we obtain the Dirac equation

$$
\begin{equation*}
\left(\gamma_{\mu} p^{\mu}-v_{e} m_{0}\right)\left|p^{(1)}, p^{(2)}, p^{(3)}, \ldots\right\rangle=0 \tag{29}
\end{equation*}
$$

with mass $m=v_{e} m_{0}$.
Recall O'Hara's extension of Pauli's principle, which says that within a state of three spinors only two spins can be correlated and the third must be statistically independent. The relation of such a configuration to the set-specific reference frame must, therefore, be described by the full $\mathrm{SO}(3)$ symmetry of the elementspecific reference frame. By using $\mathrm{SO}(3)$ instead of $\mathrm{SO}(2)$ and going through the same steps as before, we end up with the symmetry group $\mathrm{SO}(3,3)$ instead of $\mathrm{SO}(3,2)$. Instead of the factor 28 ), we must then consider the volume factor [11]

$$
\begin{equation*}
v_{p}=V\left(\frac{S O(3,3)}{S O(3,1) \times S O(2)}\right)=2^{5} \pi^{6} \tag{30}
\end{equation*}
$$

Relative to the factor we obtain

$$
\begin{equation*}
\frac{v_{p}}{v_{e}}=6 \pi^{5}=1836.1185 \tag{31}
\end{equation*}
$$

This value is remarkably close to the empirical ratio of the masses of the proton and the electron,

$$
\begin{equation*}
\frac{m_{p}}{m_{e}}=1836.15267245(75) \tag{32}
\end{equation*}
$$

This strongly supports the identification of the structure described by the three-spinor state (26) with the empirical proton.

Up to here, I have considered only a single set of binary elements. In the mathematical theory of sets, it is very common that elements belong to more than one set. Therefore, an interesting question comes up: If a binary element belongs to more than one set at the same time, how is its relation to these sets described?

The relation of a binary element to a second set can be described in the same way as to the first set, namely by a Dirac spinor. However, the orientation of the second set's semantic reference frame will, in general, be different from the first. Therefore, a momentum state $\psi(p)$, obtained, as before, in the $\mathrm{SO}(3,2)$ symmetric state space of the first set, will, in general, have a different value $p^{\prime}$ relative to the reference frame of a second set. The state must, therefore, be labeled by two parameters: $\psi\left(p, p^{\prime}\right)$. In relation to the corresponding $\mathrm{SO}(3,2)$ states these states now have a multiplicity of $v_{2}=v_{e}{ }^{2}$. Similarly, for a binary element, belonging to three different sets, the multiplicity is $v_{3}=v_{e}{ }^{3}$. There is an additional degree of freedom concerning the rotations of the basic set relative to the additional sets. These rotations, forming a subgroup of $\mathrm{SO}(3,2)$, generate the surface $S^{2}$, the sphere in three dimensions, with a volume of $4 \pi$. This factor applies to $v_{2}$ and $v_{3}$, but not to $v_{e}$, resulting in a mass relation [11]

$$
\begin{equation*}
m_{e}: m_{\mu}: m_{\tau}=1: 4 \pi\left(\frac{16 \pi}{3}\right): 4 \pi\left(\frac{16 \pi}{3}\right)^{2} \tag{33}
\end{equation*}
$$

With the experimental value mass of the electron $m_{e}$ of 0.5109989 MeV , we have $m_{\mu}=107.5916 \mathrm{MeV}$ and $m_{\tau}=1770.3 \mathrm{MeV}$. The experimental values of the muon and tauon masses are 105.658 and 1776.99.

Since in a 3-dimensional space the position of an object is determined by its relation to three reference points that do not lie in the same plane, no further information is gained by adding a fourth point. Therefore, with the relation of a binary element to up to three sets, we have obviously exhausted all relations of informational value.

The large masses of the muon and tauon allow one to expect that these configurations are not stable, but disintegrate into electrons and some other fragments, carrying away momentum and angular momentum. Within the standard model, this disintegration is described by the weak interaction. This suggests that the structures, referred to by the leptonic mass relations, also contain the germ of the weak interaction -another interesting direction for future research.

## 10 Conclusions

Sets of binary elements exhibit a surprisingly rich internal structure, emerging from two closely linked symmetries: the symmetry with respect to the association of semantics (meaning) to two abstract states (symbols) of a binary element, and, the symmetry with respect to the orientation of the element-specific semantic frame of reference relative to a set-specific frame of reference. Furthermore, these structures reflect empirically well-known structures and phenomena of elementary particle physics.

The mathematically formalized description of binary elements as carriers of information not only leads to the roots of quantum mechanics but also to Lorentz transformations, which are the core of Einstein's theory of special relativity.

There are no mysteries about the quantum mechanics of information, which is a description of the structure of information at its basic level, in a way that is covariant with respect to changes of its orientation to the semantic frame of reference. "Wave function collapse" and "quantum jumps" are natural consequences of this description. They occur whenever inexact information is replaced by exact information, and do not depend on the active role of an observer's consciousness. (Note that in this essay I have not tried to examine how information is collected or how it finds its way into the brains of intelligent beings. My considerations concern only the structure of information.)

Similarly, relativistic phenomena, as described by the theory of relativity, are obvious consequences of the fact that "information" always means "information relative to a frame of reference".

We have found the structures of electron and positron, identified by their correct electric charge and correct mass in relation to the proton mass, and of the proton, identified by its compound structure, built of three binary elements in dynamic equilibrium, and by the correct mass in relation to the electron mass. We have also identified the three generations of leptons, identified by their mass ratios.

As shown in [9, binary elements, when described in a Poincaré symmetric frame of reference, show an interaction that in structure and strength is identical to the electromagnetic interaction. For basically the same reason, electrically neutral objects exhibit a gravitational interaction [10, with a strength of the same order of magnitude as the empirical gravitational force. Furthermore, there are indications of two other interactions between certain configurations of binary elements, similar to the weak and strong interactions.

Last not least, one of the phenomenological consequences of the electromagnetic interaction is the existence of massless photons, which travel in space-time with the speed of light and interact with electrons. Thereby they allow observing objects in space-time that are made of -information.

All these results, some of them well founded, other still vague, have been obtained by mathematical deduction from the basic symmetries of binary information - without using information obtained from observation of "Nature." So I think, there is sufficient support for Wheeler's guess, "It from Bit," in the sense that theoretical physics has to be understood as a discipline that deals with the structure of information. This is very close to Bohr's dictum: "Physics is to be regarded not so much as the study of something a priori given, but rather as the development of methods of ordering and surveying human experience" [12].

What we comprehend as physical reality is, therefore, nothing other than a reflection of some predetermined informational structures that we take advantage of, in order to collect and categorize information about the physical world. The physics of elementary particles is, accordingly, a "physics of binary information" 13.

Einstein's quote, "It is the theory which decides what can be observed" 14 perfectly illustrates the role of the physics of binary information: It tells us what we see when we look at the physical world on the most elementary level of information, formed by binary information. Beyond this level, there is no further information. The physics of binary information, therefore, marks the ultimate basis of physics.

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