

## **Unravelling the Missing Physics behind Undecidability, Uncomputability, and Unpredictability**

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### **ABSTRACT**

This paper describes a potential missing physics that leads not only to the artifact undecidability, uncomputability, and unpredictability but also to the current widely known paradoxes of physics/cosmology such as dark energy, dark matter, multiverse, measurement problem (the collapse of the wave function) etc. The proposed Universal Relativity Model (URM) integrates the missing physics of the hidden variables identified as spontaneously converting (decaying and forming) mass, space, and time as per relativity theory. URM describes the fundamental relativistic mechanism governing the quantum events and as to how gravity and measurement interference could affect the wave-particle nature of the observed collapse of the wave function to classic behavior. URM predicts coherence and de-coherence boundary or collapse of the wave function affecting the limits of quantum versus classical behavior as well as the measurement uncertainty of the self-decaying/forming quantum particle. There is no inherent (Heisenberg's) uncertainty built into natural phenomena; the observed uncertainty in quantum measurements results from the measurement error caused by the destructive nature of the classical measurement process that alters the inherent relativistic mass-space-time of the measured event. URM also explains the current paradoxes of physics and cosmology such as dark energy, dark matter, multiverse, black hole singularity etc. URM consolidates and replaces many different interpretations of quantum mechanics into one coherent picture of reality within a holistic relativistic framework and model vindicated by the empirical observations of the universe.

### **INTRODUCTION**

Quantum mechanics theory, in spite of its successes, has remained enigmatic and paradoxical [[https://en.wikipedia.org/wiki/List\\_of\\_unsolved\\_problems\\_in\\_physics](https://en.wikipedia.org/wiki/List_of_unsolved_problems_in_physics)] because of a lack of understanding of its inner workings. Scientists are still puzzled over its features such as wave-particle duality, entanglement, “spooky action at a distance” and whether or not God plays dice with the universe. The behavior of the small or quantum versus the classical large objects shows uncommon features that still remain unexplained by existing theories. Even within quantum domain, massive versus massless particles are treated differently in their mathematical descriptions lacking a cohesive universal model integrating quantum versus classic behavior. Some scientists insist that quantum uncertainty is a real indeterminism in the fundamental nature of reality. Einstein disagreed with this view stating that “God does not play dice with the universe” pointing to the incompleteness of quantum mechanics theory. Common human experience does not support the intuitions of quantum mechanics. Freeman Dyson, an eminent physicist, expressed this strangeness by saying- “I understand now that there isn't anything to be understood.” Even Richard Feynman, known for his mastery of the subject, raised the question: “How does it work? What is the machinery behind the law...We have no ideas about a more basic mechanism from which these results can be deduced.” Sir Roger Penrose states –“I am going to regard the superposition of the one state plus the other as an unstable state – it is a bit like a decaying particle or a uranium nucleus or something like that, where it might decay into one thing or another and there is a certain time-scale associated with that decay. It is a hypothesis

that it (superposition of quantum states) is unstable, but this instability is to be an implication of the physics we do not understand.”

The relativistic mass-energy behavior can affect not only space-time, but also the fundamental fabric of the universe in terms of non-locality and action at a distance. This paper investigates the inner workings of quantum mechanics via developing a new wave-particle model of an unstable particle (spontaneous mass-energy conversion) based on URM, as an alternative to existing de Broglie model, representing the physics of the relativistic mass-energy effects and wave dynamics governing the quantum behavior.

## URM BASED WAVE-PARTICLE DUALITY MODEL

Bohr’s “Copenhagen interpretation” described the wave-particle duality as the basic puzzle of quantum mechanics since both of these classical conceptions of reality are required to make predictions of the observed reality. While the particle nature of light was discovered by Max Planck in 1900 and later clarified by Einstein (1905) in explaining the photoelectric effect, French physicist Louis de Broglie, in his doctoral thesis in 1924, discovered that every particle of matter also acts as a wave that guides its motion in space. Under proper conditions of size and motion of the particle and geometry of interacting bodies (such as slits), a particle will produce an interference or diffraction pattern just like a wave. Louis de Broglie proposed the following relationship between the effective wavelength and momentum of the body:

$$\lambda_{dbr} = \frac{h}{mV} \quad (1)$$

Where  $\lambda_{dbr}$  is de Broglie wavelength, m is the mass and V is the velocity of the body. Louis de Broglie derived the above equation based on classical motion of a fixed mass without any consideration of the relativistic effects. The mass of the body was assumed to remain constant irrespective of the magnitude of its velocity. The need to enhance the existing well-known de Broglie model is realized because it is unable [1 thru 4] to resolve quantum paradoxes such as non-locality, collapse of the wave-function or the measurement problem, Heisenberg’s uncertainty, and parallel universes etc.

Using the relativistic models of URM described in ref. [5 & 6], a set of relativistic relationships governing the dualistic wave-particle behavior can be derived as follows. A quantum particle such as a photon is also described as a quantum wave packet with an energy  $e$  given by the following equation:

$$e = hf \quad (2)$$

Wherein h is Planck’s constant and f is the frequency of oscillation of the wave packet. Now, from Einstein’s specific theory of relativity (ESTR), a mass m has an equivalent energy given as follows,

$$e = mC^2 \quad (3)$$

Combining the above two equations gives,

$$f = \frac{mC^2}{h} \quad (4)$$

The wavelength  $\lambda$  of the wave packet is related to the velocity V as follows,

$$\lambda = \frac{V}{f} = \frac{hV}{mC^2} \quad (5)$$

In ref. [5], for an unstable particle mass  $m$  spontaneously converting to kinetic energy and expanding radially (conserving momentum) at speed  $V$  and with rest mass  $M_0$ , URM formulations provided the following relationship,

$$m = M_0 \sqrt{1 - (V/C)^2} \quad (6)$$

Substituting the above into equations (4) and (5), we obtain the following **Relativistic Wave Particle (RWP)** model expressions for the frequency  $f_{sdm}$  and wavelength  $\lambda_{sdm}$  for an unstable particle's spontaneous mass-energy conversion during the wave-particle behavior,

$$f_{sdm} = \frac{M_0 C^2 \sqrt{1 - (V/C)^2}}{h} \quad (7)$$

$$\lambda_{sdm} = \frac{hV}{M_0 C^2 \sqrt{1 - (V/C)^2}} \quad (8)$$

### COMPARISON OF DE BROGLIE AND RELATIVISTIC WAVE-PARTICLE (RWP) MODEL

Equation (1) gives the de Broglie wavelength for a rest mass  $M_0$  as follows:

$$\lambda_{dbr} = \frac{h}{M_0 V} \quad (9)$$

Now, de Broglie frequency can be calculated as follows,

$$f_{dbr} = \frac{V}{\lambda_{dbr}} = \frac{M_0 V^2}{h} \quad (10)$$

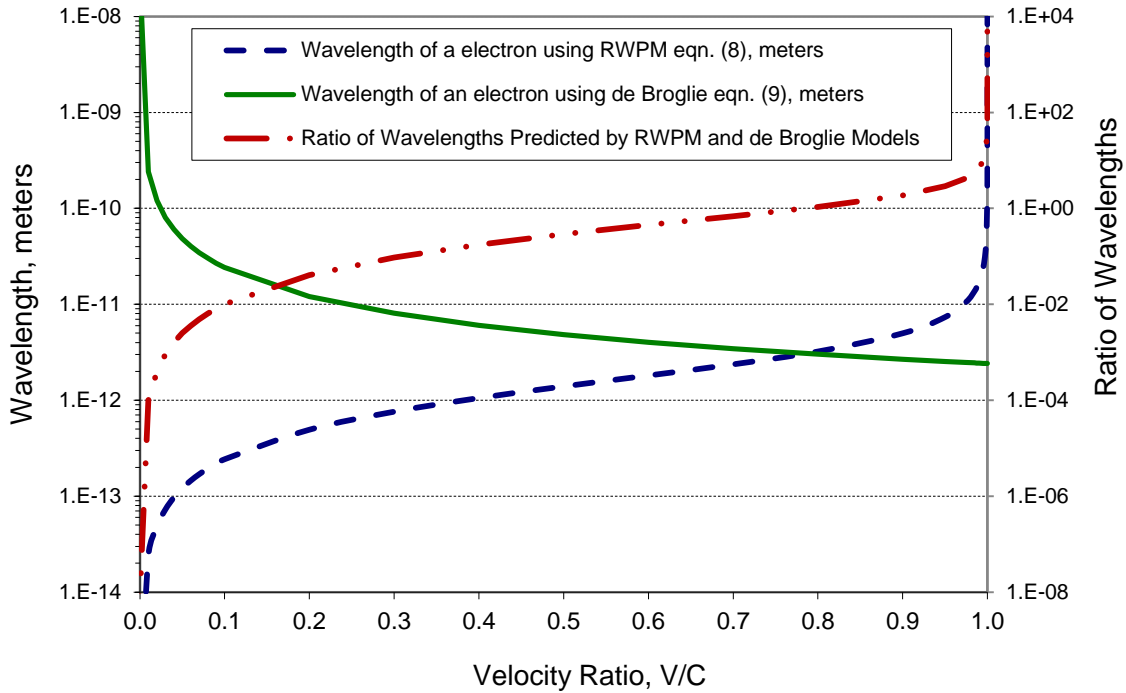
Dividing equations (7) and (8) by equations (10) and (9) respectively, we obtain:

$$\frac{f_{sdm}}{f_{dbr}} = \frac{\sqrt{1 - (V/C)^2}}{(V/C)^2} \quad (11)$$

$$\frac{\lambda_{sdm}}{\lambda_{dbr}} = \frac{(V/C)^2}{\sqrt{1 - (V/C)^2}} \quad (12)$$

Figure 1 shows wavelengths of an electron mass ( $9.1 \times 10^{-31}$  kg) calculated by RWP equation (8) and de Broglie equation (9) as a function of velocity ratio  $V/C$ . At  $V=0$ , the wavelength calculated by de Broglie model is infinite, while RWP calculated wavelength is 0. As  $V$  increases to equal  $C$ , the de Broglie wavelength decreases to reach a minimum value of  $2.42 \times 10^{-12}$  meters while the RWP wavelength increases indefinitely because of a total mass dilation. Close agreement is seen between the wavelengths predicted by two models around  $V/C$  of 0.8.

Figure 1: Comparison of wavelengths predicted by RWP and de Broglie models for an electron.



It should be noted that at small velocities ( $V \sim 0$ ), de Broglie model predicts very large wavelength even for a large mass, which is counter to the physical experience since large classical objects moving at zero or very slow speeds do not act or appear as a wave. Hence, de Broglie model does not represent physical reality at low values of  $V$ . This discrepancy is understandable since this model was originally developed for quantum particles of small masses moving at large velocities close to  $C$ .

### RWP MODEL PREDICTS PHYSICAL LIMITS OF QUANTUM VS. CLASSICAL BEHAVIOR

A quantum particle is an illusive entity that can appear from or disappear into nothingness or vacuum, and exhibits unexplained behavior that follows weird rules involving strange properties. The quantum behavior or properties are so far different from those of the real life objects that there appears to exist separate worlds or universes for the ordinary real life objects versus the quantum objects. Such weirdness is apparent in the theory of parallel universes, which is one of the highly regarded theories apparently explaining the quantum weirdness. There still exists a big gap in the fundamental understanding of the duality of wave and particle behaviors. Roger Penrose [7] states- "... I believe that the normal view of physicists is that, if we really understood quantum physics properly, we could deduce classical physics from it. .... In practice, one does not do that – one uses *either* the classical level *or* the quantum level."

In the previous section, the wave-particle behavior was mathematically described ignoring gravitational effects. In the following, we will develop a URM based relativistic understanding of the quantum behavior and derive mathematical expressions of the physical limits that govern transition between quantum and classical behavior including gravity. Equation (13) below representing URM based model of a self-decaying mass was obtained in ref. [5], eqn. (15a), from conservation of relativistic mass-energy, kinetic energy and gravitational energy,

$$(M_0 - m)C^2 = mC^2 \left\{ \frac{1}{\sqrt{1 - (V/C)^2}} - 1 \right\} + \frac{3Gm^2}{5R} \quad (13)$$

The fundamental characteristic of quantum behavior is the wave-particle complementarity, which allows a quantum entity to act as a wave or particle depending upon its environment. When the particle can move uninhibited, it generally acts or exists as a wave, and when it is intercepted via a measuring device or a fixed boundary wall or slits it appears to act as a particle. Such spontaneity of converting from a particle (mass) to wave (energy) is built into the observed wave-particle behavior in a variety of experiments involving quantum particles such as electrons, atoms or molecules. From equation (13) above, it is evident that the spontaneous mass to energy conversion, represented by equation (6), exhibiting the wave-particle behavior can occur when the gravitational energy is much smaller compared to the kinetic energy:

$$\frac{3Gm^2}{5R} \leq mC^2 \left\{ \frac{1}{\sqrt{1 - (V/C)^2}} - 1 \right\} \quad (14)$$

Radius R in the above equation can be approximated by half of the wavelength given by equation (8),

$$R = \frac{\lambda_{sdm}}{2} = \frac{hV}{2M_0C^2\sqrt{1 - (V/C)^2}} \quad (15)$$

Substituting equations (6) and (15) into equation (14) and simplifying leads to the following:

$$M_{oq} \leq \left( \sqrt{\frac{hC}{G}} \right) \left[ \sqrt{\frac{5V(1 - \sqrt{1 - (V/C)^2})}{6C(1 - (V/C)^2)^{3/2}}} \right] \quad (16)$$

Wherein,  $M_{oq}$  represents the maximum or limiting rest mass that exhibits the wave-particle duality or quantum behavior. Masses greater  $M_{oq}$  would be expected to behave as classical rather than quantum due to significant gravitational effects opposing spontaneous mass-energy conversion. It should be noted that Planck's mass is defined as,

$$M_{pl} = \sqrt{\frac{hC}{G}} \quad (17)$$

Using equation (16), we now define Planck's Mass Factor (PMF) as follows:

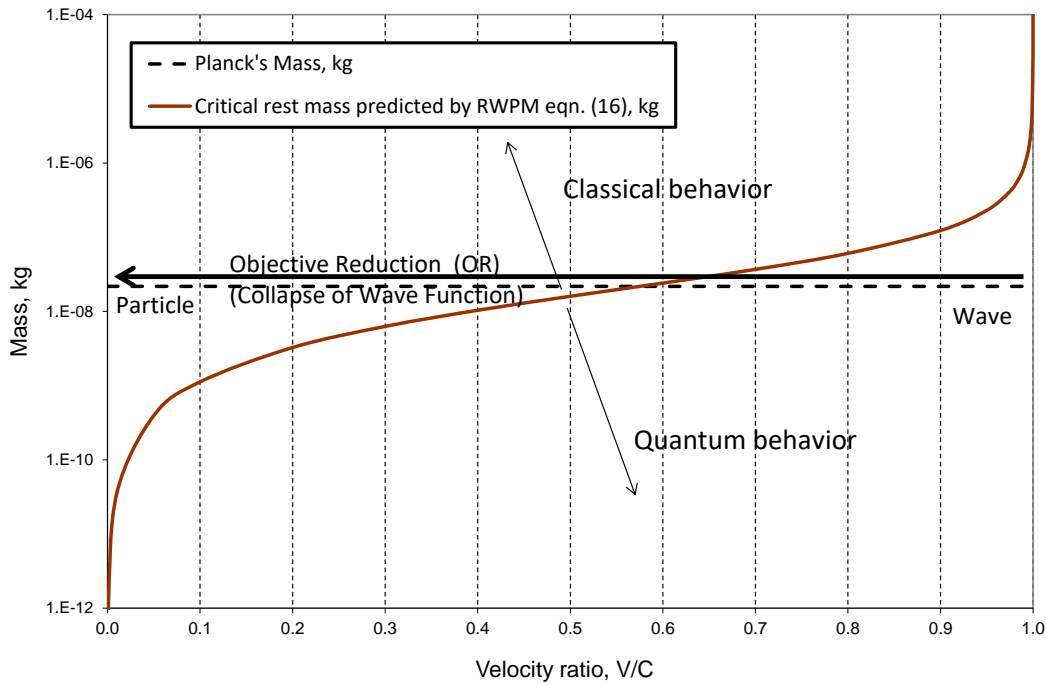
$$PMF = \left[ \sqrt{\frac{5V(1 - \sqrt{1 - (V/C)^2})}{6C(1 - (V/C)^2)^{3/2}}} \right] \quad (18)$$

Combining equations (16), (17), and (18) gives the following criterion for a mass, expressed in terms of a non-dimensional mass  $M_{oqn}$ , to act as a quantum particle:

$$M_{oqn} = \frac{M_{oq}}{M_{pl}} \leq (PMF) \quad (19)$$

Figure 2 shows the critical quantum rest mass predicted by RPW equation (16) describing the limits governing the quantum versus classical behavior. For velocities close to  $V/C = 0.6$ , the predicted critical or maximum quantum rest mass is approximately of the order of Planck’s mass,  $2.1767 \times 10^{-8}$  kilograms. For very small velocities, the critical rest mass decreases and for higher velocities close to the speed of light  $C$ , it increases by several orders of magnitude. Hence, at small velocities the quantum behavior is experienced only by very light particles and at higher velocities even a lot heavier mass can exhibit quantum properties or wave particle duality. For example, Figure 2 shows that very large mass, such as heavier galaxies at the far edge of the universe, moving close to the speed of light may exhibit quantum behavior. Such behavior is not predicted by de Broglie model lacking the relativistic physics of spontaneous mass-energy conversion exhibited during the wave-particle behavior.

Figure 2: Critical quantum rest mass and Objective Reduction (Collapse of Wave Function) of quantum to classical behavior predicted by RWP.



### RWP RESOLVES THE OBJECTIVE REDUCTION (OR) PROBLEM

The **OR problem** is also known as the **Quantum Measurement Problem or Observer’s Paradox (Collapse of the Wave Function)**. At the quantum level, the state of a system is described by a complex-number weighted superposition of all possible alternatives. The time-evolution of the quantum state or the Schrödinger evolution is obtained by the linear superposition of all possible states. Each of the individual states are assumed to evolve independently, but are superposed together with complex number weightings that are invariant in time. This linearity is built into the widely accepted formulation of quantum mechanics in Schrödinger’s wave equation, which represents a deterministic and quantitative description of quantum reality. However, as Roger Penrose [7] points out, the rules change when a measuring device or observer looks into the quantum reality and in the process converts it into a classical

reality. This process of conversion from quantum to classical reality is defined as the *Collapse of the Wave Function* or *Reduction of the State Vector*. An example of this is the observation of an electron wave as a dot when it hits the screen. Following this process, the two alternatives are no longer superposed linearly. Instead, the squares of the weighting complex numbers become the ratio of the probabilities of the two alternatives. The reality that was fully deterministic before the collapse, all of a sudden becomes non-deterministic or probabilistic after the collapse of the wave function caused by the process of measurement or observation by a conscious observer.

This paradox or mystery of quantum mechanics theory is also known as the *Measurement Problem*. The most famous example of this is the so-called *Schrödinger's Cat* paradox in which a cat can exist in a state of being both dead and alive at the same time. A cat, which is a classical object, is never seen to exist in such a quantum state in real life experience. Roger Penrose [7] has made the following remarks regarding the quantum measurement problem-

“What we do not have is a thing which I call OR standing for *Objective Reduction*.....It is a missing theory.....“I am going to regard the superposition of the one state plus the other as an unstable state – it is a bit like a decaying particle or a uranium nucleus or something like that, where it might decay into one thing or another and there is a certain time-scale associated with that decay. It is a hypothesis that it is unstable, but this instability is to be an implication of the physics we do not understand.....The thing is that, in the superposed state, you have to take into account the gravitational contribution to the energy in the superposition. But you cannot really make local sense of the energy due to (quantum) gravity and so there is a basic uncertainty in the gravitational energy...That is just the sort of thing which one gets with unstable particles.”

The mathematical formulations of RWP account for the phenomena identified above by Roger Penrose in explaining the physical basis behind the quantum measurement problem. First of all, RWP equation (13), (14), and (16) above describe the limits of classical versus quantum behavior of particles accounting for the contribution of the gravitational energy as suggested by Roger Penrose eliminating the uncertainty caused by the so-called quantum gravity absent from existing quantum mechanics theories. Secondly, equation (6) properly treats the energy conservation involving spontaneous decay of a mass. In addition to the gravitational potential energy, the kinetic energy and the mass energy are properly accounted for in RWP. This provides for a proper superposition of the mass/energy movement or conversion between various space/time states that a particle can experience before, during or following a measurement is made. In summary, RWP model integrates the missing physics described by Roger Penrose in formulating and resolving the mysterious collapse of the wave function or Objective Reduction (OR) paradox of the quantum mechanics.

Figure 2 shows a schematic of the physical process that happens during measurement of a quantum event and how a quantum wave changes to a particle for the *Objective Reduction* to occur. Let us assume that there exists a quantum entity with a rest mass equal to Planck's mass moving at close to the speed of light in free space as depicted by the right tail end of the arrow in Figure 2. When this quantum entity (existing dominantly as a wave in the free space) is interrupted by a classical measuring device, its velocity ( $V/C$ ) drops to low values as depicted by the left end of the arrow in Figure 3. As is evident, the process of measurement or any obstruction/barrier causes a sudden change or the so-called *collapse* of the quantum wave (wave function) into the region of non-quantum or classical behavior. The Objective Reduction occurs because of this sudden decrease in the quantum wave velocity close to zero leading to the collapse of the wave-function. RWP thus explains the physical process involved in the *Objective Reduction* or *Collapse of the Wave Function* that occurs during the *Measurement problem* in quantum mechanics.

As described in Appendix and ref. [5 & 6], RWP model also resolves some other well-known and as yet unresolved paradoxes related to quantum entanglement or non-locality, Heisenberg's uncertainty, and parallel universes. The overall model provides a potential quantitative mathematical and universal theory for matter-mind-consciousness as described in ref. [8].

### RWP EXPLAINS HEISENBERG'S UNCERTAINTY

The Heisenberg Uncertainty principle describes an inherent and irreducible uncertainty in prescribing both the position ( $\Delta x$ ) and momentum ( $\Delta mV$ ) of quantum particles. Such uncertainty is related to the observed dual behavior of photons and other small particles such as electron, proton etc. in the microscopic world that act both as particles as well as waves. This uncertainty is often presumed to occur due to the direct and unavoidable impact of the measuring device or process on the motion or spatial location of the measured entity itself. In classical physics and Quantum Mechanics, mass, space and time are considered to be fixed (Newtonian frame of reference) and independent entities, which are used to define velocity and momentum of the particle. Detailed calculations by Heisenberg determined the following mathematical form of his uncertainty principle,

$$(\Delta x)\{\Delta(mV)\} \geq \frac{h}{2\pi} \quad (24)$$

Since the Planck's constant,  $h$ , is very small, the uncertainties at macroscopic level or for scales of everyday large objects are negligible. However, at microscopic level wherein we deal with small particles such as electrons, protons and atoms, the uncertainty becomes significant. For particles with higher speeds approaching the speed of light, such as electrons and photons, the relativistic effects become significant and the assumption of fixed space and time does not hold. Heisenberg's principle, known as the fundamental basis for probabilistic formulation and calculation of Quantum mechanical behavior, does not specify as to how does the uncertainty get impacted by these relativistic effects.

In the following, we will use URM to reevaluate the Heisenberg's principle and calculate physical parameters and conditions under which the principle may hold and vice versa. The wavelength of a particle,  $\lambda_{sdm}$ , describes the region in space over which the particle resides, hence it represents the spatial uncertainty in the position of the particle.

$$\Delta x \approx \lambda_{sdm} \quad (25)$$

Using equation (6), the uncertainty in momentum  $mV$  can be calculated as follows,

$$\Delta(mV) = M_o \left( \frac{1 - 2\left(\frac{V}{C}\right)^2}{\sqrt{1 - \left(\frac{V}{C}\right)^2}} \right) (\Delta V) \quad (26)$$

Substituting equations (25) and (26) into (24) and simplifying, the following is obtained,



$$(\Delta x)\{\Delta(mV)\} \geq \frac{h}{2\pi} \left[ 2\pi \left( \frac{V}{C} \right) \left\{ 1 - \frac{\left( \frac{V}{C} \right)^2}{1 - \left( \frac{V}{C} \right)^2} \right\} \Delta \left( \frac{V}{C} \right) \right] \quad (27)$$

Let us define a Heisenberg's Uncertainty Factor (HUF) as follows:

$$HUF_{sdm} = \left[ 2\pi \left( \frac{V}{C} \right) \left\{ 1 - \frac{\left( \frac{V}{C} \right)^2}{1 - \left( \frac{V}{C} \right)^2} \right\} \Delta \left( \frac{V}{C} \right) \right] \quad (28)$$

Equation (24) then can be rewritten as follows:

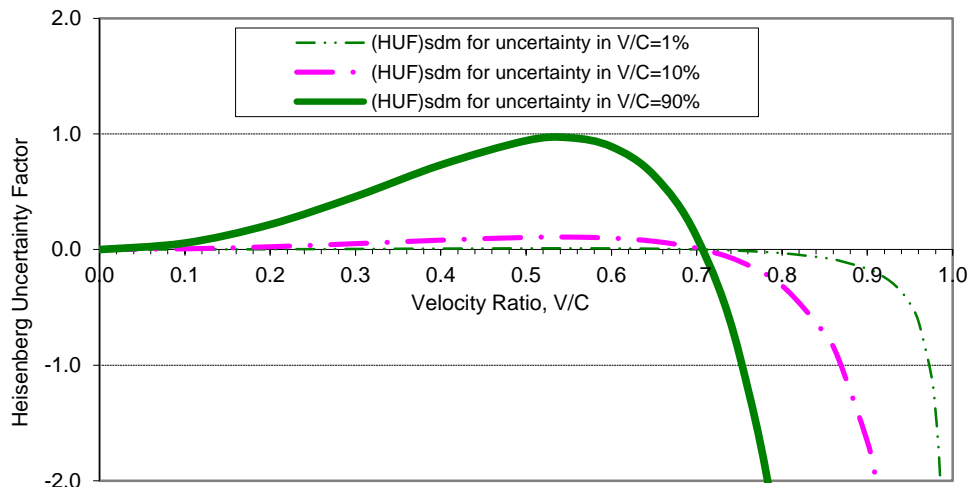
$$(\Delta x)\{\Delta(mV)\} \geq \frac{h}{2\pi} (HUF_{sdm}) \quad (29)$$

Comparing the above against the Heisenberg's principle, equation (24), the principle is satisfied when  $HUF \geq 1$  and violated when  $HUF < 1$ . It is apparent from equation (28) that HUF depends directly upon the value of velocity  $V$  and its uncertainty  $\Delta V$ . If the uncertainty  $\Delta V$  is zero, HUF will be zero and hence, the Heisenberg's principle will be violated. For non-zero  $\Delta V$ , predicted values of the HUF for a self-decaying mass are shown in Figure 3. It is to be noted that for velocities less than approximately 70% of the speed of light  $C$ , the HUF is less than 1 and the Heisenberg's principle is violated even for an uncertainty in velocity  $V$  being as high as 90%. For smaller uncertainties in  $V$ , the principle is satisfied only for much larger velocities. For 1% uncertainty in  $V$ , the minimum value of  $V$  is greater than 95% of the speed of light  $C$  for the principle to hold true.

In summary, the URM provides the physics behind the Heisenberg's Uncertainty principle and its limitations and boundaries within which it holds true:

- URM accounts for the fundamental missing relativistic physics in the formulation of the Heisenberg's principle.
- URM reveals that there is no inherent uncertainty in nature insofar as the true relativistic nature of reality is realized and treated as such.

Figure 3: Heisenberg's Uncertainty Factor for a self-decaying mass.



## SUMMARY AND CONCLUSIONS

URM describes the missing physics that reveals the hidden variables behind the incomplete formulations of quantum mechanics. These hidden variables are identified as spontaneously converting mass/energy, space, and time as per relativity theory. URM describes the fundamental relativistic mechanism governing the quantum events and as to how gravity and measurement interference could affect the wave-particle nature of the observed collapse of the wave function to classic behavior. The limits of quantum versus classical behavior are predicted in terms of the rest mass and velocity of the self-converting (decaying/forming) quantum particle. There is no inherent uncertainty built into natural phenomena; the observed uncertainty in quantum measurements results from the measurement error caused by the destructive nature of measurement that alters the dynamics and relativistic mass-energy-space-time of the measured event leading to the artifact undecidability (uncertainty), unpredictability, and uncomputability.

URM shows that the eternal wholesome fundamental reality is truncated (below Planck scale and beyond the visible universe), deformed (collapsed), and fragmented (multiple universes and dimensions) by the current incomplete quantum particles or string theories as evidenced by their well-known paradoxes (dark energy, dark matter, multiverse, black hole singularity etc.). The predicted vacuum energy by quantum theories being 120 orders of magnitude higher than the observed Cosmological Constant is a living proof of their serious incompleteness and deficiencies. The physical reality of the universe could be described as a set of infinite number of complementary relativistic mass-energy-space-time states as a function of varying velocity of the frame of reference of the observer. There is only one universe entailing multiple complementary or parallel sub-universes or worlds representing the infinite number of complementary relativistic mass-space-time states. There is no need for fine tuning (or anthropic principle) of our universe among multi-universes theory with varying universal constants. URM consolidates and replaces many different interpretations of quantum mechanics into one coherent picture of reality within a holistic relativistic framework and model vindicated by the empirical observations of the universe as described in ref. [5].

While the current widely-accepted theories may be extremely successful and justified in practical technological applications in the material world, they are shown to fall short in explaining the 96% of the observed universe. The foundations of a wholesome science must be built upon a non-destructive observation method without truncation, deformation, or fragmentation of the relativistic continuum of the fundamental universal reality. Perhaps, a meditative mind could be such wholesome observer devoid of undecidability, unpredictability, and uncomputability.

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