Four Squares

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Abstract

This essay discusses the meaning and role of the term "fundamental" as it applies to Math and Physics. The importance of Lagrange's Four Squares Theorem is also discussed. It is argued that the vacuum is a 5-D Quantum-Space-Time and that the vacuum is fundamental.

Preface

No special knowledge is required for this essay. This essay was written for the 2017 FQXi essay contest titled "What is Fundamental?".

"Nothing is particularly hard if you divide it into small jobs." - Henry Ford

Discussion

Background:

(Note - The information presented in this section was gathered from several websites - including Wikipedia - on the internet. It is widely available using a simple search on Google.)

In 1964, two scientists at Bell Labs – Arno Penzias and Robert Wilson – were studying background radio noise. They eliminated all the noise that they could but there was a radio signal that they could not eliminate or explain. What's more, the signal appeared to be uniform in strength in all directions. In 1978 they were jointly awarded the Nobel Prize in Physics for their discovery. Today, we refer to this noise as the Cosmic Microwave Background (CMB). It is essentially a black-body at 2.7 K.

Since then, scientists at NASA and other places have studied the CMB as it applies to cosmology. Perhaps the most recent of these studies is the Wilkinson Microwave Anisotropy Probe (WMAP). The inference made from these studies is that the universe is composed of 70% dark energy, 25% dark matter, and 5% mass-energy. The mass-energy component is what we are made of and it is what most of us think of when we contemplate the universe. This 5% is further broken into hydrogen and helium inter-stellar gas (4%), stars (0.5%), neutrinos (0.3%), and heavier elements (0.03%).

If we hope to understand what is fundamental in the universe, we must study and understand the 95% of the universe that is foreign to us. Let us begin.

Integers:

The modern English word "atom" is derived from the ancient Greek word "atomos" which means uncuttable or indivisible. The author readily accepts that for something to be fundamental, it must be in its most basic functional form. It must be impossible to break it into smaller components without losing its functionality or the essence of what it is. The author also believes that the term "fundamental" can be applied to either an object or an action, and that it must be understood within context.

As examples, let us consider the set of all integers and the subset of all prime integers. For the set of all integers, the values +1 and -1 are fundamental with respect to addition. These values cannot be broken into the sum of two or more smaller integers, and it is possible to generate any integer including zero by beginning with one of them and repeatedly adding either +1 or -1. Similarly, the prime integers are fundamental with respect to multiplication. They cannot be factored into the multiplicative product of two or more smaller integers, and it is possible to generate unique integer values that are the multiplicative product of two or more prime integers.

Now let us consider how to go about making a complex structure from fundamental building blocks. The important thing to remember is that only integer multiples of the fundamental building blocks can be used. So, it is impossible to make a right triangle using sides of length 1, 1, and sqrt (2) because sqrt (2) is not an integer. But it is possible to make an equilateral triangle with sides 1, 1, and 1. Similarly, it is possible to make a right triangle with sides 3, 4, and 5. In fact, that is a favorite trick used by carpenters to make a right angle. This works because $3^2 + 4^2 = 5^2$. Thank you very much Pythagoras.

So then, it is reasonable to ask if it is possible using squares to produce any arbitrary positive integer. The answer is “yes” but it is necessary to use four squares. This is called *Lagrange’s Four Squares Theorem* and it is written as follows:

Lagrange’s Four Squares Theorem:

$$a^2 + b^2 + c^2 + d^2 = e : a, b, c, d, and\ e\ are\ all\ integers$$

Now let us specify that the term on the RHS is also a perfect square. Let $e = f^2$.

$$a^2 + b^2 + c^2 + d^2 = f^2$$

Let us also specify that all the integer terms on the LHS and RHS are each multiplied by some arbitrary basis length “u”.

Equation 1:

$$(a^2 + b^2 + c^2 + d^2)u^2 = f^2u^2$$

The meaning of Equation 1 is that in a 4-D geometry, if a right triangle is constructed from an integer number of basis lengths in each of the four dimensions (a, b, c, and d), then the hypotenuse (f) that traverses through the 4-D space will also have an integer number of the basis lengths.

At first glance, it would seem obvious that Equation 1 is applicable to space-time and that the basis for length is the Plank length. But is it really? Is space-time continuous or is it discontinuous? If space-time is continuous then Equation 1 is applicable and matter must be immersed in space-time. But if space-time is instead discontinuous then Equation 1 is not applicable and perhaps matter fills the discontinuities in space-time.

In the text below, the author will bring Euler’s Equation into the discussion. In general, the cosine and sine terms will not be integers. But the circle that is their sum will be a unit circle and it can be multiplied by an integer and the result can be inserted as one of the terms of the four squares theorem.

Vibrating Strings and Space-Time:

In Quantum Mechanics, the question is often asked, "What is waving?". In the previous essay contest, the author argued in favor of a 5-D geometry composed of three spatial dimensions and two time-like dimensions (one scalar and one complex). The author will expand upon that argument herein.

When a 1-D string vibrates, it can either vibrate in the axial direction or the transverse direction. If it vibrates axially, then the system remains 1-D. However, if the string vibrates in the transverse direction, then the system becomes 2-D. Sound is an example of a vibration in the direction of the wave's motion (axial vibration). Light is an example of a vibration that is perpendicular to the wave's direction of motion (transverse vibration).

Now let us consider something like a thin piece of wire. The wire is 1-D but it can be bent into a curved shape. So, the 1-D wire exists in 2-D space. This can be extended to thin 2-D membranes and perhaps to higher dimensional objects also.

Einstein stated that matter causes space-time to bend or curve. Since space-time has four dimensions, it seems to the author that curved space-time must require five dimensions. Also, the recent LIGO experiments have confirmed gravitational waves. These are described by the LIGO scientists as vibrations in space-time itself. If these vibrations are transverse, then this would also require a fifth dimension. Therefore, it seems to the author that the vacuum is some kind of 5-D quantum-space-time and that this is the medium that is waving in QM. The author believes that this 5-D medium is the fundamental structure of the universe. Understanding and describing the vacuum should be the goal.

The Vacuum and the Electron:

$6\pi^5$. In April of 1951, Friedrich Lenz of Düsseldorf, Germany sent a letter to Physical Review noting that the ratio of the mass of the proton to the mass of the electron is very nearly $6\pi^5$. Unfortunately, no supporting theory or reasoning was provided. Perhaps this was thought to be a coincidence. Perhaps this was thought to be unimportant. For whatever reason, this observation appears to have been either ignored, dismissed, or forgotten. The author believes that this $6\pi^5$ value is a fundamental geometric ratio that describes the structure of quantum-space-time.

In a previous work[1], the author presented an argument that the vacuum is a scalar field of "potential electrons" and that electrons rise up from the vacuum when in the proximity of a proton. The essence of the concept is that electrons that are associated with electrically neutral atoms are actually stationary vibrations of the vacuum. The implications of this thinking are that the electrons associated with electrically neutral atoms do not carry momentum, that they have no kinetic energy, and that the Lorentz Transform is not applicable to them. These implications are not applicable to free electrons that are not associated with an electrically neutral atom. Protons and neutrons do move and carry both momentum and kinetic energy. Since in this model, atomic protons and neutrons move but their associated electrons are stationary, it follows that the ratios between their masses and the mass of the electron will vary due to the Lorentz Transform being applied to the protons and neutrons but not applied to the electrons.

In a continuation of that work[2], the author determined a wave-function that simultaneously satisfies both the Schrödinger Equation and the classical wave equation. This produced a quantitative model capable of making several predictions. The most interesting of these are Equation 13.4 and Equation 22.7. These are presented and briefly discussed below.

Equation 2:

$$d = 4\sqrt{3}\left(\frac{1}{mc}\right)\left(\frac{h}{2\pi}\right)\ln(\pi) = \frac{2\sqrt{3}}{\alpha}q_0$$

Equation 2 presents the diameter of the proton. The calculated value is 1.668 x 10^{-15} meters. The accepted value is 1.755(102) x 10^{-15} meters as presented by the NIST.

Equation 3:

$$m_E = \pm\left(\frac{1}{2}\right)\left(\frac{h}{2\pi}\right)\left(\frac{1}{c}\right)\left[\lim_{r\to 0}(\mathbf{\Psi_E} - \psi_0)\right]\left[\sqrt{1-\frac{v^2}{c^2}} + \frac{v}{c}i\right]$$

Equation 3 relates the mass of the electron to the properties of the vacuum. It shows that the vacuum is ideally suited to carry momentum. It also contains the Lorentz Transform and a term that might either describe QM spin or motion induced stellar aberration.

The Octonion Group:

The author proposes the following as a universal wave-function:

Equation 4:

$$\mathbf{\Psi} = e^{i\omega}e^{\mathbf{Q}} = e^{i\omega+q_0+\mathbf{q}} = e^{i\omega}e^{q_0}e^{\mathbf{q}} = e^{\mathbf{P}};\ \mathbf{P} = i\omega + \mathbf{Q}$$

Equation 4.1:

$$\mathbf{Q} = q_0 + \mathbf{q} = q_0 + q_i\mathbf{i} + q_j\mathbf{j} + q_k\mathbf{k}$$

Equation 4.2:

$$e^{i\omega} = \cos(\omega) + \sin(\omega)\,i$$

Equation 4.3:

$$e^{\mathbf{Q}} = e^{q_0}\left[\cos(\gamma_0) + \sin(\gamma_i)\,\mathbf{i} + \sin(\gamma_j)\mathbf{j} + \sin(\gamma_k)\mathbf{k}\right] = e^{q_0}[\cos(\gamma_0) + L\mathbf{u}]$$

Equation 4 has five dimensions. These are the scalar one, the complex *i*, and the unit vectors **i**, **j**, and **k**. When Equations 4.2 and 4.3 are substituted into Equation 4 and multiplied, the result is an expression with eight terms that is a subset of the octonions. The complex *i* anti-commutes with the unit vectors in this system.

In Mathematics, a group is defined as follows[3]: “Any collection of elements {A, B, …, R, …} has the group property if an associative law of combination is defined such that for any ordered pair R, S there is a unique product, written RS, which (in some agreed sense) is equivalent to some single element T which is also in the collection.”

The table below presents the multiplication table for the ordered set RS for the Octonion Group.

	S	$+1$	$+\mathbf{i}$	$+\mathbf{j}$	$+\mathbf{k}$	$+i$	$+i\mathbf{i}$	$+i\mathbf{j}$	$+i\mathbf{k}$	-1	$-\mathbf{i}$	$-\mathbf{j}$	$-\mathbf{k}$	$-i$	$-i\mathbf{i}$	$-i\mathbf{j}$	$-i\mathbf{k}$
R																	
$+1$		$+1$	$+\mathbf{i}$	$+\mathbf{j}$	$+\mathbf{k}$	$+i$	$+i\mathbf{i}$	$+i\mathbf{j}$	$+i\mathbf{k}$	-1	$-\mathbf{i}$	$-\mathbf{j}$	$-\mathbf{k}$	$-i$	$-i\mathbf{i}$	$-i\mathbf{j}$	$-i\mathbf{k}$
$+\mathbf{i}$		$+\mathbf{i}$	-1	$+\mathbf{k}$	$-\mathbf{j}$	$-i\mathbf{i}$	$+i$	$-i\mathbf{k}$	$+i\mathbf{j}$	$-\mathbf{i}$	$+1$	$-\mathbf{k}$	$+\mathbf{j}$	$+i\mathbf{i}$	$-i$	$+i\mathbf{k}$	$-i\mathbf{j}$
$+\mathbf{j}$		$+\mathbf{j}$	$-\mathbf{k}$	-1	$+\mathbf{i}$	$-i\mathbf{j}$	$+i\mathbf{k}$	$+i$	$-i\mathbf{i}$	$-\mathbf{j}$	$+\mathbf{k}$	$+1$	$-\mathbf{i}$	$+i\mathbf{j}$	$-i\mathbf{k}$	$-i$	$+i\mathbf{i}$
$+\mathbf{k}$		$+\mathbf{k}$	$+\mathbf{j}$	$-\mathbf{i}$	-1	$-i\mathbf{k}$	$-i\mathbf{j}$	$+i\mathbf{i}$	$+i$	$-\mathbf{k}$	$-\mathbf{j}$	$+\mathbf{i}$	$+1$	$+i\mathbf{k}$	$+i\mathbf{j}$	$-i\mathbf{i}$	$-i$
$+i$		$+i$	$+i\mathbf{i}$	$+i\mathbf{j}$	$+i\mathbf{k}$	-1	$-\mathbf{i}$	$-\mathbf{j}$	$-\mathbf{k}$	$-i$	$-i\mathbf{i}$	$-i\mathbf{j}$	$-i\mathbf{k}$	$+1$	$+\mathbf{i}$	$+\mathbf{j}$	$+\mathbf{k}$
$+i\mathbf{i}$		$+i\mathbf{i}$	$-i$	$+i\mathbf{k}$	$-i\mathbf{j}$	$+\mathbf{i}$	-1	$+\mathbf{k}$	$-\mathbf{j}$	$-i\mathbf{i}$	$+i$	$-i\mathbf{k}$	$+i\mathbf{j}$	$-\mathbf{i}$	$+1$	$-\mathbf{k}$	$+\mathbf{j}$
$+i\mathbf{j}$		$+i\mathbf{j}$	$-i\mathbf{k}$	$-i$	$+i\mathbf{i}$	$+\mathbf{j}$	$-\mathbf{k}$	-1	$+\mathbf{i}$	$-i\mathbf{j}$	$+i\mathbf{k}$	$+i$	$-i\mathbf{i}$	$-\mathbf{j}$	$+\mathbf{k}$	$+1$	$-\mathbf{i}$
$+i\mathbf{k}$		$+i\mathbf{k}$	$+i\mathbf{j}$	$-i\mathbf{i}$	$-i$	$+\mathbf{k}$	$+\mathbf{j}$	$-\mathbf{i}$	-1	$-i\mathbf{k}$	$-i\mathbf{j}$	$+i\mathbf{i}$	$+i$	$-\mathbf{k}$	$-\mathbf{j}$	$+\mathbf{i}$	$+1$
-1		-1	$-\mathbf{i}$	$-\mathbf{j}$	$-\mathbf{k}$	$-i$	$-i\mathbf{i}$	$-i\mathbf{j}$	$-i\mathbf{k}$	$+1$	$+\mathbf{i}$	$+\mathbf{j}$	$+\mathbf{k}$	$+i$	$+i\mathbf{i}$	$+i\mathbf{j}$	$+i\mathbf{k}$
$-\mathbf{i}$		$-\mathbf{i}$	$+1$	$-\mathbf{k}$	$+\mathbf{j}$	$+i\mathbf{i}$	$-i$	$+i\mathbf{k}$	$-i\mathbf{j}$	$+\mathbf{i}$	-1	$+\mathbf{k}$	$-\mathbf{j}$	$-i\mathbf{i}$	$+i$	$-i\mathbf{k}$	$+i\mathbf{j}$
$-\mathbf{j}$		$-\mathbf{j}$	$+\mathbf{k}$	$+1$	$-\mathbf{i}$	$+i\mathbf{j}$	$-i\mathbf{k}$	$-i$	$+i\mathbf{i}$	$+\mathbf{j}$	$-\mathbf{k}$	-1	$+\mathbf{i}$	$-i\mathbf{j}$	$+i\mathbf{k}$	$+i$	$-i\mathbf{i}$
$-\mathbf{k}$		$-\mathbf{k}$	$-\mathbf{j}$	$+\mathbf{i}$	$+1$	$+i\mathbf{k}$	$+i\mathbf{j}$	$-i\mathbf{i}$	$-i$	$+\mathbf{k}$	$+\mathbf{j}$	$-\mathbf{i}$	-1	$-i\mathbf{k}$	$-i\mathbf{j}$	$+i\mathbf{i}$	$+i$
$-i$		$-i$	$-i\mathbf{i}$	$-i\mathbf{j}$	$-i\mathbf{k}$	$+1$	$+\mathbf{i}$	$+\mathbf{j}$	$+\mathbf{k}$	$+i$	$+i\mathbf{i}$	$+i\mathbf{j}$	$+i\mathbf{k}$	-1	$-\mathbf{i}$	$-\mathbf{j}$	$-\mathbf{k}$
$-i\mathbf{i}$		$-i\mathbf{i}$	$+i$	$-i\mathbf{k}$	$+i\mathbf{j}$	$-\mathbf{i}$	$+1$	$-\mathbf{k}$	$+\mathbf{j}$	$+i\mathbf{i}$	$-i$	$+i\mathbf{k}$	$-i\mathbf{j}$	$+\mathbf{i}$	-1	$+\mathbf{k}$	$-\mathbf{j}$
$-i\mathbf{j}$		$-i\mathbf{j}$	$+i\mathbf{k}$	$+i$	$-i\mathbf{i}$	$-\mathbf{j}$	$+\mathbf{k}$	$+1$	$-\mathbf{i}$	$+i\mathbf{j}$	$-i\mathbf{k}$	$-i$	$+i\mathbf{i}$	$+\mathbf{j}$	$-\mathbf{k}$	-1	$+\mathbf{i}$
$-i\mathbf{k}$		$-i\mathbf{k}$	$-i\mathbf{j}$	$+i\mathbf{i}$	$+i$	$-\mathbf{k}$	$-\mathbf{j}$	$+\mathbf{i}$	$+1$	$+i\mathbf{k}$	$+i\mathbf{j}$	$-i\mathbf{i}$	$-i$	$+\mathbf{k}$	$+\mathbf{j}$	$-\mathbf{i}$	-1

When James Clerk Maxwell first formalized electro-magnetism, he did so using quaternions. Hamilton had recently published his development of quaternions and they seemed ideally suited to describe 3-D space. The relevance to this essay is that half of the terms in the Octonion Group are a quaternion and the other half of the terms are produced by multiplying the complex *i* by a quaternion. This multiplication transforms the quaternion terms into “something else” but this “something else” is still part of the group. By multiplying by the complex *i* a second time, the opposite of the original quaternion is returned. To paraphrase Dirac, it is the square root of geometry. Therefore, the Octonion Group as described here can encompass all of electro-magnetism and “something else”. It does so in a way that is internally consistent and self-interacting. The challenge then is to understand the physical meaning of the complex quaternion terms. The author is actively studying this question.

Matrix Operations:

The material in this section is developed more completely by the author in reference [4]. Now let us consider a bi-quaternion form of octonion $\mathbf{O_Q}$ as follows:

Equation 5:

$$\mathbf{O_Q} = e^{i\omega}\mathbf{Q} = [\cos(\omega) + i\sin(\omega)]\mathbf{Q} = \cos(\omega)\,\mathbf{Q} + i\sin(\omega)\,\mathbf{Q}$$

The conjugate of this expression is:

Equation 5.1:

$$\mathbf{O_Q^*} = \cos(\omega)\,\mathbf{Q}^* - i\sin(\omega)\,\mathbf{Q}$$

This is easily proven by multiplication of these two expressions. Note that the conjugate uses both **Q** and **Q***.

$$\mathbf{O_Q^*}\mathbf{O_Q} = [\cos(\omega)\,\mathbf{Q}^* - i\sin(\omega)\,\mathbf{Q}][\cos(\omega)\,\mathbf{Q} + i\sin(\omega)\,\mathbf{Q}]$$

$$\mathbf{O_Q^*}\mathbf{O_Q} = \cos\omega\,\mathbf{Q}^*\cos\omega\,\mathbf{Q} - i\sin\omega\,\mathbf{Q}\cos\omega\,\mathbf{Q} + \cos\omega\,\mathbf{Q}^* i\sin\omega\,\mathbf{Q} - i\sin\omega\,\mathbf{Q}i\sin\omega\,\mathbf{Q}$$

$$\mathbf{O_Q^*}\mathbf{O_Q} = cos^2\omega\,\mathbf{Q}^*\mathbf{Q} - i\sin\omega\cos\omega\,\mathbf{QQ} + i\cos\omega\sin\omega\,\mathbf{QQ} - i^2 sin^2\omega\mathbf{Q}^*\mathbf{Q}$$

$$\mathbf{O_Q^*}\mathbf{O_Q} = (cos^2\omega + sin^2\omega)\,\mathbf{Q}^*\mathbf{Q} + i(-\sin\omega\cos\omega + \cos\omega\sin\omega)\mathbf{QQ}$$

Equation 5.2:

$$\mathbf{O_Q^*}\mathbf{O_Q} = \mathbf{Q}^*\mathbf{Q} = q_0^2 + q_i^2 + q_j^2 + q_k^2$$

There is something that is very interesting about this expression. The phase angle ω has disappeared from the expression. Therefore, the result is the same irrespective of the phase angle ω that is used. In fact, the complex plane has disappeared from the expression. Essentially, the phase angle ω has become a hidden variable. In the author's opinion, this behavior mimics the behavior of wave-function collapse. This also provides a method to return to the four squares theorem as discussed above.

Now let us multiply a pair of octonions that satisfy Equation 5.

$$\mathbf{O_A O_C} = (\mathbf{A} + i\mathbf{B})(\mathbf{C} + i\mathbf{D})$$

$$\mathbf{O_A O_C} = \mathbf{AC} + i\mathbf{BC} + \mathbf{A}i\mathbf{D} + i\mathbf{B}i\mathbf{D}$$

$$\mathbf{O_A O_C} = \mathbf{AC} + \mathbf{B}^* i\mathbf{C} + \mathbf{A}i\mathbf{D} - \mathbf{B}^*\mathbf{D}$$

$$\mathbf{O_A O_C} = \mathbf{AC} - \mathbf{B}^*\mathbf{D} +$$

$$\mathbf{B}^* i\mathbf{C} + \mathbf{A}i\mathbf{D}$$

Equation 5.3:

$$\mathbf{O_A O_C} = \begin{bmatrix} +\mathbf{A} & -\mathbf{B}^* \\ +\mathbf{B}^* & +\mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{C} \\ \mathbf{D} \end{bmatrix}$$

Look at how simple it is to express this by using Hamilton's quaternions! This can then be presented as an 8x8 matrix multiplication by substituting the submatrix that is associated with each quaternion.

Equation 5.4:

$$\mathbf{O_A O_C} = \begin{bmatrix} +a_0 & -a_i & -a_j & -a_k & -b_0 & -b_i & -b_j & -b_k \\ +a_i & +a_0 & -a_k & +a_j & +b_i & -b_0 & -b_k & +b_j \\ +a_j & +a_k & +a_0 & -a_i & +b_j & +b_k & -b_0 & -b_i \\ +a_k & -a_j & +a_i & +a_0 & +b_k & -b_j & +b_i & -b_0 \\ +b_0 & +b_i & +b_j & +b_k & +a_0 & -a_i & -a_j & -a_k \\ -b_i & +b_0 & +b_k & -b_j & +a_i & +a_0 & -a_k & +a_j \\ -b_j & -b_k & +b_0 & +b_i & +a_j & +a_k & +a_0 & -a_i \\ -b_k & +b_j & -b_i & +b_0 & +a_k & -a_j & +a_i & +a_0 \end{bmatrix} \begin{bmatrix} +c_0 \\ +c_i \\ +c_j \\ +c_k \\ +d_0 \\ +d_i \\ +d_j \\ +d_k \end{bmatrix}$$

And the inverse matrix is as follows:

Equation 5.5:

$$[m]^{-1} = \frac{1}{\|\mathbf{Q}\|^2} \begin{bmatrix} +a_0 & +a_i & +a_j & +a_k & +b_0 & -b_i & -b_j & -b_k \\ -a_i & +a_0 & +a_k & -a_j & +b_i & +b_0 & -b_k & +b_j \\ -a_j & -a_k & +a_0 & +a_i & +b_j & +b_k & +b_0 & -b_i \\ -a_k & +a_j & -a_i & +a_0 & +b_k & -b_j & +b_i & +b_0 \\ -b_0 & +b_i & +b_j & +b_k & +a_0 & +a_i & +a_j & +a_k \\ -b_i & -b_0 & +b_k & -b_j & -a_i & +a_0 & +a_k & -a_j \\ -b_j & -b_k & -b_0 & +b_i & -a_j & -a_k & +a_0 & +a_i \\ -b_k & +b_j & -b_i & -b_0 & -a_k & +a_j & -a_i & +a_0 \end{bmatrix}$$

Where:

Equation 5.5.1:

$$\|\mathbf{Q}\|^2 = \|\mathbf{A}\|^2 + \|\mathbf{B}\|^2$$

As a reminder, the coefficients in the above matrices must satisfy the following:

Equation 5.6:

$$a_0 = \cos(\omega)\, q_0;\ a_i = \cos(\omega)\, q_i;\ a_j = \cos(\omega)\, q_j;\ a_k = \cos(\omega)\, q_k$$

$$b_0 = \sin(\omega)\, q_0;\ b_i = \sin(\omega)\, q_i;\ b_j = \sin(\omega)\, q_j;\ b_k = \sin(\omega)\, q_k$$

Conclusions

The universe is huge, unimaginably so. Our Hubble Bubble has a radius of roughly 13.8 billion light-years. Most of this space appears to us as empty vacuum. Yet, Maxwell showed that two properties of the vacuum define the speed of light. Paul Dirac once described spin as the square root of geometry. Therefore, there is reason to believe that that this vacuum has structure. The 5-D Octonion Group presented here preserves these features and allows sufficient space for both QM and GR to exist. The author argues that the vacuum is the most fundamental structure in the universe and that the Octonion Group describes the vacuum.

Acknowledgements

The author thanks Wikipedia.

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