Nature's Information and Harmonic Proportion

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Abstract

The history of science is polarized by debates over Plato and Aristotle's holism versus the atomism of Democritus and others. This includes the complementarity of continuous and discrete, one and the many, waves and particles, and analog or digital views of reality. The three-fold method of the Pythagorean paradigm of unity, duality, and harmony enables the calculation of fundamental physical constants required by the forces of nature in the formation of matter; thereby demonstrating Plato's archetypal viewpoint.

## 1. Introduction: Unity, Duality, and Harmony

As the Pythagoreans understood, even the simplest mathematical concepts – one, two three, four, straight lines, circles – are nontrivial; in fact, the are deep and mysterious. – William Byers *How Mathematicians Think* [1]


Unity, Duality, and Harmony

The Pythagorean paradigm included both the Orphic and Egyptian tradition. From Aristotle the *quinta essentia* is Plato's fifth element, related to the dodecahedron and described as the "one in four" in alchemy. The fifth Platonic solid, the dodecahedron, has 12 faces, 20 vertices, and 30 edges. The dodecahedron has been associated with the elemental aether, classical quintessence as a unified force, and the icosahedral symmetry group. The dodecahedron and its "embroidered figures" represented the whole universe in Plato's natural philosophy. Aristotle, in his *Metaphysics*, finds the Pythagoreans

"...consider that number is the principle both as matter for things and as forming both their modifications and their permanent states, and hold that the elements of number are the even and the odd, and that of these the latter is limited, and the former unlimited; and that the One proceeds from both of these (for it is both even and odd), and number from the One; and that the whole heaven, as has been said, is numbers."
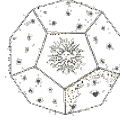
Kepler's Dodecahedron Universe

The faces of a regular dodecahedron are pentagons, and the diagonal of the regular pentagon with side equal to one is ϕ, the golden ratio. The golden ratio can be found from the 3, 4, 5 triangle construction. Plato described the basic geometric proportions of the Cosmological Circle in his allegorical "likely story" of the ideal City of Magnesia.
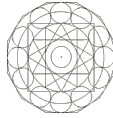
The Cosmological Circle

Michael Schneider finds "The celestial design canon contained in the Delphic riddle of the squared circle served as the pattern for twelve part temples, myths, measures, and societies throughout the world over many centuries." [2]. The dodecahedron has been considered a three dimensional version of the Cosmological Circle, with its twelve pentagonal faces. Euler's equation is a well-known favorite in the history of mathematics,

$$e^{i\pi} + 1 = 0. \tag{1}$$

Natural logarithm base, $e \simeq \sqrt{5+\sqrt{4+\sqrt{3}}}$ and $\sqrt{3+\sqrt{4+\sqrt{5}}} \simeq e^{\pi}/\pi^2$. Another geometric relationship, recalling Euler's equation and previous work shows a connection between the golden ratio and dodecahedron representing Aristotle's quintessence [3]. The initial numerical form of quintessence, $Q \simeq 1.019$, was from Malcolm Macleod's geometric model of momentum and later approximated by the heptagon and the Cosmological Circle [3].

$$e^{iQ} \simeq \phi^2/5 + \sqrt{5}i/\phi^2, \tag{2}$$

$\cos^{-1}(\phi^2/5) \simeq 58^o$, $\sin^{-1}(\sqrt{5}/\phi^2) \simeq 59^o$ which is nearly the hexagon angle of 60° and the sum of both is the dodecahedron dihedral angle $\approx 117^o$. The dihedral angle is the internal face angle where two adjacent polyhedron faces meet. Kepler writes, "Finite things which are circumscribed and shaped can also be grasped by the mind: infinite and unbounded things, insofar as they are such, can be held in by no bonds of knowledge, which is obtained from definitions," and continuing, "For shapes are in the archetype prior to

their being in the product; in the divine mind prior to being in creatures, differently indeed in respect of their subject, but the same in the form of their essence." [4]. In Michael Schneider's Platonic viewpoint [2], "The wonder of twelve is that it has so many divisors (one, two, three, four, and six), making it the supreme number appointed at the archetypal level of mathematics as the natural framework of arithmetic and geometry."

$$Q \simeq (\tan{(2\pi/7)})^{1/12}, \tag{3}$$

the quintessence geometry. $Q \simeq 377/370 \simeq 12\pi/37 \simeq 1.019$, and $\tan(2\pi/7) \approx \sqrt{\phi}$. The large triangle in the regular heptagon gives an approximation of two "Kepler triangles" $(1)^2 + (\sqrt{\phi})^2 = (\phi)^2$ with the golden ratio, $\phi = (1+\sqrt{5})/2$. The inner radius r_i of the regular dodecahedron is $r_i = (250 + 110\sqrt{5})^{1/2}/20 \simeq 7/2\pi \simeq \pi/eQ^2$, see reference [3]. 6, 12, and 37 are most prominent in the ancient canon and oral tradition of geometry [3].

$$Q^{-1} \simeq 3/(666\,999)^{1/12}, \tag{4}$$

$18 \times 37 = 666$, and $27 \times 37 = 999$. In previous work the twelfth or one-twelfth power, and division or multiplication by three or one-third was applied to fundamental physical constants to reveal a connection with classical harmonics. The classical harmonic is shown as the focus of interest for the "meaningful information" of harmonic proportion [5, 6] (or power of ten) in data relevant to the Cosmological Circle or Foundation Stone. For the question of fundamental units see the Eq. (14) discussion, [3, 7] and references.

2. Silver Constant and the Regular Heptagon

Seven and twelve are associated so often in myth and legend it becomes obvious that they refer to a system of symbolic philosophy. – Michael Schneider
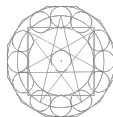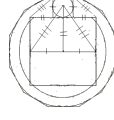

Star Heptagon in the Cosmological Circle

$$s = 2 + 2\cos{(2\pi/7)}, \tag{5}$$

the silver constant s with the heptagon angle $2\pi/7$. $s \cong 3.246\,979\,603\,717$. s is the seventh Beraha constant [8]. The approximate chord of the regular heptagon, $1.802 \simeq \sqrt{s} \simeq 7\phi/2\pi \simeq r_i\phi$. $s \simeq 2\tan Q \simeq \tan\sqrt{\phi} \simeq 2\phi$, ϕ is the golden ratio, and ϕ^2 is the fifth Beraha constant. If $x = \sqrt[3]{7+7\sqrt[3]{7+...}}$, then $s = 2 + (x+2)/(x+1)$. $s \simeq 250/77 \simeq 120/37 \simeq \sqrt{4\sqrt{7}}$. and $37 \simeq 14\sqrt{7}$. 77 is the small circle diameter in the harmony pattern of the Foundation Stone, see Eq. (10) discussion. 7 is the basic heptagon radius in the Cosmological Circle, and 14 is the diameter. $Q \simeq (887/84s^2)^{12}$, $7 + 77 = 84$, and $110 + 777 = 887$. Next, the mid-radius of the regular dodecahedron, $r_m = (3+\sqrt{5})/4 = \phi^2/2 \simeq 3\sqrt{2}/s \simeq \ln(1+\phi^2)$.

3. Golden Ratio and the Fine-Structure Constant

They call the Pentad …Pallas, because it reveals the fifth essence. – Anatolius

Kepler's triangle 1, $\sqrt{\phi}$, ϕ *and* 3, 4, 5 *top triangles in the Cosmological Circle*

Fifth essence, quintessence: $(3 \times 4 \times 5)/(3+4+5) = 5$, the harmony archetype. Relating quintessence, the golden ratio, and Pythagorean harmonics from the Cosmological Circle with the fine-structure constant; with the value for α that is determined from Eq. (7).

$$Q \cong 1 + \alpha(\phi + 9\alpha/110\phi)^2 \cong 1.019\,113\,431\,9, \qquad (6)$$

$Q \simeq 1 + \alpha\phi^2 \simeq \sqrt{e}/\phi$. $3^6 = 9^3 = 360 + 369 = 729$ and $729 - 619 = 110$ from the basic Foundation Stone dimensions of ancient Greek geometry, their template for the formation of matter and closely related to the Cosmological Circle [3]. 729 is the perimeter of the three circles, the harmony pattern, in the Foundation Stone and approximate harmonic of the fine-structure constant. Also, $10 \times 11 = 110$. 11 is the diameter of the circle inscribed in the basic square of the Cosmological Circle. The fine-structure constant is $\alpha \simeq (73/110)^{12}$ and $110 - 37 = 73$. $\alpha = e^2/\hbar c$, in cgs units. $\hbar = h/2\pi = h$-bar, the reduced Planck's constant, c the speed of light constant, and e is elementary charge [9].

$$\alpha^{-1} \cong ((111\pi^2/8) + (2/21)) \cong 137.035\,999\,16, \qquad (7)$$

$3 \times 37 = 111 \simeq 180/\phi \simeq s^4$ with ϕ the golden ratio and s the silver constant. $3 \times 7 = 21$. 111 is significant in the Cabalah as the mind-body-spirit version of the Pythagorean unity-duality-harmony. 8 is the harmonic mean between 6 and 12, $2 \times 6 \times 12/(6+12)$.

$$\alpha^{-1} \cong 222.333/\beta \cong 137.035\,999\,09, \qquad (8)$$

where $\beta \cong (\phi + ((4-s)/(60s - 24))) \cong 1.6\,22\,44\,22\,88\,66 \simeq s/2$ showing the golden ratio and silver constant. $6 \times 37 = 222$, $\alpha^{-1} \approx 222/\phi$, $1/(3 + 1/333) = 0.333$, and $9 \times 37 = 333$. Next, the universal parabolic constant is $P \cong 2.295\,587\,149\,392 \simeq \sqrt[3]{12} \simeq 6/\phi^2 \simeq s/\sqrt{2}$. The parabolic constant P is related to the mid-radius of the dodecahedron as $P \simeq r_m + 1$.

$$\alpha^{-1} \cong ((12 - 10/3P)/7)^{12} \cong 137.035\,999\,111, \qquad (9)$$

$$NIST\,2002\,[3]\,\alpha^{-1} = 137.035\,999\,11\,(46).$$

$P = \sqrt{2} + \ln(1 + \sqrt{2}) \simeq \ln(\pi^2) \simeq \ln(7\sqrt{2}) \simeq$ diameter of the heptagon. $P \simeq \csc(\pi/7)$. $111 + 729 = 7 \times 10 \times 12 = 21 \times 40 = 840$. $7 + 77 = 84$. The dodecahedron outer radius, $r_u = (\sqrt{15} + \sqrt{3})/4 = \sqrt{3}\phi/2 \simeq 7/5 \simeq \pi_Q/P$. Silver number $\mathcal{P} = \sqrt[3]{1 + \sqrt[3]{1 + \sqrt[3]{1\ldots}}} \cong 1.324\,717\,957\,244 \simeq (1 + s/10)$, which gives $\alpha^{-1} \cong (47s/(84\mathcal{P} - 10))^{12} \cong 137.035\,999\,09$.

The silver number or plastic constant, $\mathcal{P}$ is the limiting ratio of successive terms in the Padovan sequence and the Perrin sequence. The fine-structure constant, determining the strength of the electromagnetic interaction, is related to the least-action principle as a product of energy and time, with origins in the foundations of variational calculus and the cycloid curve of the brachistochrone problem; which in turn is related to the regular heptagon and squared circle. The sum of $1 + \sqrt{2} = \delta_s$, named the silver ratio, is found in the Sacred Cut pattern with the 3, 4, 5 triangle proportion and golden ratio ϕ [10].


Sacred Cut in the Square

According to Tons Brunés the "Sacred Cut represents the main factor in ancient geometry." [10]. Of the three silver proportions, the silver ratio δ_s, $\cosh(\ln \delta_s) = \sqrt{2}$, notice that the equation representing $\sqrt{2} = x + 1/x$ has complex roots $(1 \pm i)/\sqrt{2}$; also, $i = \phi i + 1/\phi i$, and $0 = i + 1/i$ [3, 10]. $(i + 1/i)^i \simeq \exp(i(2 + \pi)/14)$, $(s + 1/s)^s \simeq 80 - 14\mathcal{P}$, and $(\pi + 1/\pi)^\pi \simeq 56 - 5\mathcal{P} \approx 7^2$. The reciprocal $7^{-2} = 0.02\,04\,08\,16\,32\,...$, and 2, 4, 8 form a side of Plato's Lambda or Heptachord; with 1 at the top and 3, 9, 27 on the other side.

4. Harmonic of the Speed of Light Constant

The Triad has a special beauty and fairness beyond all numbers, primarily because it is the very first to make actual the potentialities of the Monad – oddness, perfection, proportionality, unification, limit. – Iamblichus
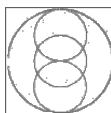
Foundation Stone Geometry

The Pythagorean Triad was the progression of unity, duality, and harmony. The speed of light harmonic is $c \simeq \sqrt{3\sqrt{4\sqrt{5}}}$, $c^{1/12} \simeq \phi\pi \simeq 5Q$, $1 + \phi^2 \simeq \sqrt{5\sqrt{4\sqrt{3}}}$, and $r_m \simeq \ln(1 + \phi^2)$. From a combination of the Heptachord numbers 3, 9, 27, and 2, 4, 8; with $40 + 27 = 67$:

$$c \cong (30(9s + 8)/(67s + 2))^{12} \cong 299\,792\,457.4, \tag{10}$$

$$NIST\ 2006\ [9]\ \ c = 299\,792\,458\,ms^{-1}.$$

From the Foundation Stone and All-Seeing Eye $\sqrt{77^2 - (77/2)^2} \simeq 67$, $67s \simeq 222/Q$ [3]. $30 + 37 = 67$, $67 + 70 = 137$. Harmonic $c \cong (7/729\alpha)^4 - (2/7.29)^{12}$ and $7.29 \simeq 9s/4$. In Michael Schneider's view, "These number correspondences are not coincidental but recur wherever the cosmos is perceived as an integrated whole." This is the Pythagorean paradigm of harmonic proportion which also includes a mysterious qualitative meaning.

5. Geometry and the Gravitational Constant

The importance of the quaternary obtained by addition is great in music because all the harmonies are found in it. – Theon of Smyrna

The heptagonal seven and the silver constant s are found in the harmonic of Newton's universal gravitation constant of the equation for gravitational force: $F = GMm/r^2$. The harmonic of G is found in the All-Seeing Eye construction just mentioned above [3].

"Lapis Philosophicus" inner circle reproduction

by Sir Isaac Newton from "The Crowning of Nature"

$$G \cong (7 + s/70)^{-12} \cong 6.674\,26 \times 10^{-11}, \tag{11}$$

$$NIST\,2006\,[9]\,G = 6.674\,28\,(67) \times 10^{-11}\,m^3 kg^{-1} s^{-2}.$$

$180 - 110 = 110 - 40 = 137 - 67 = 77 - 7 = 70$. The harmonic equation, $G \simeq \hbar c^3/\pi^2 C^4$, shows C grid light harmonic: $C^2 = Q^2 + Q^2 \simeq m_P c/\pi$, where m_P is Planck mass [3]. The connections of the regular heptagon with the foundations of the least action principle, the universal parabolic constant, and the dodecahedral harmonic proportions all provide another way of understanding the geometric origin of Feynman's path integral approach to gravitation that allows both a discrete and a continuous space-time formulation.

6. Planck's Constant and Sidharth's Equation

The Dyad is also an element in the composition of things, an element which is opposed to the Monad, and for this reason is perpetually subordinate to the Monad as matter is to form. – Anatolius as recorded by Iamblichus

An approximate harmonic of Planck's constant from the classical Foundation Stone is $h \simeq 729/110$ and $h^2 \simeq 44$, perimeter of the basic earth square from the Cosmological Circle. The Pythagorean Tetrad is the *analogia* accounting for the formation of matter.

$$h \cong (s^3(17 + (40\sqrt{70}\pi^2)^{-1}))^{-12} \cong 6.626\,068\,96 \times 10^{-34}, \tag{12}$$

$$NIST\,2006\,[9]\,h = 6.626\,068\,96\,(33) \times 10^{-34}\,Js.$$

$17 + 60 = 40 + 37 = 77$, $17 + 67 = 84$. $h/2\pi \simeq s^2/10$, $\alpha^{-1} \simeq 4s^3$. As Paul Valéry states, "The introduction of quanta, or indivisible units of energy, gave integers a role of utmost importance in a cosmos that until then had been dominated by the continuous."

Malcolm Mac Gregor found elementary particle masses in quantum chromodynamics are α quantized beginning from the lowest mass levels formed by the triad of electron, pion, and muon. The fine-structure constant "...generates the interactions between electrons and all of the other massive elementary particles." [11]. Pions are the lightest mesons,

$$m_\pi \simeq (7^2/10s)^{12}, \tag{13}$$

the harmonic of the pion, pi meson π^+ mass, m_π. With quintessence, pion harmonic $m_\pi \simeq 137Q$ and $\alpha^{-1} \approx (s^2/7)^{12}$. Burra Sidharth's formula with different integral values of l and n reproduces the approximate masses for all known elementary particles [12],

$$m_P \simeq l\,(n + 1/2)\,m_\pi, \tag{14}$$

where m_P is the mass of particle P. The existence of "approximation" in Sidharth's equation and harmonic proportion is significant to the formation of matter. Mendel Sachs says, "What I have found in my research program is that the mathematical structure of quantum mechanics (the Hilbert function space) emerges as a linear approximation for a generally covariant field theory of the inertia of matter. ...a theory based on continuity and holism in the laws of nature." [13]. What appears to be discrete is in reality [14] a dynamical approximation from symmetrical reflective properties and the self-referential nature of quintessential light. The Greek Omphalos is the Solar Foundation Stone, the center of the world and the Seat of Apollo at Delphi; the Monad, first causal seed of the universe. Apollo personified the harmony of the "golden mean." To the Foundation Stone four more circles are added to give seven circles, and the Seed of Life pattern.
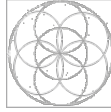
Seed of Life in the Foundation Stone

From ancient geometry, $7 \times 9 \times 9 = 567$ is the archetypal solar mass harmonic [3]. $567/729 = 7/9$. With the $3 \times 6 \times 9 = 162$, the life generating phi harmonic, giving $162 + 567 = 729$, the tri-circle perimeter of the Foundation Stone. 729 was the "solar number" as it was associated with the days and nights of the solar year. $9 \times 37 = 333$, and $567/333$ is proportional to the archetypal earth mass harmonic [3]. In modern astrophysics the black hole entropy and the information entropy involve the Boltzmann constant, a proportionality between the temperature and energy; with the harmonic of $k_B \cong (35s\sqrt{3\sqrt{\phi_f}/11})^{-12} \simeq (35P)^{-12}$, ϕ_f is the reciprocal Fibonacci constant [3]. Also, $k_B \cong (78.8\,44\,33\,55\,Q)^{-12} \cong ((70 + 8\sqrt{11}/3)Q)^{-12}$. In the Zero Zone System of units, conversion operator Q, $Q[K] \cong ((111\mu - 20)/6s)^{12}$; thermodynamic temperature K harmonic (k_B/h), and μ is Soldner's constant (root of the logarithmic integral, $\text{li}(x) = 0$). $Q[m_e] \cong (4(31\mathcal{P} - e)/s)^{12}$ where m_e is electron mass. $Q[E] \cong ((84 + (37 + \ln 2/e)^{-2})/s)^{12}$ where $E = mc^2$ is converted into dimensionless harmonics [15]. See discussion of Eq. (9) for 111, silver number $\mathcal{P}$, and 84 again. For helpful discussions of the Zero Zone System of units read Timothy Desmond and other essays in the *Journal of Futures Studies* [16].

7. Conclusion: The Quintessential Harmony

> *The idea of order is intimately connected with Limit (peras), the opposite of which is the Unlimited (apeiron), and these are the two most basic, and hence most universal, principles of Pythagorean cosmology.* – David Fideler [17]

Werner Heisenberg is often quoted in support of Platonic views, "Perhaps we should approach Plato and indeed state that in the beginning was the symmetry." [18]; "... In the philosophy of Democritus the atoms are eternal and indestructible units of matter, they can never be transformed into each other. With regard to this question modern physics takes a definite stand against the materialism of Democritus and for Plato and the Pythagoreans. ... I think that modern physics has definitely decided in favor of Plato. In fact the smallest units of matter are not physical objects in the ordinary sense; they are forms, ideas which can be expressed unambiguously only in mathematical language ...." Thus, "Unity and complementarity constitute reality." Paul Valéry contemplates, "The universe is built on a plan the profound symmetry of which is somehow present in the inner structure of our intellect." The archetypal geometry from the seed of life to the flower of life and the fruit of life generate the Platonic solids and classical elements. Plato advises that, "Their combinations with themselves and with each other give rise to endless complexities which anyone who is to give a likely account of reality must survey."

Da Vinci′s drawing of Platonic Solids

Nature's quintessence is a fundamental harmony of wholeness [19]. For Keith Critchlow harmonic proportion is "... key to arriving at a transcendental unity from the polarity of existence." [20]. Sri Aurobindo Ghose considered, "This world is a vast unbroken totality, a deep solidarity joins its contrary powers." The quintessential light of aetheric nature is also the *Lightness of Being.* Aristotle again in *Metaphysics*, "Then comes the most difficult of all questions, whether unity or being, as the Pythagoreans and Plato said, is not a particular something at all, but is the very being of any being." Finally, philosopher Franklin Merrell-Wolff concludes, "With Plato we have the clear emergence of the conception of ideal elements as ontologically significant and determinant." [21].

Acknowledgments

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