Watching a falling apple atom by atom

A classical unification scheme

$$4GmM = F_{Planck} \times r_{sm}\, r_{sM}$$

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Prelude: The material presented in this essay is a part of the author's unfinished work for the past few years. The reasons for getting involved with this project are beyond the scope of this essay. Suffice to say that it all started with a few questions with regard to the nature of space, time, and gravity, where I felt the need for a little education on the subject. Lacking the mathematical tools, not having much background in physics, and so much technical jargon in the literature forced me to find a way to get around the difficulties. Since then I have been busy crunching the numbers on a hand calculator and putting down the interesting and unusual results in several notebooks. The major part of the work revolved around the nature of gravity in classical terms, the completeness of the most familiar equation in gravitation, $F = GmM/r^2$, whether this equation is a stand-alone equation, part of more complex equation, or both; and if the Universal Gravitational Constant (G) has a mathematical origin. A couple of lucky assumptions lead to the results that will follow. I had been planning to share the results with the experts in the field in a proper venue once finished. Not a long time ago, I stumbled upon the site for Foundational Question Institute (FQXI) and found the subject of the essay contest surprisingly very relevant to my work. Therefore, I decided to submit it for the contest, even though it may not look like an essay.

"No great discovery was ever made without a bold guess"

Isaac Newton

Abstract: Close examination of a nondimensionalized Newton's equation for universal gravitation written for the Bohr hydrogen atom provided the basis for the assumption that $F = GmM/r^2$ may be a reduced form of a more complex equation. This assumption lead eventually to the discovery (invention?) of a previously unknown, fundamental equation in classical gravitation. The fundamental nature of the equation is demonstrated by:

- Deriving the equation for Newton's law of universal gravitation, revealing the mathematical origin of the universal gravitational constant G, and the peculiar dimensionality of this constant.
- Deriving the exact solution (Schwarzchild solution) of Einstein's General Relativity equations.
- Establishing a relationship between nuclear and gravitational potential energies, showing that nuclear and gravitational potential forces are the same, and are differentiated only by the virtue of their distances.
- Deducing Einstein's mass-energy equivalence equation.
- Proving that the nuclear and the Planck forces are the same and should be considered a basic universal constant, rather than a derived quantity.

- Deriving some of the Planck units using $\boldsymbol{F_{planck}}$, $\boldsymbol{\hbar}$, and $\boldsymbol{c}$ without $\boldsymbol{G}$.
- Demonstrating that the gravitational potential energy of the solar system planets may be proportional to the sum of the potential energy of their constituent atoms.

1. Introduction

It is possible to reduce, extend, and/or modify the Newton's equation for gravitation in several ways, thus creating an array of new equations as exemplified by the following equations, where $\boldsymbol{Q}$ stands for a quantity.

$$\boldsymbol{mM = Q \times r^2} \qquad Q = F/G \qquad Q \text{ dimensions: } kg^2m^{-2} \qquad (1.1)$$

$$\boldsymbol{mM \times Q = mv^2 \times r^2} \qquad Q = G \times r \qquad Q \text{ dimensions: } kg^{-1}m^4s^{-2} \qquad (1.2)$$

$$\boldsymbol{mM \times Q = mv^4 \times r^2} \qquad Q = G \times r \times v^2 \qquad Q \text{ dimensions: } kg^{-1}m^6s^{-4} \qquad (1.3)$$

$$\boldsymbol{mM \times Q = (mv^2)^2 \times r^2} \qquad Q = G \times r \times v^2 \times m \qquad Q \text{ dimensions: } m^6s^{-4} \qquad (1.4)$$

$$\boldsymbol{mM \times Q = mv^4 \times r/G} \qquad Q = v^2 \qquad Q \text{ dimensions: } m^2s^{-2} \qquad (1.5)$$

(1. 1) is known as the nondimensionalized Newton's gravitational equation. Multiplying both sides of the equation (1.5) by $\boldsymbol{G}$ followed by dividing to $\boldsymbol{m}$ and $\boldsymbol{r}$ one ends up with $\boldsymbol{GM\alpha_0 = v^4}$ which is Milgrom's modified version in MOND for predicting the galaxy rotation curves without invoking the dark matter. The rest were created by the author as a part of his self-instigated project. The question is whether these equations, while mathematically legitimate, could be considered legitimate gravitational equations or not, and if legitimate then to what extent one can stretch this equation. Moreover, is there any usefulness in doing so or the whole operations will be mere redundancy?

2. A clue from Bohr hydrogen atom

The truth is that (1.4), an extended Newton equation for gravitation, was originally arrived at through a totally different approach. It was arrived at as a consequence of the author's attempt to write a nondimensionalized gravitational equation for the Bohr hydrogen atom. I entertained the possibility that the resulting number (underlined) obtained by division of the product of the masses of proton and electron by the Bohr radius squared could be a squared number itself (2.1). Such an assumption puts the second root of the number (underlined) in the vicinity of kinetic energy or the charge of the electron.

$$\boldsymbol{m_pm_e = Q \times r^2 = 1.5\times10^{-57} = \underline{5.3\times10^{-37}} \times (5.3\times10^{-11})^2 = \underline{(7.3\times10^{-19})^2} \times (5.3\times10^{-11})^2} \qquad (2.1)$$

Knowing that the force between proton and electron, obtained using the equation $\boldsymbol{F = kq^2/r^2}$ should be around $\boldsymbol{8.2\times10^{-8}}$ Newton, and the fact that for a circular motion $\boldsymbol{U = F \times r = 2E_k}$, one could rewrite the equation (2.1) such that the mentioned conditions for the system are satisfied (2.2).

$$\boldsymbol{m_pm_e \times 36 = (4.4\times10^{-18})^2 \times (5.3\times10^{-11})^2} \qquad (2.2)$$

This equation can take the general form as (2.3), which itself is a special case of (1.4).

$$m_p m_e \times Q = (m_e v^2)^2 \times r^2 \qquad (2.3)$$

$$mM \times Q = (mv^2)^2 \times r^2 \qquad (1.4)$$

It is noteworthy that equation (2.3) can be written in a different form using the Planck equation as shown in equation (2.4).

$$m_p m_e \times Q = (hv)^2 \times r^2 \qquad (2.4)$$

3. Deriving the equation for Newton's law of universal gravitation and Schwarzchild radius

Now if we go back and apply this to any two body cosmic system, we have

$$mM \times Q = (mv^2)^2 \times r^2 = U^2 \times r^2 = F^2 \times r^4 \qquad (3.1)$$

Using numerical values, it is easy to see that the second root of the equation is equal to ***GmM*** by magnitude and $\mathbf{kgm^3s^{-2}}$ dimensionally. Extracting ***mM*** (kg^2) from the second root of the equation renders the newton's equation for gravitation.

$$(mM \times Q)^{1/2} = mv^2 \times r = U \times r = F \times r^2 = GmM \qquad (3.2)$$

Considering the constancy of the speed of light in vacuum, ***Q*** which has the m^6s^{-4} dimensions can be divided by c^4 resulting in the following equation.

$$mM \times c^4 \times d^2 = U^2 \times r^2 = (mv^2)^2 \times r^2 \qquad (3.3)$$

A careful examination of d^2 shows that it is ¼ of the product of the Schwarzchild radii of m and M. I will use the r_{sm} and r_{sM} to differentiate the radii obtained in this article from that of Schwarzchild radius. Thus,

$$mM \times c^4 \times r_{sm} r_{sM} = (mv^2)^2 \times r^2 \qquad (3.4)$$

Dividing both sides of the equation by m^2 (3.4) followed by taking the second root (3.5), we have the Schwarzchild radius equation divided by two.

$$c^4 \times r_{sM}^2 = v^4 \times r^2 = G^2M^2 \qquad (3.5)$$

$$(c^4 \times r_{sM}^2)^{1/2} = (v^4 \times r^2)^{1/2} = (G^2M^2)^{1/2} = c^2 \times r_{sM} = v^2 \times r = GM \qquad (3.6)$$

4. Other implications of the equation (3.3)

Had we not known the Einstein's mass- energy equivalence equation ($E = mc^2$) from the Special Theory of relativity (STR), we would have arrived at the conclusion that mMc^4, and subsequently mc^2 and Mc^2 must represent a new type of potential energy, simply by juxtaposing the two sides of the equation (3.3) or (3.4). Fortunately, we do not have to worry about the naming of this potential energy now. It is known to us as nuclear potential energy. Hence, we can write the equation (3.4) as:

$$E^2_{nuclear} \times r_{sm}\, r_{sM} = E^2_{gravitational} \times r^2 \qquad (4.1)$$

Since $U = F \times r$, accordingly, for the nuclear potential energy

$$E_{nuclear(m)} = F_{nuclear(m)} \times r_{sm} \qquad (4.2)$$

It turns out that nuclear force is a constant and is equal to Planck force, therefore,

$$E_{nuclear} / r_{sm} = F_{nuclear} = F_{planck} \qquad (4.3)$$

Subsequently,

$$F_{Planck} \times r_{sm}\, r_{sM} = F_{gravitation} \times r^2_{gravitation} \qquad (4.4)$$

A crucial piece of information is obtained by dividing the nuclear force or the Planck force by speed of light in vacuum squared (c^2) or just the speed of light in vacuum. In doing so, we have established a relationship between mass/time and mass/distance, exactly, like that of distance/time in the speed of light in vacuum. Hence, closing the mass/distance/time cycle. This piece of information also provides a bridge between cosmos and the Planck scale world. Since,

$$F_{nuclear}/c^2 = kgms^{-2}/m^2s^{-2} = kgm^{-1} = 1.35x10^{27} \qquad (4.5)$$

$$F_{nuclear}/c = kgms^{-2}/ms^{-1} = kgs^{-1} = 4x10^{35} \qquad (4.6)$$

The same quantities are obtained by dividing the Planck mass to Planck length and Planck time.

5. Deriving some of the Planck units using F_{planck}, $\hbar$, and c without G.

Planck mass, Planck length and Planck time can be calculated using F_p, $\hbar$ and c only, as shown

$$F_p \times \hbar = kg\ m\ s^{-2} \times kg\ m^2\ s^{-1} = kg^2\ m^3\ s^{-3} \qquad (5.1)$$

$$(kg^2\ m^3\ s^{-3} / c^3)^{1/2} = m_P \qquad (5.2)$$

$$\hbar / F_P = m \times t \qquad (5.3)$$

$$(m \times t \times c)^{1/2} = l_P \qquad (5.4)$$

$$l_P / c = t_P \qquad (5.5)$$

Obviously, by having these basic units one can calculate other Planck units as well.

6. Different aspects of the equation for the Bohr hydrogen atom

It should be noted that the product of the masses of proton and electron can be broken down in a different way than the equation (2.1), as depicted in the following two equations.

$$m_p m_e = 1.523x10^{-57} = (1.602x10^{-19})^2 \; x \; (2.436x10^{-10})^2 \tag{6.1}$$

$$m_p m_e = 1.523x10^{-57} = (3.204x10^{-19})^2 \; x \; (1.218x10^{-10})^2 \tag{6.2}$$

Whether we take $(1.602x10^{-19})^2$ to represent charge or energy, we have to accept that there are some hidden dimensions on the mass side of the equation, whose magnitude is unity and has the dimensionality of m^6s^{-4} in case we consider this number to represent energy. Notice that r here is very close to Van der Waals diameter (6.1) and the radius (6.2) of hydrogen molecule. Interestingly, if the quantity 36 (35.1 more accurately) in equation (2.2) is incorporated into Bohr radius squared the radius would be very close to permittivity of free space by magnitude (6.3).

$$m_p m_e = (4.37x10^{-18})^2 \; x \; (8.93 \; x \; 10^{-12})^2 \tag{6.3}$$

an interesting aspect of equation (6.1) is that if we take $1.602x10^{-19}$ as charge and replace it with the Planck charge then Q will be equal to the fine structure constant (6.4).

$$m_p m_e \; x \; 137 = 1.523x10^{-57} \; x \; 137 = (1.875x10^{-18})^2 \; x \; (2.436x10^{-10})^2 \tag{6.4}$$

Last not the least, if the velocity of electron in equation (2.3) is replaced by the speed of light in vacuum, the calculated r is equal to Planck mass squared (6.5).

$$1.52x10^{-57} = (\, m_e \; x \; c^2 \,)^2 \; x \; (\, 4.76x10^{-16} \,)^2 = (\, m_e \; x \; c^2 \,)^2 \; x \; (\, 2.18x10^{-8} \,)^4 \tag{6.5}$$

7. Equivalency of the Planck equation and Newton's equation for universal gravitation

A dimensional analysis of the Planck equation written in the following form

$$E \; x \; \lambda = hc \tag{6.6}$$

shows that hc has the same dimensions as the GmM (kgm^3s^{-2}) and $F \; x \; r^2$ in Newton's equation for universal gravitation, therefore, dividing hc by mM, F and r^2 should give G, r^2 and F respectively. Thus,

$$hc/m_{Pl}^2 = 3.16x10^{-26}/(\, 2.18x10^{-8} \,)^2 = 6.65x10^{-11} \tag{6.7}$$

$$hc/F_{Pl} = 3.16x10^{-26}/1.20x10^{44} = (\, 1.60x10^{-35} \,)^2 \tag{6.8}$$

$$hc/l_{Pl}^2 = 3.16x10^{-26}/(\, 1.60x10^{-35} \,)^2 = 1.20x10^{44} \tag{6.9}$$

7. Gravity of the planets of the solar system on the atomic scale

The average gravitational potential energy of the individual constituent atoms of each planet could be calculated by dividing the gravitational potential energy of that planet divided by the number of atoms in the planet, or simply, dividing the velocity squared divided by the Avogadro number. These numbers for the planets Mercury, Venus, Earth and Mars are shown by the equations (7.1), (7.2), (7.3) and (7.4) respectively.

$$\mathbf{(\ 47872.0\ )^2 / 6.0x10^{26} = 3.8x10^{-18}} \quad (7.1)$$

$$\mathbf{(\ 35021.4\ )^2 / 6.0x10^{26} = 2.0x10^{-18}} \quad (7.2)$$

$$\mathbf{(\ 29785.9\ )^2 / 6.0x10^{26} = 1.5x10^{-18}} \quad (7.3)$$

$$\mathbf{(\ 24130.9\ )^2 / 6.0x10^{26} = 9.7x10^{-19}} \quad (7.4)$$

As mentioned before, equations (2.2) is a specific case of general equation (2.3), assuming that $\boldsymbol{Q = n^2}$, with $\boldsymbol{n}$ being an integer (7.5), one can calculate the energies and subsequently the velocities associated with those energies for n = 1-4.

$$m_p m_e \text{ x } 36 = (4.4x10^{-18})^2 \text{ x } (5.3x10^{-11})^2 \quad (2.2)$$

$$m_p m_e \text{ x } Q = (m_e v^2)^2 \text{ x } r^2 \quad (2.3)$$

$$m_p m_e \text{ x } n^2 = (4.4x10^{-18})^2 \text{ x } (5.3x10^{-11})^2 \quad (7.5)$$

n = 1	$E = 4.4x10^{-18}$	***v = 51475***	***vs. 47872 for Mercury***	*(7.6)*
n = 2	$E = 2.2x10^{-18}$	***v = 36398***	***vs. 35021 for Venus***	*(7.7)*
n = 3	$E = 1.5x10^{-18}$	***v = 29719***	***vs. 29786 for Earth***	*(7.8)*
n = 4	$E = 1.1x10^{-18}$	***v = 25737***	***vs. 24131 for Mars***	*(7.9)*

The proximity of the calculated velocities with the observed ones seems consistent with the assumptions made.

8. Conclusion

Making a simple assumption on the completeness of one of the foundational equations in physics provided a platform for unifying the gravitational force with the other fundamental forces in Nature. This was done using a few simple equations in classical physics. Additionally, I have presented circumstantial evidence that the gravitational potential energy of the planets in the Solar system may equal the sum of the potential energy of the particles making up the planets.