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Lorentz symmetry broken

As the concept of symmetry in physics has developed by full swing in the twentieth century, the extension of the concept of continuous symmetry from "global" symmetries to "local" symmetries has been at its heart. The principle of local Lorentz invariance is shared by general relativity and particle physics, which in contemporary sense enwrapping the theory of special relativity, which has been viewed as global[1] [2].

A new evaluation is proposed to manifest that in specific cases Lorentz violation occurs related to special relativity for observers with low velocity in about inertial frames that perform aligned and synchronized observations to frames approaching relativistic velocities. These observers perceive Galilean transformation rather than Lorentz transformation, which disagrees with special relativity and Lorentz symmetry that basically state that the laws of physics look identical to any (local) inertial observer. In other words the outcome of physical experiments observed by different observers contradicts Lorentz symmetry, and there might exist an uncertainty about prediction of events depending on how observations are carried out. Additionally it is concluded that clocks in such frames can be synchronized as no length contraction and time dilation takes place in the mentioned frames which also controvert special relativity.

Generally gauge transformation approach is exploited which incorporates with the principal of general relativity with reference to general coordinate transformations in the essence of invariant under continuous reparameterizations of space-time in conjunction with the topological arrangement of events through space time and as well as the additional assumption of general relativity that each infinitesimal small region of space approaches flatness with metrical properties of special relativity. This stand point could also be viewed as incorporation of Lorentz transformation and gauge transformation on the same basis [3]. This means that, by iterating infinitesimal symmetry transformations for a finite transformation, in combination with change of basis and gauge transformation, will take us to the Galilean foremost.

Introduction

This paradox is to be categorized with other Lorentz violations that would contradict measurements of quantities such as geometry, energy and momentum among different frames, inconsistence with SR and Lorentz symmetry prediction.

In mainstream physics Lorentz violation refers to theories which are approximately relativistic when it comes to experiment (and there are quite a number of such experimental tests)[4][5] but yet contain tiny or hidden Lorentz violating corrections.

Symmetry and gauge transformation

Introduction

It is essential to discuss the subjects that are relevant to this paradox, to be able to comprehend it better. Additional supportive parts are included in the Appendix part.

Galilean transformation

Without making any contestation about the classical meaning of Galilean transformation, Galilean transformation in its contemporary sense is the limit for Lorentz transformation where none-relativistic speeds are concerned as length contraction and time dilation will be vanishing.

Inertial system

Many terrestrial experiments adopting approximate symmetry as coordinate frame fixed in the earth is not inertial, due to the earth's rotation but it does not alter the outcome of experiments sufficiently to cause concern.[6] For the reason mentioned a frame with an infinitesimal movement ϵ compared to a stationary frame can with good approximation be considered inertial.

It is often misapprehension that Special Relativity is not able to treat accelerating objects or non-inertial reference frames (see also Thomas precession and Thomas-Wigner precession). Consequently the conclusion drawn is that general relativity is compulsory because of special relativity inability of handling accelerating frames, which is not factual. Special relativity just handles accelerating frames in a different manner but it is still able of handling that. [7] [8]

A number of authors have discussed rotational disks [9] [10] [11] in combination with Relativity inclusive idea of none-Euclidian nature of such systems. These discussions could also be extended on experiments or paradoxes of rotating disk inter alia Sagnac effect or Ehrenfest paradox etc. However, it is known that, if the light propagates in inertial frames, it won't be any anisotropic effects.

The thought experiment overview

Experiment arrangement

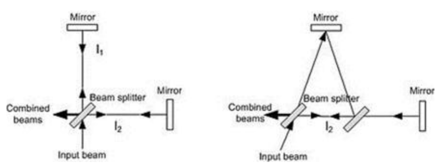

Figure 1. *Michelson Morley Experiment: interferometer of equal length arms l_1 and l_2 which are perpendicular: rest frame on left-hand side; moving frame on right-hand side; given any velocity v, the light paths would be unequal for these two frames.*

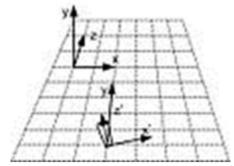

Figure 2. *illustrates the plane that hosting (x,z), (x',z') and an observer seeing the experiment along z'.*

In this experiment, the arrangement of Michelson Morley experiment will be in such way that the experiment is observed by two different observers (refer to *figure 2*). See also: Michelson–Morley experiment

Observer *O* (inertial frame) is stationary with regards to the laboratory that moves with velocity v_x along *x* axis which performs the experiment.

Furthermore the *(x,z)* coordinates are in same plane as *(x',z')* and *y* axis is parallel with *y'* axis. Additionally *y* and *y'* are in same plane and are parallel during the experiment in such way that this plane has a fixed point in *O'* frame center point (i.e. point *C* or spatial origin and is centered during the experiment), which will rotate as *O* frame progresses in *x* direction.

In other words, observer *O'* has a circular motion compared to *O* and is viewing the experiment by a rotational movement synchronized and aligned with the center point of the experiment, additionally the center of its rotation is stationary compared to frame *O*. Let *r(t)* be the distance of *O'* frame to experiment center point (i.e. beam splitter), in addition *r(t)* lies in the *(x,z)* plane.

Also consider the line that crossing the beam splitter i.e. l_2 is overlies with *x* axis (*figure 1*). Furthermore consider the l_1 (*figure 1*) overlies with *y* coordinate which lies also in *y* and *y'* plane as mentioned above. As regards observer *O'* as the experiment proceeds; as rotation and boost take place in mentioned coordinate systems which will then be transformed into *O'* coordinate system. The observer *O'* will follow the experiment as it rotates in such way that its line of site to the experiment's center point is synchronized with the velocity (v_x) of the experiment frame *O*, i.e. rotates with constant angular velocity ω.

Lorentz Transformation

Let rotation matrix (R) in *y* direction be denoted by:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & 0 & \sin(\theta) \\ 0 & 0 & 1 & 0 \\ 0 & -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \qquad (1\text{-}1)$$

R is orthogonal and its inverse $R^{-1}$is equal to its transpose R^T. Furthermore Lorentz boost in x direction is denoted by:

$$B = \begin{bmatrix} \gamma_x & -\beta_x\gamma_x & 0 & 0 \\ -\beta_x\gamma_x & \gamma_x & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (1\text{-}2)$$

Where $\boldsymbol{\beta_x} = -v_x/c$ (also called relative velocity and interchanging the observers will change its sign) and $\gamma = 1/\sqrt{1 - v^2/c_2}$ is the Lorentz factor (means γ_x wherever no index written). Furthermore B is symmetric and unimodular with $det=1$.

By adopting Polar Decomposition theorem of linear algebra, i.e. product of an orthogonal matrix and a positive-define symmetric matrix, any arbitrary Lorentz transformation could be broken into a unique decomposition, as product of rotation and boost, which generally don't commute, i.e. $\Lambda = R.B = B_1.R$ and $B, B_1, R \in SO[3,1]$ (see also [12] [13]).

The product of R_θ and B_x matrices according to the proper Lorentz group would be:

$$R_\theta B_x = \begin{bmatrix} \gamma_x & -\beta_x\gamma_x & 0 & 0 \\ -\beta_x\gamma_x\cos(\theta) & \gamma_x\cos(\theta) & 0 & \sin(\theta) \\ 0 & 0 & 1 & 0 \\ \beta_x\gamma_x\sin(\theta) & -\gamma_x\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \qquad (1\text{-}3)$$

The product with R^{-1}or R^T will give a symmetrical matrix. As already mentioned the product of two Lorentz transformations is another Lorentz transformation (see Appendix A). In this case the product of the two consecutive Lorentz transformations will also satisfy (see also [14] [15] [16] [17]):

$$\Lambda^T \eta\, \Lambda = \eta \qquad (1\text{-}4)$$

Under Special relativity (resp. more generally) we have:

$$s^2 = \eta_{\mu\nu}\, x^\mu x^\nu \;\; ; \; s^2 = g_{\mu\nu}\, x^\mu x^\nu \qquad (1\text{-}5)$$

Furthermore as:

$$s^2 = s'^2 \qquad (1\text{-}6)$$

Then one can conclude:

$$s'^2 = -\gamma^2[ct - \beta x]^2 + [x\gamma\cos(\theta) - ct\gamma\beta\cos(\theta) + z\,sin(\theta)]^2 + y^2 + [ct\gamma\beta\sin(\theta) - x\gamma\sin(\theta) + z\,cos(\theta)]^2 \quad (1\text{-}7)$$

In above expression, it is assumed that the frame *O'* is stationary in global manner (see also [16]). Referring to expression (A1-5), we have:

$$\lambda_x = -\gamma_x\,\beta_x \quad (1\text{-}8)$$

If $\beta_x \ll 1$ then the series expansion of expression (1-8) will be:

$$-\gamma_x\,\beta_x = \frac{\frac{v_x}{c}}{\sqrt{1-\frac{(v_x)^2}{c^2}}} \approx \frac{v_x}{c} + O(\frac{v_x}{c})^3 \quad (1\text{-}9)$$

Recalling expression (A1-4), the infinitesimal space time interval will be denoted by:

$$ds'^2 \approx -[cdt - \lambda dx]^2 + [\lambda cdt + dx + \theta dz]^2 + dy^2 + [-\theta dx + dz]^2 \quad (1\text{-}10)$$

Gauge transformation and Lorentz transformation

As already mentioned gauge and Lorentz transformation are put on same basis (also called time-dependent Lorentz transformation [18]) concerning the experiment, indeed both gauge and Lorentz group also corresponding to the finite-dimensional Lie Group. This means, we make local spatiotemporal parametrization of the global symmetry transformation in conjunction with topologically connected point coincidences which then can be transformed linearly while performing change of basis. In essence, the generalization of Lorentz transformation (also denoted as **Ĺ**) unlike SR are not constant, i.e. referring to expression (A1-2), these transformation applies only to infinitesimal displacements as in our case. Having said that, as the experiment proceeds at any given time, the reference frame *O'* which has its origin at the fixed point *C* makes an infinitesimal rotational displacement in the lab system's direction in a synchronized manner and indeed at each point a linear transformation can be performed. Considering a principal frame bundle as there is a well-defined connection between the coordinate bases, then one can glue every four-vector (or labeling every infinitesimal or inertial event by a position vector) during the time evolution and map them to a base until the full iteration is completed. Furthermore it can be assumed that each coordinate basis will be associated with one fiber where a symmetry transformation can be performed as there exists one-to-one-correspondence. The corresponding four-velocity of e.g. the first time instance in frame *O'* is:

$$\tau = \frac{ds'}{c}\,; U' = \frac{dx'}{d\tau} = \frac{\gamma dx'}{t'} \quad (2\text{-}1)$$

Referring to frame *O'* as the coordinate basis makes an infinitesimal rotation, the local time for new basis ($t'_1 = t'_0 + \delta t'$) can be synchronized to the old basis and the transformation to the new basis will be according to:

$$x'_1 = R_y(\theta_{t'})x'_0 \quad (2\text{-}2)$$

This implies that for an infinitesimal rotational displacement, the four velocity of *O'* by assuming *c=1* and $U'_t = dt'/d\tau$ will be:

$$U' = U'_t\,(1, -\omega, 0, \omega) \qquad (2\text{-}3)$$

Furthermore, the lab systems angular velocity corresponds to infinitesimal rotation of the angle $\delta\theta$ which also can be linked by the relative linear velocity $v_x = c\beta_x$.

Referring to *figure 3*, and accepting the fact that flat spacetime is homogeneous and isometric, then it is obvious with enough large distances and infinitesimal angles, we can have simple trigonometric relations for distances *r* and *r'* respectively *AB* and *A'B'* while assuming $r(t) \approx r$ overlaying on *BC* line during the time evolution. The essential fact is, that observer *O* in its proper frame will measure length *AB* while it has a velocity v_x relative to e.g. origin *C*. This also corresponds to the angular velocity $r(t)\omega$ with reference to origin *C* as already mentioned. Same length *AB* will be measured contracted by γ^{-1} by a stationary observer residing at origin *C* (or in its nearest neighbourhood), as a consequence the measured arc *AB'* by this observer will also be contracted by γ^{-1}. Similarly as time measured by the stationary observer residing at origin *C* will be dilated by γ^{-1} then it can concluded that in the frame of the stationary observer *C*, the measurements in combination with Lorentz transformation corresponds to:

$$AB' = r\delta\theta_o = r\omega_o t_0 \;\; ; \;\; r\delta\theta_o = \gamma^{-1} r\delta\theta_c \qquad (2\text{-}4)$$

Since $\gamma^{-1}t_c = t_o$ then it implies that:

$$\omega_o = \omega_c \qquad (2\text{-}5)$$

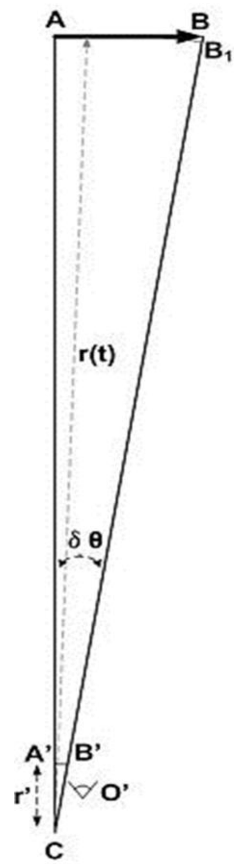

***Figure 3**, Showing observations linking of frames O and O' by angular velocity*

Which as expected means that the two observers agree about their angular velocity with reference to the origin *C*, to be the same. As observer *O'* is in closed neighborhood of origin *C*, it will also measure the same angular velocity.

This means if frame *O'* makes an infinitesimal circular movement e.g. in clockwise direction, this is equal to displacement $\delta\theta r' \approx A'B'$ while frame *O* makes a displacement *AB* during same time. This means also that there is a straight line of sight from origin *C* to point *B* or simply *r(t)* is a straight line. The obvious conclusion is if both *O'* and *O* in the limit approach same angular velocity, then there is a straight line of sight e.g. *r(t)* which coupling the two frames at each particular time, otherwise there will be existing anisotropic effects.

Since the relative angular velocity of the two frames is $\Omega = \omega_o - \omega_o$ then for the instantaneous linear velocity of the lab *v*, the condition below will be satisfied:

$$v_{\Omega} = r_t.\Omega \rightarrow 0 \qquad (2\text{-}6)$$

Similarly, this must also satisfy the condition that *O'* must reside on *r(t)* for isotropic properties that was already discussed. At this point, It is obvious that with regards to local parametrization, as each trajectory from *O* to *O'* is straight and passing through origin *C* during the experiment, then we can make linear symmetry transformations at each particular time instance for the entire iterations over the finite time interval while the observer *O'* also changes its coordinate basis by infinitesimal angle θ around y axis at each time instance.

Referring to expression (A1-5) then we'll obtain the following expression:

$$\epsilon = \begin{bmatrix} 0 & \lambda_{\Omega} & 0 & 0 \\ \lambda_{\Omega} & 0 & 0 & \theta \\ 0 & 0 & 0 & 0 \\ 0 & -\theta & 0 & 0 \end{bmatrix} \qquad (2\text{-}7)$$

Where we have:

$$\lambda_{\Omega} = -\gamma_{\Omega}.\beta_{\Omega} \qquad (2\text{-}8)$$

But as regards the metric interval by considering expressions (A1-4), (2-7) and (2-8), expression (1-10) is reduced to:

$$ds'^2 = -[cdt]^2 + [dx + \theta dz]^2 + dy^2 + [-\theta dx + dz]^2 \qquad (2\text{-}9)$$

This also means the Lorentz transformation for full iteration by considering expression (A1-4), (2-9) will be:

$$x'^{v} = \sum_{v} \Lambda^{v}_{\mu}\, x^{\mu} \qquad (2\text{-}10)$$

Conclusion

As the metric interval is presented in its general form, by suppressing any dimension(s), the evolution can be calculated in particular direction. Comparing expression (2-9) with (1-10), we can easily and not surprisingly conclude that observations in global and local manner will be different with regards to our particular case.

$$s'_{global} \neq s'_{local} \qquad (2\text{-}11)$$

As we consider the inertial frame *O* and applying a gauge transformation, the physical content should be the same which is also in accordance with gauge principal. Furthermore on the whole, with enough large distances observer *O'* makes a negligible displacement which is closed to a stationary and non-rotating observer e.g. *O''*. Now it is clear that we have a Galilean transformation while making a gauge transformation as the lab proceeds in an inertial manner and as already mentioned the none-linear part of the gauge transformation doesn't impact the physical reality as spacetime is homogeneous and isotropic as expected. In another word if we consider *figure 1*, left hand frame, then firstly *O'* will see no Lorentz contraction or time dilation secondly it agrees that speed of light is the same as in *O* frame as if the observation was analogous to the observation made by frame *O* itself. If we consider the other inertial frame *O"* with low or zero velocity, we can agree that *O'* and *O"* frames with low speed will observe the fast-moving frame *O* differently for instance *O'* can agree to Galilean transformation while *O"* agrees to Lorentz contraction:

This disagreement observed by *O'* and *O"*, is contradictory to special relativity and Lorentz symmetry. Imagine if an actual annihilation take place in frame *O*, then *O'* and *O"* will measure different energies for created photons as the wave lengths would be measured differently among mentioned frames. It seems that it governs an uncertainty about predictions of events among various frames.

Clock Synchronization

The proper time which is invariant is defined by:

$$d\tau'^2 = d\tau^2 = dt^2 - dx_i^2/c^2 \qquad (3\text{-}1)$$

As regards Lorentz transformation in combination with *figure 1* right hand side, we would have the time difference for light traveling fort and back along the rods l_1 and l_2 as:

$$\Delta t_\perp = \frac{2l_1}{c\sqrt{1-v^2/c^2}} \qquad (3\text{-}2)$$

Respectively we have:

$$\Delta t_\parallel = \frac{2l_2}{c(1-\frac{v^2}{c^2})} \qquad (3\text{-}3)$$

For the stationary frame (*figure 1*, left hand side) the time difference along l_1 reduces to:

$$\Delta t_\perp = \frac{2l_1}{c} \qquad (3\text{-}4)$$

and consequently:

$$\Delta t_\perp = \Delta t_\parallel \qquad (3\text{-}5)$$

As regards invariant metric interval we have $ds'^2 = ds^2$ which is specially consistent with our case while applying gauge transformation. By comparing expression (A1-1) and (2-9) one can conclude that both time *t* and *t'* as well as *y* and *y'* are intact and equal as expected, while *x* will depend on angle θ in *O'* frame, and as already discussed no length contraction will take place. Considering the Experiment Arrangement and constructing a rod clock (see also source [19]) oriented transverse to frame *Os* direction of motion, i.e. along *y* axis, in such way that the rod has one mirror at each end that reflecting light pulses between the mirrors, it can be concluded that Frame *O* and *O*’ can synchronize their clocks, as there won’t be any length contraction and the light path observed by *O*’ is not oblique in our specific case (see *figure 1*, left hand side), and both frames will measure light pulses of equal length.

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Appendix

Lorentz symmetry

Lorentz symmetry, the feature of nature that says experimental results are independent of the orientation or the boost velocity of the laboratory through space. According to SR the laws of physics are invariant under Lorentz transformations, and indeed under the full Poincaré group of transformations. [20]

The space and time coordinates *(x, y, z, t)* in a Minkowski space-time is the invariant space time interval which in differential form is:

$$ds^2 = -[ct]^2 + x^2 + y^2 + z^2 \qquad (A1\text{-}1)$$

Under a general coordinate transformation we will have $x_\mu \rightarrow x'_\mu$. The coordinate interval dx_μ transforms in contravariant manner. Respectively under transformation law the coordinate systems can also transform in covariant manner (or a mixture of those) (see also [20]).

The vector transformation law (which is in accordance with Lorentz transformation) can be denoted by:

$$V_\mu = \frac{\partial x_\mu}{\partial x_{\mu'}} V_{\mu'} \qquad (A1\text{-}2)$$

The metric transforms as:

$$g_{\mu'\nu'} = \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} g_{\mu\nu} \qquad (A1\text{-}3)$$

As Λ transformation not necessarily leaves the interval unchanged unless the interval invariance can be satisfied by Lorentz transformations; which forms a set i.e. Lorentz group. The set of both translations and Lorentz transformations (boosts and rotations) is a ten-parameter set i.e. the Poincaré group which consists of proper Lorentz transformation plus all translations and their products, and are generally part of Lie group. Moreover two successive Lorentz transformations, if set of Lorentz transformation Λ comprise a group to be represented by **Ĺ** then we have a group property $\Lambda_1 \Lambda_{2=} \Lambda_3 \in$ **Ĺ** for any Λ_1 and $\Lambda_2 \in$ **Ĺ**. Generally products of boosts and rotations are both proper and orthochronous and form a group and leave their overall space time interval invariant. [21][22]

Infinitesimal Lorentz transformation

By iterating infinitesimal transformations it is possible to recreate finite transformations [23] [24]. Considering Lorentz generators of group transformations, with regards to rotation, the three generators may be written as the antisymmetric tensor, similarly generators of the Lorentz boosts can be constructed by its respective Lorentz generators. Generally we will have:

$$x'_\mu = \Lambda_{\mu v} x_v \,;\, \Lambda_{\mu v} = \delta_{\mu v} + \epsilon_{\mu v} \qquad (A1\text{-}4)$$

Where $\epsilon_{\mu v}$ is infinitesimal matrix element and $\delta_{\mu v}$ is Kronecker Delta. One can construct ϵ and by denoting λ and θ to boost respectively rotation vectors, which are set of linearly independent real parameters of related to the group elements and the dimension of the group:

$$\epsilon = \begin{bmatrix} 0 & \lambda_1 & \lambda_2 & \lambda_3 \\ \lambda_1 & 0 & -\theta_3 & \theta_2 \\ \lambda_2 & \theta_3 & 0 & -\theta_1 \\ \lambda_3 & -\theta_2 & \theta_1 & 0 \end{bmatrix} \qquad (A1\text{-}5)$$

Furthermore the commutation relation defines the Lie algebra of the Lorentz group:

$$[S_i, L_j] = \epsilon_{i,j,k} K_k \qquad (A1\text{-}6)$$

Where $\epsilon_{i,j,k}$ are the structure constants which define the multiplication properties of the Lie group. Furthermore infinitesimal coordinate transformation leaves the Hamiltonian unchanged, and hence is associated with a conserved quantity [25].

Gauge transformation

The approach to the experiment is that Lorentz transformation (also called local Lorentz transformation [26] [27]) and gauge transformation will be cohesive on same foundation and in combination with point coincidences and iteration of infinitesimal transformations to get a finite transformation. Generally gauge transformations are not inertial by nature; in general they transform inertial reference frames to non-inertial networks of local reference frames as they introduce fictitious interactions. As the gauge transformation preserves the field theory and result in identical observable quantities by performing change of basis while underlying non-gauge-invariant quantities won't alter any invariance. In most gauge theories, the set of possible transformations of the abstract gauge base at an individual point in space and time is a finite-dimensional Lie group.

Roughly speaking gauge groups involving reparameterizations of fiber bundle which are topological or differentiable affinity between different fibers. Furthermore fiber bundles are manifolds which are locally products of a base manifold B with a fiber manifold F (union of the base manifold with the internal or spatial vector spaces) and globally they may have different structures. It is to say that continuous variations of the base point should result in continuous variations in the fiber.

A principal bundle which is naturally twisted with gauge theories is a special case of a fiber bundle where the fiber is a group *G* (usually a Lie group).

The action of Lorentz group *L* preserves the fiber and freely acts on every fiber, which also means that *S* is a principal fiber bundle with base *M* and structural group *L*. In other words this means that it is possible to perform a Lorentz transformation at every point in space.