My God, It's Full of Clones: Living in a Mathematical Universe

Marc Séguin

Physics Department, Collège de Maisonneuve, Montréal, Québec, Canada

ABSTRACT

Imagine there's only math — physics is nothing more than mathematics, we are self-aware mathematical substructures, and our physical universe is nothing more than a mathematical structure "seen from the inside". If that's the case, I will argue that it implies the existence of the *Maxiverse*, the largest imaginable multiverse, where every possible conscious observation is guaranteed to happen. I will attempt to explain why, of all the worlds in the infinite Maxiverse, we happen to live in one that can be understood by physical laws simple enough to be discovered (or, at least, approximated well enough for predictive and technological purposes). I will consider the question of personal identity in the context of a Maxiverse that contains an infinite number of exact clones of myself, and whether I should expect my future subjective experience to be unbounded. I will also consider the question of whether the Maxiverse hypothesis makes predictions that can be put to the test.

1. Math from nothing and your physics for free

Let's suppose that the relationship between mathematics, the study of abstract structures, and physics, the study of the physical world, can be represented by the equation

$$Physics = Math + ?$$

where "*?*" stands for one or more other fundamental ingredients which make up physics. Modern physics reveals that matter is made of fundamental building blocks, essentially electrons and quarks in the case of ordinary matter. Most of the properties that we associate with matter at our scale (like texture and color) are *emergent* properties that do not exist at the level of electrons or quarks, which are "wave-particles" without precise shapes, positions or trajectories. To be fair, some properties like mass and electric charge do exist at the level of electrons and quarks, and we can quantify them with a number — but beyond that, we don't really know what they truly are. In quantum field theory, wave-particles are excitations of fields that fill all space and are so "ethereal" that they might as well be purely abstract structures. Therefore, perhaps the "*?*" in our equation stands for nothing:

$$Physics = Math + nothing\ else$$

If this **Mathematical Universe Hypothesis (MUH)** is true (Max Tegmark [1], [2], [3]), our physical universe is *nothing more* than a mathematical structure: the patterns and laws which make up the mathematical description of our physical world *are physical reality itself*, the physicality of our universe being completely accounted for by the emergent properties of the underlying abstract mathematical structure. According to the MUH, physical reality is a web of relationships between entities that are themselves purely abstract: it's "all structure, no stuff ", a view that Jim Holt [4] calls **cosmic structuralism**.

An interesting parallel can be made between the MUH and the "Physical Life Hypothesis",

$$Biology = Physics\ (Chemistry) + nothing\ else$$

which has replaced (at least, among scientifically minded people) the once popular "Vitalism Hypothesis",

$$Biology = Physics\ (Chemistry) + \text{"}Life\ spark\text{"}$$

Many people intuitively reject the MUH because of a "gut feeling" that mathematical structures and physical structures cannot be equivalent: after all, mathematical structures are abstract, while physical structures are, well, "physical", which means "concrete", "tangible", "material". But if you accept that a living being can be thought of as nothing more than a complex arrangement of atoms obeying the laws of physics, is it really that hard to accept that a physical universe can be thought of as nothing more than a complex mathematical structure?

2. What part of ∃ don't you understand? Welcome to the Maxiverse

Cosmic structuralism as expressed by the MUH explains in a simple way why our universe exists: if the basic level of reality is an abstract mathematical structure, *our universe just has to exist*, since all mathematical structures exist by themselves, in a "timeless" and "eternal" way. On the other hand, if the basic level of reality was made of some physical stuff, to make a universe, you would need to get the stuff first: even if it turns out that our universe sprang from a tiny fluctuation in some primordial "false vacuum" quantum field, you would still need to get some false vacuum from the store!

Of course, if math is all it takes to have a universe, it makes no sense to believe that some universes exist while others don't. Therefore, the MUH implies the existence of an enormous *multiverse* that contains all the universes generated by all the mathematical structures which have the right properties to be physical universes: some of these universes are quite similar to our own, while others contain exotic phenomenon which obey completely different laws of physics. Tegmark calls this the **Level IV multiverse** (to distinguish it from lesser multiverses which regroup universes that share some similarities to our own). Every universe in the Level IV multiverse is as "real" as our own: what we consider to be our universe is simply the particular mathematical structure that we happen to find ourselves in.

A popular criticism of any hypothesis which implies a multiverse is saying that it violates Occam's razor, because it postulates an enormous, potentially infinite number of unobservable universes to explain our observable reality. But it all depends on what you try to minimize: the number of things that exist, or the number and complexity of the principles that define their existence. If you want to explain why one or only a few universes exist, you must specify the precise laws they obey and their initial conditions (at least). You must also specify and justify the rules which select these universes to be real while relegating all other possibilities to the dustbin of existence. Specifying the initial conditions alone might necessitate a mind boggling amount of information. On the other hand, to describe completely the Level IV multiverse, one short sentence is enough: *the collection of every mathematical structure which has the correct properties to correspond to a physical reality*.

The idea that every possible universe is as real as our own has been proposed by several philosophers before Tegmark's formulation of the MUH. (Robert Nozick [5] calls it the *principle of fecundity* and David Lewis [6] calls it *modal realism*.) Indeed, for some philosophers, the idea that everything exists is nothing less than a *logical inevitability*. No matter what the ultimate cause of existence is, we know that it has been able to create an actual world at least once, since we observe such a world. What could prevent this cause from acting again to create another world, and another, and another? And even if a given cause eventually "runs out of steam", being an ultimate cause, it exists by itself: if it instantiated itself once, what could prevent it from instantiating itself once more, creating other worlds? How could this process fall short of creating an ensemble of worlds which encompasses all possibilities? The MUH only makes it easier to believe in such an abundance of worlds, because all you need is math, and "math is cheap", existing by itself without using up any "limited natural resources".

It is interesting to note that Tegmark, although he believes in an enormous Level IV multiverse, takes great care to explain that he does not believe that every *imaginable* universe exists, because "humans can imagine many things that are mathematically undefined and hence don't correspond to mathematical structures" ([3], p. 351). In [2], Tegmark explains that although a mathematical structure is made of objects with relations between them, not all theories are mathematical structures, because some "objects", like God in the theory "God created Adam and Eve", may not be definable, even in principle, in a rigorous enough way to serve as objects within a mathematical structure.

I disagree with Tegmark on this issue, because I do not think it's possible to imagine an abstract structure which could not, in some way, be described by mathematics: there is no "Level V" made of non-mathematical structures, because mathematics is the general study of structures. Of course, one can imagine a gigantic, convoluted, ugly, unwieldy and irregular structure which would appear, at first glance, unmathematical. But from the point of view of an infinitely intelligent mathematician, even such a structure would be describable in a mathematical way. (According to Gödel's incompleteness theorem, there exist true mathematical statements that can never be proven by a finite set of axioms manipulated by a finite mind, but I do not think it makes the MUH ill-defined, and I do not believe, like Tegmark does, that we have to restrict the MUH to finite "computable" functions to make it work.)

Another divergence between my opinions and those of Tegmark concerns the issue of infinity. Tegmark is very cautious when it comes to infinity, because it makes probabilities within the multiverse virtually impossible to compute in a non-arbitrary way (more on this in section 3). In my view, the MUH implies the existence of an *infinit*e multiverse, a *maximal cosmological playground* that contains every imaginable physical reality and generates every imaginable conscious observation: I propose to call it the **Maxiverse**. In the rest of this paper, I will discuss some issues raised by the Maxiverse hypothesis.

3. Why is our world so lawful and simple?

In his 1999 book *Robot: Mere Machine to Transcendent Mind* [7], computer scientist and artificial intelligence pioneer Hans Moravec was one of the first to explore the idea that the Maxiverse might be real. In an interview from that same year published in [8], Moravec describes one of the biggest challenges of the Maxiverse hypothesis:

> So if our world exists […] in a sea of other possibilities, you then have to ask the question: Why is our world so boring? In the space of all possible worlds, there's a world in which in the next second you sprout wings on your head and your nose grows into an elephant's trunk. […] So why doesn't that really happen to us?

Our world is clearly regular: it obeys stable physical laws, and those laws are relatively simple, in the sense that we can at least approximate them in such a precise way that we can predict the evolution of physical systems and build technological contraptions which exploit that knowledge. (For instance, we can plan years in advance for something as complex as a robot rover mission to Mars, and carry out the plan with success.) In the cosmological smorgasbord of the Maxiverse, where every imaginable universe exists, we could argue that baroque, irregular and chaotic worlds should greatly outnumber lawful and predictable worlds like ours: our type of universe would then be highly unlikely, which would make the Maxiverse hypothesis highly suspect. One way out of this dilemma is to suppose that some worlds in the Maxiverse are more probable than others, and that regular worlds like ours have a higher *measure* (probability) than irregular worlds where you sprout wings on your head.

The problem is that, in an infinite ensemble of universes, the notion of what is likely or unlikely becomes ill-defined. In a finite ensemble, it is straightforward to evaluate the fraction of its members which have a given property: for instance, if you have a bag which contains a finite number of black or white marbles, the question "What fraction of balls is white?" has a definite answer, because you can sort the balls in two piles and count them. On the other hand, in an infinite ensemble, the question "What fraction of the members of the ensemble has some property X?" is not well defined. Tegmark ([3], p. 313) considers the example of the infinite set of all positive integers (1, 2, 3, 4, 5, 6…). It seems obvious that half its elements are even: if you analyze any portion of the list, you observe that odd and even numbers alternate, and you naturally conclude that half of the numbers are even. The problem is that, *because the set is infinite*, you can imagine other systematic orderings which do not lead to the same conclusion. For instance, if you make a list by starting with 1, 2, 4 and extend it by always adding the next larger *odd number* followed by the next two larger even numbers, you get the sequence *1*, 2, 4, *3*, 6, 8, *5*, 10, 12, *7*, 14, 16, *9*, 18, 20… This infinite list is complete, because no positive integer is left out. But now, if you try to evaluate the fraction of even numbers by analyzing some portion of the list, you conclude that *two thirds* of the numbers are even! This example may appear contrived, because it seems "obvious" that the order 1, 2, 3, 4, 5, 6… is the most natural one. But if you try to list universes in a set that contains an infinite number of them, there

will not be any obvious, natural and unique way to order them, and it will be impossible to unambiguously calculate the fraction of these universes associated with a given property. This ambiguity concerning probabilities within infinite sets is known as the **measure problem**, and it is the main reason why Tegmark hopes that his MUH can be reined in to imply only a finite set of finite universes.

Even if we do not know how to solve the measure problem, I believe that we can explain the lawfulness and simplicity of our universe by invoking something similar to the **anthropic principle**, the somewhat tautological statement that it's impossible to observe a universe whose properties are incompatible with life, since we couldn't exist there in the first place. In [8], Moravec explains that we observe that our universe stays lawful and predictable, even if there are many scenarios where it doesn't, *because in these scenarios, our consciousness immediately ceases to exist*:

> A lot of your experiences depend on everything working just the way it does. If the speed of light were to change, certain chemical reactions [in your brain] would alter, and your consciousness would probably be gone. But pretty much if the laws of physics were altered in any way, your consciousness would no longer work the way that it does. In those other worlds, if that's all that happened, you would no longer exist. So you can't find yourself in those worlds. Maybe some other things could change that bring back your consciousness, but that would be like another coincidence that would have to happen. […] So the most likely world that you will find yourself in in the next moment is one that's just a continuation of the world that you're in right now, because nothing has to change. […] Probably it is the case that this is the simplest world, the world that required the least number of coincidental starting positions to produce us.

4. Lost in the Maxiverse

Where am I in the Maxiverse? According to the MUH, my consciousness, my sense impressions and my memories are nothing more than a complex but finite self-aware substructure. In the infinite ensemble of all possible mathematical structures, there exists an infinite number of exact copies of this finite substructure. Each of these "clones" has exactly the same experiences that I have, and believes itself to be me. Each clone is embedded in a different larger mathematical structure, in a different "physical world". But these differences do not have any practical impact on my clones, since they are all identical to me. (There exist also an infinite number of slight variations of my substructure, and an infinite number of slightly more different variations, ad infinitum, but let us concentrate on my exact copies.) Some of my clones are part of mathematical structures which correspond to 13.8 billion-year-old infinite universes embedded in post-inflationary bubbles: details of these universes that do not influence the clones (like what happens outside the cosmic horizon in each universe) differ from universe to universe. I have some clones that are part of mathematical structures which correspond to powerful computer simulations run by posthuman historians to better understand a crucial part of human history, the pre-singularity decades at the beginning of the 21st century. I also have clones that are part of mathematical structures which correspond to the playgrounds of mad transdimensional superintelligences, where they play the role of existential pets.

Fundamentally, I believe that *all these clones are me*, and that I live simultaneously in an infinite number of larger contexts which differ only in unobservable ways. From one perspective, each of my clones is embedded in a different larger structure, which should make it possible to distinguish between them. But from another perspective, my self-aware substructure completely defines me (because it includes my consciousness and my sense impressions), and this substructure is a unique mathematical structure. There is no way to tell which perspective is the correct one: my clones are a single mathematical structure, that occupies "one spot" in the Maxiverse, but at the same time, they are scattered in an infinite number of copies throughout the Maxiverse. In the non-space of all mathematical structures, it is impossible to self-locate: we are, each and every one of us, fundamentally lost in the Maxiverse.

If you don't see how it's possible to live simultaneously in an infinite number of different but indistinguishable contexts, the analogy of a stretch of road that carries two different road numbers might help. In Quebec, highways 20 and 55 share the same stretch of road for a few kilometers. Suppose you

BOBBY HIDDY, WIKIPEDIA

wake up in the passenger seat on that stretch of road, with no memory of getting into the car. It makes no sense to argue about which road you *really* are on. Of course, once you get to the exchange when the two roads go their separate ways, you will wind up on one and only one road. Suppose the driver continues on highway 55: you could conclude that you have been on highway 55 since you woke up, *but only in retrospect*. In the same way, if you consider all the ways that your self-aware substructure can be embedded in a larger mathematical structure, you exist right now in a 13.8 billion-year-old universe, on a rock and metal and water planet where it is 2015 on the local calendar, but also inside a powerful computer running an historical simulation in a remote solar system where the planets have long ago been dismantled and converted into computronium. And there are many, many other scenarios that are equally true of your current situation. It makes no sense to ask which one is correct, even though, if you are lucky, you might eventually be able to eliminate some possibilities… *in retrospect.*

5. Life and death and life in the Maxiverse

I go to sleep on the night of March 4, 2015, right after submitting my FQXi essay. What can I expect tomorrow morning? If the MUH is true and implies the existence of the Maxiverse, there are many, many possible answers. In some universes, my self-aware substructure has been terminated during the night by an unlucky meteor strike (a small meteorite, sufficient to break through a roof and kill a person in his sleep, would not be detected beforehand). In other universes, a wave of decaying vacuum travelling at the speed of light has instantaneously disintegrated the Earth while I slept, taking humanity by surprise and painlessly wiping it out. In another universe, I was part of an historical simulation that got axed because of budget cuts, and I have been terminated and erased during the night. Of course, I cannot ever become aware of these possibilities: I can only be aware of the scenarios where I continue to exist.

Tomorrow morning (from my perspective), I can be any of the self-aware substructures in the Maxiverse that remember going to sleep as me: let's call them my **F-clones** (F for "future"). Some of my F-clones are very surprised to discover that something drastic has happened during the night. Some find themselves in a strange setting, with an error message floating in mid-air explaining that their historical simulation is being shut down, but that they will be taken care of according to the ethical rules of the posthuman research institute that ran the simulation (budget permitting, of course). Some of my F-clones find themselves in a mathematical structure where the Christian Last Judgment has begun during the night, and they stand in line at the Pearly Gates in freshly tailored white and gold robes (or are they blue and black?), waiting to be processed. But of course, I know (based on my previous experiences) that these unusual scenarios are unlikely: I expect to wake up in my bed and lead a more or less ordinary day, which must indicate that somehow (despite the measure problem), my F-clones which correspond to these ordinary scenarios greatly outnumber the other ones.

As long as waking up normally tomorrow morning has a reasonable probability of happening, I should expect that my future subjective experience will remain bound to the lawful and regular universe that I have gotten to know. But if I'm very old or terminally ill, at some point, some other category of scenarios will become more probable. In [9], Moravec describes how, in the space of all possible worlds, our subjective experience can always find a way to continue (an idea I will refer to as the **Maxiverse Immortality Hypothesis**):

> Our consciousness […] continues from moment to moment most simply if those laws continue to operate as they have in the past. Thus, with overwhelming probability, we find the laws are stable. In the space of all possible universes, we are bound to the same old one. As long as we remain alive. When we die, the rules surely change. As our brains and bodies cease to function in the normal way, it takes greater and greater contrivances and coincidences to explain continuing consciousness by their operation. We lose our ties to physical reality, but, in the space of all possible worlds, that cannot be the end. Our consciousness continues to exist in some of those, and we will always find ourselves in worlds where we exist and never in ones where we don't. […] Perhaps we are most likely to find ourselves reconstituted in the minds of superintelligent successors, or perhaps in dreamlike worlds (or AI programs) where psychological rather than physical rules dominate.

Because he doesn't think that the MUH implies an infinite Maxiverse, but only a finite Level IV multiverse (to avoid the measure problem), Tegmark has doubts about the soundness of this immortality argument. Nevertheless, in [3] (p. 220), he writes:

> But who really knows? When one fateful day in the future, you think that your own life is about to end, remember this and don't say to yourself *There's nothing left now* — because there might be. You might be about to discover firsthand that parallel universes really do exist.

6. What is the Maxiverse good for?

In the preceding sections, I have tried to build a case for the MUH and the Maxiverse hypothesis, but I am well aware that many readers will consider this essay a meaningless pipe dream that only someone who has lost contact with reality could entertain. Indeed, there are many reasons to reject the Maxiverse hypothesis. One major reason is the unfortunate fact that it cannot make any predictions that can be tested, since, in the Maxiverse, any imaginable observation occurs somewhere. Consider the most bizarre, seemingly illogical and far-fetched observation that you can think of. Because parts of the Maxiverse correspond to simulated, "fake" worlds that have been designed by twisted programmers with a weird sense of humor, no observation, no matter how crazy, could contradict the Maxiverse hypothesis. On the other hand, one can hope that some observations could at least strengthen its likelihood. For instance, if, after your death, you find yourself in some afterlife, you could interpret this as a confirmation of the Maxiverse Immortality Hypothesis presented above. Unfortunately (or fortunately?), as Tegmark and others [10] have pointed out, the Maxiverse does not guarantee immortality in all cases: if, instead of an abrupt transition between life and death in this universe, you undergo a gradual loss of mental faculties (for example, because of Alzheimer's disease), you may reach a point, before your death in this universe, where you no longer have any memories of the life you just lived. In this case, there is no F-clone in all the Maxiverse that can meaningfully carry your subjective experience into the future. If you don't even remember your life while still alive, what kind of meaningful afterlife could you possibly have?

Another argument against the Maxiverse hypothesis (in fact, against any theory which incorporates seriously the notion of a multiverse) is the belief that it critically undermines the future of theoretical physics. The danger, of course, is to invoke the Maxiverse as an easy way out whenever a given property of our universe seems too difficult to explain. For instance, the measured value of the cosmological constant is much, much smaller (by roughly 120 orders of magnitude) than the theoretically predicted "most probable" value. Some cosmologists seriously consider that the way out of this dilemma is to invoke the anthropic principle applied to a multiverse where universes with every value of the cosmological constant exist: only in universes where the cosmological constant is much, much smaller than the expected value would the conditions be suitable for the emergence of life. But other cosmologists consider that it is too soon to suppose that we will never discover some new physics which will explain naturally the observed value of the cosmological constant. I agree that it is not good scientific practice to appeal to the anthropic principle by default, but it doesn't mean that I think we shouldn't explore the implications of the Maxiverse hypothesis — in the same way that I believe we should not build nuclear weapons, but it doesn't mean that I think there shouldn't be any nuclear physicists.

The Maxiverse hypothesis, although not falsifiable in the usual sense, can motivate some legitimate scientific research. Since the measure problem is such a nuisance in any theory which postulates an infinite reality, more fundamental research at the intersection of probability theory and the study of infinity would obviously be helpful. If the MUH is correct, everything is a mathematical structure, including our minds, and it is our self-awareness and our observations of the world that give it its "physicality": without conscious observers, it wouldn't mean anything to say that some mathematical structures correspond to physical universes, because there would be no one to "feel" the "potential physicality" of these abstract structures. Therefore, the more we learn about the fundamental nature of consciousness, the better we will be able to understand the deep relationships between mind, matter and mathematics.

Another criticism of the Maxiverse hypothesis comes from the domain of human values. The mathematical structures which make up the Maxiverse exist by themselves, in a "timeless" and "eternal" way. Therefore, when we succeed at something in our universe (raising a child, inventing a new medicine, discovering a new theory), we do not change the Maxiverse in any way, we merely "visit" preexisting mathematical structures that have always been part of the Maxiverse. Moreover, while we succeed in our universe, some of our clones experience failure in theirs. Does this mean that we should stop trying and stop caring, since all outcomes exist anyway in the Maxiverse? A similar existential dilemma occurs with the issue of free will, given the fact that, from the point of view of 4-dimensional space-time, the past, present and future of our lives correspond to completed, fully determined 4-dimensional braids. Free will makes no sense from such a perspective, but from *our* perspective of 3-dimensional beings experiencing time in a sequential fashion, free will does have meaning. In the same way, in the context of the Maxiverse, your actions have no impact overall. But from your point of view, limited to one universe, your actions do matter, and you should care. In fact, free will acquires *more* meaning within a Maxiverse, since your inability to self-locate within it means that, essentially, there is no way to predict what you will do next. Some of your clones will do one thing, some will do something else, but since before the fact *you are all of these clones simultaneously*, no one, even an omniscient intelligence, can predict in advance what a particular you will do and experience in a particular universe.

If all this makes your head spin, you're not alone. If you no longer know what to make of the MUH and the Maxiverse hypothesis, you might consider the position defended by Piet Hut in [11]. Hut argues that we know way too little about mathematics, physics and consciousness to be able to have a coherent discussion about their ultimate nature and relationship. He believes humanity's knowledge might one day be sufficient to successfully tackle these issues: it's just that, at our current level of scientific and philosophical sophistication, we're just not worthy of such deep questions — yet. Maybe he's right, but this will not stop speculative cosmologists from arguing about these deep questions anyway, because that's what speculative cosmologists do. In the end, I believe in the Maxiverse because it is the ultimate playground for the curious mind. Living forever… across wildly divergent realities… who could ask, literally, for anything more than the Maxiverse? And if I'm right, somewhere within its infinitely complex simplicity, one of my F-clones is having a drink with one of your F-clones, and we're having a big laugh about it all. Cheers!

References

[1] Tegmark, Max. "Is "the theory of everything" merely the ultimate ensemble theory?", Annals of Physics **270**, 1-51, 1998, http://arxiv.org/pdf/gr-qc/9704009v2.pdf

[2] Tegmark, Max. "The Mathematical Universe", Foundations of Physics **38**, 101-150, 2008, http://arxiv.org/pdf/0704.0646v2.pdf

[3] Tegmark, Max. *Our Mathematical Universe*, New York: Alfred A. Knopf, 2014.

[4] Holt, Jim. *Why Does The World Exist?* London: W.W. Norton, 2012.

[5] Nozick, Robert. *Philosophical Explanations.* Cambridge, Massachusetts: Harvard University Press, 1981.

[6] Lewis, David. *On the Plurality of Worlds.* Blackwell, Oxford, 1986.

[7] Moravec, Hans. *Robot: Mere Machine to Transcendent Mind.* Oxford: Oxford University Press, 1999.

[8] Brown, David Jay. *Conversations on the Edge of the Apocalypse,* chapter "Robots and Children of the Mind — Hans Moravec", New York: Palgrave Macmillan, 2005.

[9] Moravec, Hans. "Simulation, Consciousness, Existence", Carnegie Mellon University, 1998, http://www.frc.ri.cmu.edu/~hpm/project.archive/general.articles/1998/SimConEx.98.html

[10] Standish, Russell K. *Theory of Nothing*. Charleston, S.C.: Book Surge, 2006.

[11] Hut, Piet, Mark Alford and Max Tegmark. "On Math, Matter and Mind", Foundations of Physics **36**, 765-794, 2006, http://arxiv.org/pdf/physics/0510188v2.pdf