

A Musing Upon the Nature of Fundamental Objects vis-a-vis the Point and the Plane

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1) A Point and a Plane

I had a conversation going on in my mind some time ago. If I were to explain Geometry to someone who didn't know about the subject at all, then how would I begin? I'd tell them that the fundamental element upon which all of Geometry is built upon is a Point. But then they'd ask, "What is a Point?" I'd say that a Point is an infinitesimally small object lying on a Plane. Then they'd ask, "What is a Plane?" I'd say, a Plane is a surface composed of infinitely many Points! Whoops. In order to explain the nature of a knowledge system, I attempted to take the reductionists route so that I could reduce the entire game down to its fundamental postulate(s) —only to find out that at the very bottom, what I end up with isn't a single fundamental building block, but rather an object whose existence is intertwined with the surface it is woven into. For what does a single Point even mean? It means nothing, since size is a relative concept, and for an infinitesimally small object to exist, it must exist alongside or in this case lie upon a larger object that gives its size and separate existence a context.

This imagined conversation made me ponder upon the circularity inherent in our definitions of the fundamental rules of the world, or rather the fundamental axioms of our knowledge systems.

2) Approximation taken as Certainty for the sake of Workability

i) What is a Point?

Some time after this argument popping up in my head, I searched on the Internet to find out whether this debacle has been talked about before.

In my quest, it turned out that Euclid—considered to be the Father of Geometry by many—seemed to have known that his 'Point' was an impossible object, but went ahead with it anyway in order to build up his axioms of Geometry, that he compiled in his compendium of books, called The Elements.

According to the very first definition in the compendium, Euclid describes the object of the Point thus; 'A point is that of which there is no part.'¹

What object contains no part though? That which can't be divided further. This means that the 'Point' as defined by Euclid is the fundamental unit of space—just as was the case in my

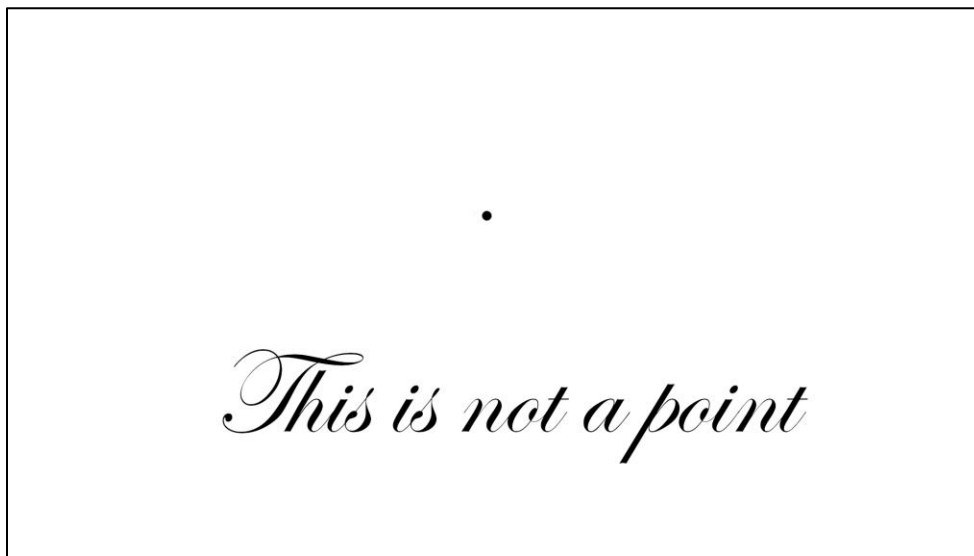
¹ <http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf>

thought experiment. This means that space is composed of these Points. Hence, space is discretized into Points. Now either these various Points have boundaries between each other, or they do not.

According to Definition 4 of The Elements, 'A straight-Line is (any) one which lies evenly with Points on itself.'²

I would assume this 'even'-ness to be a lack of any roughness in the Line, implying that a Line is smooth in nature. The object of the Line should hence be continuous. This presents a problem then. If a Line is continuous in nature, then how can it be divided into infinitely many Points? How can a smooth object be discretized thus? And taking this argument further, shouldn't Space itself be that which has no part then? The Point is then either a density which can't be divided further, or a dimension-less void. Space being composed of fundamental units makes sense only when one is able to create a rip in it to arrive at non-Space. Neither Euclid ever talked about non-Space (or non-Plane) in his treatise, nor have physicists found out such an entity yet.

But these definitions aside, I would like to point to what a Point was for me when I was first introduced to Points and Lines in 4th grade. It was a miniscule dot. I even remember our Maths teacher being particularly strict about us drawing Points as miniscule dots, not thicker than the Line segment drawn between them.



So I would propose that a 'primitive notion'³ such as a Point is a fictitious entity that is somehow related to how the human brain makes sense of the world on a biological level.

² <http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf>

ii) What even is a 2-dimensional Plane?

Definition 2 of The Elements says that, 'And a Line is a length without breadth.' And as per its Definition 5, 'And a surface is that which has length and breadth.'⁴

Let's logically work backwards to understand the structure of Euclid's system. The definitions cited above imply that Euclid was building a world which is an abstraction of our everyday physical reality, such that it could be broken down into slices without depth. These slices could further be divided into Lines with neither breadth nor depth. And the Line could ultimately be divided into a Point with neither length, nor breadth nor height.

If we try to explain Points, Lines and Planes in terms of dimensions, then a Point has to be a 0-dimensional object. A Point is a dimensionless void then. The description of a Point as '...that of which there is no part,' indicates that Euclid was treating a Point not as an object having length, width, or breadth, but as an indivisible location.

If these properties are taken literally, then, following Definition 3, 'And the extremities of a line are points.'⁵ building a Line segment from 2 Points would mean that it contains 2 nothings. And as per Definition 4 pertaining to a Line already stated above, a Line should be composed of an infinitely many number of these voids! That's downright absurd! How can a smooth, continuous object be entirely composed of infinite voids? This is clearly a paradox; a paradox hidden in the fundamental definitions with which Euclid was starting his system!

Forget 4-dimensional objects; how could even a 1-dimensional Line possibly exist? As soon as a Line has any thickness, it extends into the axis perpendicular to its propagation, which turns it into a 2-dimensional object. If a Line were to be a 1-dimensional object, then it would, just like a true Point, not have any thickness or rather size whatsoever-it would only propagate continuously without thickness. A Line then, literally, would be the propagation of nothingness. How does that make any sense? Let's say that a Line is 2-dimensional then. But how could an object in a 3-dimensional world existing in the X-axis and Y-axis, not have a thickness in the Z axis, however small? It's ridiculous to talk of either a 1-dimensional or a 2-dimensional Line then. It seems that the 3-dimensional world cannot be broken down into fundamental elements of lesser dimensions. Yet this bizarre and abstract system of Geometry has worked for us in both building things and understanding Nature. Co-ordinate Geometry has worked well for us over centuries now, both to understand Nature and to create workable technology and

³ https://en.wikipedia.org/wiki/Primitive_notion

⁴ <http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf>

⁵ <http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf>

functional architecture. It has helped us solve Mathematical mysteries and even made calculations to launch us into space.

3) What even should be considered to be the Fundamental Categories?

In the first section, I had talked about a Point and a Plane—in other words, a particle and a space.

Now, rethinking the fundamental object, a doubt arises in my mind. What should be considered the fundamental properties of physical existence—The Distance between Points or the Points in the space? Should a relationship, or perhaps a configuration be regarded as the fundamental object, or should particles enmeshed in these relations be regarded as the fundamental objects? And how could one even be separated from the other, when both are mutually dependent? A pragmatic answer could be that what we consider be the fundamental object(s) is/are dependent on the problem in question which needs to be solved. But I wonder though—is there a real answer independent of any particular knowledge system? Or is it that it's impossible for the human mind to make sense of the world as it is without fictionalizing it down to neat, artificial mutually exclusive categories?

Is it even possible to come to an understanding of the world without fictional cognitive categories? Could we somehow come to comprehend the world through its actual organizational structure, instead of comprehending it through human-made fictional categories as we are doing now? Can we really understand the world as it is, rather than in terms of its abstractions created by the human mind?

I have argued above that the fundamental building blocks of Geometry are fictional ideas born of the human mind. They seem to be impossible ideas that have translated into letting us know how the world is really organized. Therein lies a perplexing reality of the human mind—that in order to make sense of the whole, we must break down an object into imaginary categories.

Also, when we're talking about fundamental categories in any subject then, we're talking about fundamental objects of cognition for the human mind, and not the fundamental building blocks of the object that that subject aims to study. Are the fundamental objects just a creation of our mind to understand a seamless whole, owing to the cognitive limitation of the human mind?

An interesting question to ask and explore would be—What, if any, is the relationship between the cognitive objects of the human mind and that of Nature? Since we are ourselves made out of Nature, could our reductive 'primitive notions' in fact have some sort of real mapping onto Nature?