Is the Effectiveness of Mathematics Unreasonable?

Abstract:

In a well-known essay[1], Wigner took the position that the effectiveness of mathematics in the natural sciences was “unreasonable,” even miraculous. Within a basically Realist[2] perspective, a change of focus to view “object” as a secondary concept, and “pattern” as the central concept, has the somewhat surprising effect of making the effectiveness of mathematics in the natural sciences seem utterly unsurprising, even unavoidable.

Mathematics is the study of patterns, as patterns, without reference to meanings, that is to say, without reference to objects, except insofar as they stand for aspects of the pattern. These abstract patterns are not based on the objects except, perhaps, historically. The objects are aspects of the pattern.

Science, on the other hand, is a way of studying the patterns found in Nature. These concrete patterns are based on objects but they are patterns first and foremost, and it would be astonishing if they were somehow incapable of being studied in the abstract: as syntax without the semantics.

What is pattern?

Since I am taking “pattern” as a basic concept, I will not attempt to define it in terms of something more basic. My usage is pretty much in line with normal usage, but a partial clarification of what I mean by it may be in order. Roughly then, anything which is not utterly patternless, that is to say, not entirely random, exhibits a pattern.[3]

What is mathematics?

Abstract, or “pure,” mathematics is the study of the possibilities and limitations of patterns as pure patterns. In fact, mathematics studies the contours of patterns, without reference to what “objects” may be the pattern-holders. Basically that means it is about syntax not semantics.[5] As Bertrand Russell put it[6] “*… mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true*.” Not knowing what we are talking about means we don’t care (in pure mathematics) what the objects are…, or whether they exist in Nature. We study the intricacies of the structure without reference to meaning.

For example, there are five regular solids in (the abstract pattern called) a Euclidean 3-space. This is a robust logical construction and surely “exists”[4a] as pure pattern (if “exist” is taken as the word to describe what abstract patterns do.) This has nothing to do with the question of whether Mother Nature has utilized a Euclidean 3-space anywhere in nature, that is, it has nothing to do with whether a Euclidean 3-space “exists”[4b] in Nature.

More subtly, what we think of us as the objects of Euclidean 3-space don't need to exist, even in the abstract, as actual objects. They surely "exist" as aspects of the pattern, but the pattern has no real need for objects to form it, so it is not necessary to posit that they exist in any other sense. They are not really necessary.

This is in some ways analogous to understanding a curved space of (say) 4 dimensions. It is not really necessary to have a 5^{th} dimension for the other 4 to curve around. The extra dimension is completely optional…, and similarly, objects can be optional. Pattern, on the other hand, is not optional. The pattern <u>is</u> the mathematics.

What about Nature?

Nature in general, and natural phenomena in particular, exhibit patterns. We generally describe this as "nature follows laws." The phrase "laws of nature" is a commonplace, but "law" sounds algorithmic to me, and I'm not interested in opening that completely different can of worms, so I will content myself with the lesser claim that nature exhibits patterns, and not worry about whether all the patterns are necessarily algorithmic.

What is science?

Science is a way of studying Nature. It has been characterized many ways, by looking at it from different perspectives. The above generalization from "natural laws" to "patterns" leads to the following, approximate characterization of the activities of science by dividing them into several, somewhat arbitrarily chosen baskets:

- looking for natural phenomena that appear to exhibit patterns,
- accumulating data about the phenomena,
- looking for patterns in the data,
- choosing candidate abstract patterns as models for the concrete patterns found in the data.
- looking for aspects of the abstract pattern (model) that make predictions about as-yet-unstudied aspects of the concrete, natural pattern
- accumulating new or more accurate data, often data that is relevant to this new aspect of the pattern
- updating the choice of model with a new abstract pattern if needed
- (and go around the circle again…, and again)

Of course we, homo sapiens, being a brand new species, have only just started exploring the "library" of abstract, mathematical patterns, so sometimes "choosing candidate abstract patterns" involves doing something completely new rather than just "choosing" a pattern off the rack.

What is a model?[7]

A model of a natural phenomenon is an abstract mathematical pattern that fits the data we have about the phenomenon. That is, it's an abstract pattern used as a representation of fact.

At any given state-of-our-knowledge (of a given set of natural phenomena,) we have only finitely many data points and each one is of finite accuracy so, technically, there is an intractably large, infinite class of mathematical systems that are candidate models, that is: they fit the already-known data. Some of them are more interesting than others, in fact some are utterly uninteresting.

What makes a model interesting?

Many things make a model interesting, and different scientists will surely have different lists. A few rather obvious points are:

The aesthetic criterion of simplicity is perhaps more than merely aesthetic. Naturally we should only be interested in models whose axiom sets are dramatically smaller than the data set we are working with…, models that can be largely encapsulated in a few "laws." The fewer the axioms the more interesting the model.

Any model (mathematical pattern) will also make predictions about aspects of the natural pattern (phenomenon) for which we don't yet have data points. The more accessible the predictions, and the more testable with today's technology, the more interesting the model.

Frequently, an attractive model doesn't quite fit the data. An anomaly or two is not a fatal flaw to a model, but the better the fit the better. In a more limited context George Box[8] said "All models are wrong but some are useful." If there is an anomaly, it is helpful to know some limited domain of validity within which the model seems to be reliable.

From the above perspective, since we expect to always have only finitely many data points, we can never know if a model is <u>the</u> correct model, but usefulness is certainly another desirable trait.

For example, a few centuries ago, there was an ongoing argument about whether light was a particle or a wave. Newton preferred a particle model, Huygens preferred a wave model, but each model was useful because it rather quickly became clear to physcists when they should use one model and when the other.

In contrast, today's most popular economic models are in a somewhat messier state. It appears that Keynes' model and Hayak's each have domains of validity, but it is far less clear when to use one model and when to use the other one.

Some differences between the “hard” sciences and the “soft” sciences

As with the above examples, in the “hard” sciences, it is more frequently clear which model to use on which occasion.

In both “hard” and “soft,” there typically will be a staggeringly large number of independent variables, but in the harder sciences, more than the softer sciences, one can shorten the list to a tractable number and know when one will still get a highly accurate approximation (a useful model.)

In either case, there is an abstract model chosen to fit the data as well as possible, at least in some limited domain. Put differently, “…there is an abstract pattern, chosen to match the natural pattern as well as possible….”

Is effectiveness unreasonable?

The aspect of natural phenomena that science studies is the natural laws. If one changes perspective slightly and generalizes from “natural laws” to “natural patterns,” it seems pretty reasonable that mathematics, the study of abstract patterns, should be a useful tool.

If we find geometric patterns in nature they can also be studied in the abstract. If we find symmetries, they too can be studied in the abstract, and of course, if we find numerical patterns in nature, they can be studied in the abstract as well.

Furthermore, most of the patterns we are looking at will heavily involve numbers because any time we are talking about finding patterns in datapoints and measurements we are talking about finding patterns in numbers.

Notes

1) ..that is also part of the inspiration for this essay contest

2) Perhaps this position should be viewed as fitting inside Structural Realism

3) In fact, in a perhaps idiosyncratic aspect of my usage, “completely random” would be the limiting case, and still a “pattern.” But that will not be relevant to this discussion.

4a & b) I have put “exist” in quotes in this paragraph, because the two uses clearly have very different meanings, and I am not particularly interested in the terminological discussion of whether one should use the word “exist” to refer to that which abstract objects do.

5) One might need to generalize beyond an algorithmic idea of syntax

6) “*Mathematics and the Metaphysicians*” See, for example, http://www.readbookonline.net/readOnLine/22895/

7) This does not coincide with the usage of “model” in the branch of mathematical logic known as model theory, which is talking about something else entirely.

8) See, for example, http://www.quantumdiaries.org/2014/07/04/wrong/