Is 1+1=2 an empirical proposition?

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1 Meaning in mathematical propositions.

Imagine someone demands from us a proof of the truth that $1 + 1 = 2$. Certainly, our reaction will depend on whether we are asked by a child or a mathematician. While in the former case we can act by showing that one coin put together with another coin, when counted again are two coins. In the latter case, it is not clear what we are meant to show. Is a syntatic calculation in an abstract setting showing the truth about acausal, non-temporal, and non-spatial objects what is required?

Consider again the child learning arithmetic. At what point, does she grasp the same truth the mathematician is trying to prove in her theorems. Or is it necessary to be familiar with Peano Axioms and first order logic to understand the true meaning of $1 + 1 = 2$? We become bewitched by what we believe lies behind the symbols. It seems like the meaning of the mathematical proposition is covered by some fog and dispersing the fog is the work of the mathematician.

But what is meaning in the first place? How do symbols, for example words, acquire meaning? We follow Wittgenstein in ascribing meaning to a large class of words as the way we use the word in language. For example, the meaning of the word "table" is not a mental or physical representation of a four legged object; the meaning of the word table is the way in which we use it as in "Put the essay on the table" or "the data is shown in table 1". We can only say we understand the meaning of the word if, in the former sentence we act by putting the essay on the table, and in the latter by looking at the correct place. If someone says "I feel very table." we will demand some explanation as the word is used in a manner which doesn't fit any use that we are aware of. We don't understand the use of the word table in that context. The meaning goes astray.

Can this idea of meaning be taken to mathematics? Or in other words: is the use in language of the mathematical proposition what gives them meaning? An analysis between empirical statements and mathematical propositions is needed here. The truth of a mathematical proposition is independent of any empirical phenomena. The use of mathematical prepositions supports

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this premise. For example, if we see two drops of water coming together and forming one, we are not tempted to claim that $1 + 1 = 2$ is false. Rather, we conclude that this is not an example of the application of the proposition $1 + 1 = 2$. Imagine that actually all objects behave in this peculiar way by sometimes merging sometimes splitting such that the proposition $1 + 1 = 2$ can never be an example of something physical. In that case, the truth or falseness of arithmetic propositions won't matter anymore as the whole use of arithmetic will reduce to that of a game of symbols. While the truth or falseness of mathematical proposition are independent of empirical phenomena, the sense (or meaning) of mathematical propositions is not.

The fact is that nature doesn't behave in this way and that empirical regularities appear constantly. We as humans become aware of this brute fact and act accordingly. We employ initially the symbols to show the regularity and then "harden" them into mathematical propositions. This is the mathematical practice. The mathematicians job is to see that a stick used to measure the length of the objects, can become a ruler. The usefulness then of mathematics relies on us behaving the same way when presented with the same 'mathematically' related situations (arranging, sorting, recognising shapes, performing one-to-one correspondences, and so forth.) This is not an agreement of opinion, but an agreement in what being a human is. This is what gives mathematics its objectivity.

2 Physics is an empirical science.

Physics is the empirical science concerned with the behaviour of inanimate matter. The term empirical science refers to the fact that physics relies solely in the ability to test claims about the world with an experiment. Nevertheless, physics is a word of the English language and has different uses which means it has different meanings. For example, we can also describe physics to be what physicist do. Before getting into a loop and describe a physicist as someone who does physics, we can define a physicist as those members of society which work in physics departments and receive funding to do physics. In this sense, physics is defined as anything this group of people do. These two meanings are interwoven, but they are not equivalent. Our first definition describes an activity, the second one describes an activity done by certain people.

If we are interested in our first definition of physics, we may ask: how is theoretical physics possible with the picture of mathematical propositions presented above? A contradiction seems to appear: if mathematics are hardened empirical regularities what then is the tool theoretical physics uses to describe 'yet to be found' empirical regularities. Isn't the achievements of theoretical physics to be able to go from mathematical propositions to empirical predictions?

The contradiction can be resolved by simply accepting that theoretical physics doesn't start from mathematical propositions. The objective of a theoretical physicist is not to only find rules that agree with empirical data, but ways to find new empirical data. The mathematical symbolism a theoretical physicist uses is closer to a symbol game than mathematics. Even if the symbols are the same. One can for example imagine a game that uses the letters of the alphabet, nevertheless we are not inclined to think of this as English.

Again, what gives life to the symbol game and eventually transforms it into mathematics is the empirical regularity that must be associated with the symbols. Theoretical physics then is a no-man's-land in the junction of mathematics, physics and philosophy where everything is allowed as far as one is able to predict something. Then, if the prediction becomes a regularity it is physics, and if the regularity is "hardened" so empirical data can't change it anymore it becomes mathematics.

If the more sociological point of view towards physics is taken, then theoretical physics is defined by the practitioners. Is this then just a matter of agreement between a community? Compare with the mathematicians accepting something is proven: what the community is agreeing to is not an opinion, but of the same way of acting. In the same way, we breath the same, our hearts beat the same, etc. Nevertheless, are the agreements in the case of theoretical physicist equally grounded as in the case of the mathematicians? The distinction between them is a philosophical task that must be done in order to avoid misunderstanding. Mathematical meaning can go astray too.

3 Ordinal arithmetic and String theory.

The discussion above provides initial ideas where theoretical physics and mathematics can be discussed in terms of use and meaning. The main shift one would like to achieve is to move from the dicotomy of "true" and "false" propositions to the notions of "sense" and "nonsense". What one needs to see is the background which allow both activities to have meaning. In particular, there are two examples which provide a formidable starting point of analysis: "Ordinal Arithmetic" and "String theory".

Ordinal arithmetic seems to put heavy tension on the claims made before. Where is the empirical regularity that makes ordinal arithmetic true? The temptation to make this question must be avoided. We need to ask: "Where is the empirical regularity that makes ordinal arithmetic have sense?", or "How is ordinal arithmetic used?". The analysis then should go to understand the connections between ordinal arithmetic and the rest of mathematics. Notice that the relevance of the concept of infinity for the discussion has been shifted from a metaphysical discussion to a pragmatic one. The question we are interested in should be: How are we using infinity in order to have useful statements in ordinal arithmetic? But, what if applications are lacking and theorems do not have implications to any other part of mathematics. The answer must be similar to: "What is the use of a cog that spins vigorously but is not attached to the rest of the machine?"

String theory provides an excellent case of analysis because of the multiplicity of meanings. It is hard to draw a line between what is mathematics and what is theoretical physics in String theory. However the must important part is to be sure that there is not mathematical symbols out of the appropriate context and therefore nonsense. This is what the analysis should do. The claims that string theory is a part of theoretical physics and therefore an approach to describe some empirical regularity of the Universe must be independent of the mathematical machinery developed. Arguing otherwise, is close to misunderstanding the difference between physics and mathematics (the differences as a human activity). In this case, string theory as a theoretical physical tool can only be evaluated in the light of usefulness to explain and predict empirical phenomena. An experiment is needed to settle the question.

The claim that string theory contains mathematical propositions has also to be put in order. This means mathematical propositions in string theory must be approached with the same mathematical attitude as when one is doing mathematics. This analysis must put in clear view the empirical content of the theory against the mathematical propositions no empirical phenomena can change. However, there might be also the case that String theory is a new kind of activity. Which is neither physics, nor mathematics. Only the dialogue between philosophers, mathematicians and physicists can shed light on this delicate matter.

The work here presented has taken as a basis the ideas of Ludwig Wittgenstein in Philosophical Investigations and Remarks on the Foundations of Mathematics. All the thoughts that correspond to Wittgenstein should be atributed to him while the ones than depart from him should be atributed to the author. Also the work of Sorin Bangu in http://www.iep.utm.edu/wittmath/ was influetial for the work.