Truth or Dare: Possible Mathematical Representation of Reality

Tapio Salminen[1]

Abstract

In this essay I consider reality as understood in verificationism or in pragmatism such that it is "the end of enquiry". In this framework reality can be given a one to one correspondence with the ontology of a hypothetical best possible theory of physics capable of answering all possible questions as long as "best" includes the condition of being the simplest. If this theory can be written in the language of mathematics, as will be motivated by its need to describe information, then reality admits a mathematical representation.

---

[1] tapio.salminen@iki.fi

1 Introduction

What does reality consist of? What exists and what is true? Is reality mathematical? There are several ways of interpreting the semantics in these types of questions and accordingly several answers can be given, differing not only in content but also on what level the question is understood. I will begin by contrasting two alternative definitions of reality

1. There exists an objective, absolute reality, independent of any observer. Our observations and scientific theories describe this reality.
2. Reality consists of everything that is in principle verifiable; hence reality can be different for different observers. Our observations and scientific theories describe this reality.

The first item is one way in which *scientific realism*[2] is defined [1]. The claim is that there is an absolute, objective, reality which we describe. I will argue below that we are physically unable to give a description of an absolute reality (let alone observe one) and as an unobservable, indescribable element of even our best possible theory the usefulness of the concept should be weighed carefully. For the most part I will consider instead item 2: reality as understood in (some forms of) *verificationism* or *pragmatism* and state that what exists needs to be in principle verifiable [2], thus denying the existence of anything that is not even indirectly observable. The idea is poetically captured by the pragmatist catchphrase:

> "Truth is the end of enquiry."

The difference between this and the realist belief is that for the realist everything in the ontology of our best theories exists, also the unobservable. Experimentally this debate is firmly on safe ground since what we are debating is by definition beyond falsifiability.

I will further argue that *if* it is possible to write down a final theory of physics capable of answering all answerable questions, then the simplest such theory will match the verificationist reality exactly. Starting from Wheeler's idea that the world can be fully described in terms of information, I will motivate why such a theory should be expected to be written down in mathematical language and hence be a mathematical representation of reality. On a more speculative note, taking cues from our best possible theories at present, I will discuss what such a reality might be made of; most importantly that we should use relative rather than absolute notions.

Since any consideration of a reality that is not by definition objective and absolute is met with objections such as *"Idealism!"* or *"Solipsism!"* I will add a disclaimer for the rest of the text:

> *Disclaimer: Every physical system is an equivalent observer.*

Setting all physical systems on equal footing as observers does away with the (here irrelevant) concepts of awareness, sensory etc qualities of an observer. Any physical system is capable of interacting with

---

[2] I am using philosophical terms in a crude way - in the philosophical literature all of these terms are used to describe a variety of different ideas and nuances. In this essay *realism*, *verificationism* and *pragmatism* are to be understood only to mirror the two definitions of reality and truth given above.

any other system and is thus a possible observer. Thus the main difference between a person and an electron as an observer is the amount of information they are able to store, not their self-awareness. For example, if we believe the world to be made of quantum systems, *all systems need to be quantum*; there is no divide into the microscopic world that is observed and the macroscopic observer. Certainly there is no denial of a reality outside the observers mind (they need not have one), there is simply the claim that what is true for an observer need not be true for everyone else. This requires an obvious consistency condition: if observers compare their data, these need to match exactly.

2 The Limits of Our Knowledge

Suppose we would have the best possible theory for describing our world; a theory that is (in principle) able to answer all answerable questions and would thus exhaust everything we can know about our world - an end to enquiry. To follow tradition let us call this theory the Theory of Everything (ToE) and intuitively conceive it as being a working theory of all physical interactions, perhaps a fusion of our current best theories of physics General Relativity (GR) and the Standard Model of Particle physics (SM); reducing to these in some appropriate limits.

Such a theory would complete the gaps in the ontology we infer from GR and SM relieving our ignorance and reconciling the contradictions that arise when we try to superimpose the two. In a sense, such a theory would allow us to dispense with *belief* in favor of knowledge as far as is physically possible. How far would this take us?

According to our verificationist premise, reality is made up of elements we can in principle verify. On the other hand, the ToE by definition describes everything we could even in principle verify. To say that therefore the ontology of the ToE can be taken to describe reality perfectly, to be a *representation* of reality, we need the further assumption that there is nothing more in the ontology than what can be verified. Thus I will argue for the following

Simplicity Principle: Physical theories should only posit the existence of verifiable things,

i.e. (even in principle) unobtainable information must be redundant in the description of the world[3]. If we adhere to this principle we are able to conclude that the ontology of the ToE is a valid representation of reality and it follows that if the ToE is written down in the language of mathematics, reality admits a mathematical description - answering the question posed for the essay.

I also wish to explore why I expect such a theory to be possible in the first place, why it should be written down in a mathematical language and what would be the limits of its description. The central concept for all of this is information.

3 Information

> "[...] all things physical are information-theoretic in origin and this is a participatory universe." – John Wheeler [3]

---

[3] *The Simplicity Principle* does away with the infamous Russell's teapot in all of its forms.

Physics is about describing the world we live in. Although it is sometimes debated [4] (see also [5]), this description is to be based on observations and observability, past, present or future; direct or indirect. Since observability is fundamentally about what we can know, it is natural to follow Wheeler and use information itself to describe the world.

We start from this epistemic angle with the following assumption about knowledge

> *Assumption 1: Everything any observer knows (and can know) can be represented as information in an array of bits* $|b_1, \ldots, b_n\rangle$.

This clarifies what is meant by the term "information" here: it does not entail any interpretation or meaning given to this information. In conjunction with the above, and at the core of this essay, I would like to invoke the following (converse of Wheeler)

> *Assumption 2: Information needs to be encoded in something physical.*

These assumptions implicitly entail a materialistic basis for epistemology: All knowledge can be viewed as information, information corresponds to something physical[4]. In this way of talking about information, it is always something that someone *has*, whether it is any observer in the universe or a hypothetical super-being outside it. Although there is no need to assume this here, we can further note that since there is expected to be a finite minimum physical size information requires [6], in any spacetime of finite size the number of bits should be finite.

Since we are considering reality to be made of only verifiable things, then all obtainable information exhausts reality. In fact, since this is the case, on a meta-level this implies (as long as we stay away from any interpretational issues) that the rules of mathematics and physics have no external existence and are instead characterizable as information that different observers have and that form a coherent set of beliefs, in line with the coherence theory of truth. If all obtainable information exhausts reality, and information follows a mathematical language and logic, then so does reality.

Different Types of Knowledge To simplify the following discussion let us classify information into the categories

(0) Self-measurement
(1) Data - Information we have of the world
(2) Inferred information - Ontology posited by our best theory
(3) Ontology of superior theories we hope to discover

We use physics to describe the world from an outsider's point of view - the description is always that of an outside observer and therefore the problem of self-measurement rarely arises. This is an interesting open question, but for simplicity I will (for the most part) assume that when talking about the information we have, we exclude all information about our own state and therefore consider (0)

---

[4] It does not entail anything about the interpretation of this information; hence for example it does not make the claim that (physically ambiguous, user-defined) concepts such as "love" need to be describable in terms of information contained in field degrees of freedom.

as information that is not available to an observer. That being said, allowing self-measurement would not significantly modify the conclusions.

The meaning of (1) is clear: an observer gains information about another system and according to the assumptions above stores it in a way that is representable by a collection of bits $|b_1, \dots, b_n\rangle$. In the same way as (1) contains all the directly observed information we have of the world, (2) represents all the indirect information we have. (2) is everything we can infer from our "best theories" based on the data we have - in the way we "know" there were dinosaurs based on us having dinosaur bones and a theory that posits the existence of these now extinct creatures (and nothing to make us think we should doubt this inference).

The status of Data is clearly stronger than that of Inferred information; we are more likely to believe that there were no dinosaurs than that there are no huge bones in our museums. We (rightly) hold reservations about (2) leaving room for (3). This insecurity is why (3) is included in the list though it is clearly hypothetical; it's taking into account all possible mistakes we make when we follow another catchphrase and "believe the ontologies deposited by our best scientific theories". All Data is by nature definite knowledge whereas the inferred information can be indefinite as regards observability. That is to say, Inferred information can be divided into the directly observable, the indirectly observable[5] and the unobservable. A realist would argue that everything posited by our best theories exists, perhaps insofar as this is not contradicted by another one of our best theories, whereas a verificationist wants to exclude everything unverifiable, i.e. everything that is not even indirectly observable.

Let us suppose we are some day able to write down a Theory of Everything, i.e. we could dispense with (2) and only consider (1) and (3) In this case we could predict everything that is predictable, infer all that is inferable, and a description of the world in terms of this information, direct and inferred, would be the best possible one[6]. Now let us consider one important aspect guaranteed by our *Assumptions* above: since the inferred information is nevertheless information, that too will be represented by bits and having that information needs to correspond to something physical. Also, since information is something a system has of another system, it is a relative notion; (0)-(3) are all relative rather than absolute. Let us see what this entails for the best possible description by considering a simple toy example.

3.1 A Simple Observation About Information

To make the idea of "having information" more explicit, consider a toy universe with three particles {A,B,C} each with one degree of freedom that corresponds to one bit of information, say an arrow pointing up/down or a number {0,1}. Since the universe by definition is all there is, to describe physics inside it we would need to *be* either A, B or C. Now if this is the case it is obvious that we can only *have* one bit of information, say we are A and interact with B to learn B's state w.r.t. our own B_A. If we raise the number of bits in A to two we can have information of both B and C and thus exhaust everything there is to know about the universe, for A. In this setting it seems natural to say that A

---

[5] For example a wave function in quantum mechanics is not directly observable, but its existence is indirectly verifiable via the double slit experiment.

[6] Note that this description, nor the set of information, need not be unique.

cannot have information about herself, as there is no natural way for A to interact with herself. If that's the case, it is impossible for anyone to describe the entire universe, even in principle, as the only entity that could have 4 bits of information is the union of A,B,C measuring itself.

There is of course no problem for any outside observer from knowing everything there is to know about this simple system (like we do, right now), but that will require the existence of a super-observer outside the universe. Thus it follows that in any universe where the entities can be described in terms of information *complete information about the universe is impossible to acquire.* Importantly, in such a world, it is impossible even to describe the universe as seen by an outside super-observer - no one has enough bits to *have* that description. Hence, a full description of an absolute reality is physically impossible. Nevertheless, there is a most-complete-possible description of reality available to each observer, which therefore corresponds to reality as "the end of enquiry". Since the information one has depends on the questions one asks, what is true can never be an absolute, unique concept.

Note that this is not confined in any way to a quantum description or simple systems. We can imagine Laplace trying to describe a deterministic universe by "knowing the positions and three momenta of all of its constituents at a given instant" - he simply cannot, because in order to be able to he would need to be outside the universe (or *be* the entire universe and perform a perfect self-measurement). Either way, since information needs to be encoded in something physical he would need to have as many degrees of freedom as the universe itself.

4 Relativity Theory

In the rest of the essay I wish to discuss two topics: (a) Why *the Simplicity Principle* could be seen as a guide in deriving better theories of physics and (b) What have our best theories taught about the world and therefore what kind of properties could we expect the hypothetical ToE to have. Let us therefore have a brief look at the best theories at present: General Relativity and Quantum Theory.

4.1 Learning from Simplicity

A theory can be superseded by a "better" one in different ways. There is a straightforward way: a theory posits the existence of something observable with specific properties, which is then directly disproven by experiment - a better theory needs to be build to accommodate for the new data. Then there is a subtle way: we build a simpler theory, one with better predictive power and look for differences in the predictions of the two theories. Better predictive power can often be achieved if we are able drop an unphysical element from the theory, which in turn, for example, leads to a reduction in the number of necessary initial conditions to predict the evolution of a system. Since these elements correspond to something superfluous, they are to be found in the class of unverifiable, unobservable ontology inferred from the theory.

As an example you could take Newtonian gravity, drop (the assumption of) the existence of absolute space that is something *by definition unobservable*, i.e. the absolute positions and momenta of particles in favor of their relative separations and momenta in the spirit of Mach; therefore reducing initial data required as input and thus increasing the predictive power of the theory. Following this line of thought, with enough patience, you could hope to eventually discover General Relativity, where the notions of absolute space and time have disappeared altogether and what remains are only dynamical fields w.r.t. other dynamical fields. As is nicely put in [7]:

> "The clean way of expressing Einstein's discovery is to say that there are no space and time: there are only dynamical objects. The world is made by dynamical fields. These do not live in, or on, spacetime: they form and exhaust reality."

There is no background geometry, there are only fields that exist w.r.t. other fields with which they interact. In Wheeler's "participatory universe" these should in turn be described in terms of information.

The gravitational field is expected to be no different from the other fields and indeed can be thought of as given by the other fields by the Einstein's equations. Since all of the other fields are shown to require a quantum description, so too should the gravitational field. We have not yet been able to write down a consistent description of it, but nevertheless there are several features the quantized gravitational field is expected to have, for example [8]:

> "[Einstein's equations with quantized matter fields] ...implies a modified ontology for general relativity, whereby matter has indefinite properties, and whereby spacetime has indefinite properties as well."

When we describe physical systems in terms of information, and especially on the quantum level, what we should keep in mind is that in addition to necessarily accommodating for indefinite properties in the ontology, we need to remember that things *exist* with respect to other things and not absolutely.

5 Quantum Theory

Quantum mechanics taught us that there is an important distinction to be made between the things that are directly observed and those that are not. In classical (i.e. non-quantum) theories - motivated by *the philosophy* that reality should be observer-independent, objective and absolute - it is assumed that there is no difference whether something inferred from the theory is directly observable or not; A particle has a definite trajectory whether we observed it or not, a cat is obviously dead or alive whether this is an observable property or not. As it turned out however, it does matter whether we have information of a system or not; the way in which things exist depends on what we know about them. From this it follows that because different observers in general have different information, their description of reality will be different.

5.1 Implications of Quantum Theory

As the ontology of the SM is less discussed than it's baby sister, Quantum Mechanics (QM), let us discuss a practical lesson the latter has taught us about reality.

Since we are describing reality in terms of information, a good place to start is Relational Quantum Mechanics (RQM) [9–11], an effort to describe QM solely in terms of information. Paraphrasing the main observation of [9, 10] let us consider the following

> *Relational Principle: Wave functions, measurement outcomes and quantum events are only defined w.r.t. an observer.*

For example, as discussed in detail in [9], upon interaction between any observer O and any observed system S the wave function of S collapses w.r.t. the observer O, but not w.r.t. to anyone else. Thus the description of S given by O i.e. $\psi_O(S)$ is different from that of someone (O') who hasn't measured S (or know the result O acquired) and hence w.r.t whom the wave function of S has not collapsed, i.e. clearly we must have $\psi_{O'}(S) \neq \psi_O(S)$[7]. The collapse of the wave function of S, i.e. the non-unitarity of the evolution of S, w.r.t. O can then be interpreted as the inability of O to have a complete description of the interacting system $O + S$, because O is denied from knowing its own quantum state (by self-measurement).

Since this applies to all observers, the important question is: *if there is an absolute, objective, true way of the world, then is the system S in a superposition state or not?*

To accommodate realism (or "strict realism") in the sense of there being an absolute, objective reality we describe, we only need to make a simple extension of the description of RQM. Since in the description of an outside observer all interactions proceed in a well defined unitary Schrödinger evolution, the only thing we need to do to make this true in an absolute, objective sense is to consider the description of a super-observer outside the universe; in his view everything is absolute, objective and unitary. This description is that of the wave function of the universe $|\Psi\rangle$ that appears in the many-worlds interpretation of quantum mechanics [12, 13][8].

To put the idea in simple terms, it is posited that reality is to be described by the totality of things: $|\Psi\rangle$ contains all possible measurement outcomes in a branching structure and the apparent subjectivity of wave functions (for example) is just an artifact of invoking an observer performing measurements and thus ending up on a specific branch of the structure. If on the other hand one describes reality via $|\Psi\rangle$, there is no problem with objectivity, and neither is there a problem with unitary time evolution, i.e. the "measurement problem" of QM disappears.

Limits of Description So, in a sense, the Everettian interpretation amounts to saying that the *correct description* is the description that an all-knowing super-observer (outside the universe) in RQM would have. Hypothetically that is, since in RQM the existence of such a super-observer is denied as unphysical. In the framework of this essay, the existence of such an unobservable super-observer is clearly against *the Simplicity Principle.* More importantly, it is against *Assumption 2* we made about

---

[7] As it is easy to misinterpret the relational nature of existence in QM, it is perhaps good to stress that whenever O and O' compare their data (this too is a quantum interaction!), the two match exactly; as stated in [10]: "It is clear that everybody sees the same elephant. More precisely: everybody hears everybody else stating that they see the same elephant they see." This consistency is guaranteed by the machinery of QM, a seemingly miraculous feat that gives one the feeling that QM has captured something fundamental about reality.

[8] In fact Wheeler, Everett's thesis advisor, called it "the relative-state formulation of quantum mechanics".

information; if information needs to correspond to something physical, *it is physically impossible to have the full description of such a super-observer* - as in Chapter 3, we see that the absolute reality is beyond a possible description, whereas there are no problems for a verificationist account. Thus, in this framework we need to discard the many-worlds interpretation in favor of RQM and accept the fact that the best description of the world includes relational notions.

Rephrasing the above: In QM the universal wave function is needed to accommodate for the *belief* that reality is objective and absolute. Without it, the argument goes, there would be no physics since the predictions would be subjective and could thus not be compared. It would appear however, that this assumption is unnecessarily strong because it fails to take into account the fact that *comparing* is itself a quantum interaction. Due to the miraculous way QM guarantees the consistency of information upon comparison one can drop the existence of an absolute reality as superfluous and still maintain the predictability of the theory *for every observer*. Once again, there is no denial of a reality outside the observers mind, there is simply the claim that what exists for different observers need not coincide unless the observers compare their notes.

6 Mathematical Reality

We have considered reality in the sense of being composed of only verifiable things or more poetically by the pragmatist motto as “the end of enquiry”. We have then considered the possibility whether such a reality could be thought of as a mathematical entity and to that end assumed that

(a) It is possible to write down a theory that is able to answer all answerable questions; able to describe all the information that is possible for an observer to have.
(b) This theory can be written in the language of mathematics.
(c) We can remove everything unobservable from this theory as superfluous.

None of the three assumptions seem impossible or even unreasonable, and in the case that all three are met, we would have a one to one correspondence between this theory and the reality as understood above. Therefore the theory is a representation of reality and hence reality can indeed be ascribed a mathematical description.

To motivate (a) and (b) we have taken the view that we should describe the world in terms of information, and to look at the limits of this description we have assumed that information needs to be encoded in something physical. In particular, it becomes physically impossible to have a full description of an absolute reality and, as also suggested by our best current theories, we should keep in mind that information and hence reality is expected to be relational rather than absolute in nature.

References

[1] C. Wright, “Truth and Objectivity” Cambridge, MA: Harvard University Press (1992).

[2] C. Hartshorne, P. Weiss and A.W. Burks (eds.), “The Collected Papers of Charles Sanders Peirce” Cambridge, MA: Harvard University Press (1958).

[3] J. A. Wheeler, “Problems In The Foundations Of Physics. Proceedings, 72nd International School Of Physics, Varenna, Italy, July 25 - August 6, 1977” ed. G. Toraldo Di Francia, Amsterdam, Netherlands: North-holland (1979) 395-492.

[4] R. Dawid, “String Theory and the Scientific Method” Cambridge University Press (2014).

[5] P. Woit, “Not even wrong: The Failure of String Theory & the Continuing Challenge to Unify the Laws of Physics” London, UK: Cape (2006).

[6] J. D. Bekenstein, Contemp. Phys. **45** (2003) 31 [quant-ph/0311049].

[7] C. Rovelli, “Philosophy and Foundations of Physics: The Ontology of Spacetime” ed. D. Dieks, Elsevier (2006) 25-36.

[8] S. Weinstein, “Einstein Meets Magritte: An Interdisciplinary Reflection on Science, Nature, Art, Human Action and Society” Vol 6, Springer Netherlands (1999) 267-279.

[9] C. Rovelli, Int. J. Theor. Phys. **35** (1996) 1637 [quant-ph/9609002].

[10] M. Smerlak and C. Rovelli, Found. Phys. **37** (2007) 427 [quant-ph/0604064].

[11] B. Van Fraassen, Found. Phys. **40** (2010) 390.

[12] H. Everett, Rev. Mod. Phys. **29** (1957) 454;
H. Everett, “The Many-Worlds Interpretation of Quantum Mechanics” (1973).

[13] D. Wallace, “The Emergent Multiverse: Quantum Theory According to the Everett Interpretation” Oxford University Press (2012).