Tachyons in monistic space-time geometry

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Abstract

It is shown that geometry of Minkowski $\mathcal{G}_{\mathrm{M}\sigma}$, constructed by a metric approach in terms of world function, differs from usual geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$, constructed as a Riemannian geometry on the basis of the linear vector space, although world functions of both geometries are the same. In $\mathcal{G}_{\mathrm{M}\sigma}$ amplitude of the tachyon world chain wobbling is infinite, and a single tachyon cannot be detected. Tachion gas in $\mathcal{G}_{\mathrm{M}\sigma}$ has very large pressure and can form halo of the dark matter around some galaxies. Tachyons do not exist in the space-time geometry $\mathcal{G}_{\mathrm{M}}$. Such cosmological problems as dark matter and dark energy (cosmological antigravitation) appear to be solved freely on the level of the space-time geometry without any additional suppositions.

1 Introduction

Space-time geometry is a ground of particle dynamics. But the boundary between geometry and particle dynamics is mobile. For instance, motion of charged particles in the space-time geometry of Minkowski is described as conditioned by interaction with electromagnetic field which is considered as a self-sufficient essence. In the 5-dimensional Kaluza-Klein space-time geometry [1] the same motion of charged particles in the electromagnetic field is considered as a free particle motion, and electromagnetic field is considered as an attribute of the space-time geometry. The same is valid for the gravitational field which is also considered as an attribute of the space-time geometry. Thus, the particle motion in macroscopic fields can be reduced to a free motion in the space-time geometry which is more complicate, than the geometry of Minkowski.

A work with one complicated essence is more simple, than the work with several simple essences, and one may try to use geometrization of physics also in microcosm. Unfortunately, the Riemannian space-time geometry which is used now for the microcosm description is insufficiently general. We show this in the example of a discrete geometry.

In the discrete geometry $\mathcal{G}_{\mathrm{d}}$ the following relation takes place

$$|d\left(P_0, P_1\right)| = \left|\sqrt{2\sigma_{\mathrm{d}}\left(P_0, P_1\right)}\right| \notin \left(0, \lambda_0\right), \quad \forall P_0, P_1 \in \Omega \tag{1.1}$$

Here $\sigma_{\mathrm{d}}\left(P_0, P_1\right) = \frac{1}{2}d^2\left(P_0, P_1\right)$ is the world function and d is the distance between points $P_0, P_1 \in \Omega$. Here Ω is the set of points (events) in the space-time. It means that in the space-time there are no distances, which are shorter, than elementary length λ_0. The value $\sigma_{\mathrm{d}}\left(P_0, P_1\right) = 0$ is possible, if $P_0 = P_1$, or P_1 lies on the light cone with the vortex at point P_0. Condition (1.1) is a constraint on the world function σ_{d}, but not on the set Ω, although one considers usually (1.1) as restriction on the set Ω. Considering Euclidean geometry (or geometry of Minkowski) on a lattice, one thinks that he considers a discrete geometry, because in this case the restriction (1.1) is fulfilled.

We shall consider condition (1.1) as a condition for the world function, given on the manifold of Minkowski Ω. In this case the simplest discrete analog $\mathcal{G}_{\mathrm{d}}$ of the geometry $\mathcal{G}_{\mathrm{M}}$ of Minkowski is described by the world function

$$\sigma_{\mathrm{d}}\left(P_0, P_1\right) = \sigma_{\mathrm{M}}\left(P_0, P_1\right) + \frac{\lambda_0^2}{2}\operatorname{sgn}\left(\sigma_{\mathrm{M}}\left(P_0, P_1\right)\right) \tag{1.2}$$

where σ_{M} is the world function of $\mathcal{G}_{\mathrm{M}}$. There exist three representation of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ [2]: E-conception, V-conception and σ-conception. Contemporary physicists use mainly V-conception which equipped by such a structure as linear vector space $\mathcal{L}$. The Riemannian geometry is a set of many infinitesimal Euclidean spaces with structure $\mathcal{L}$ on each space. A coordinate system, which is an attribute of structure $\mathcal{L}$, unites all these infinitesimal spaces. Coordinateless description is impossible in the V-conception.

Generalized geometries constructed by means of the σ-conception do not contain the linear vector space $\mathcal{L}$, generally speaking. In the σ-conception the geometry is described by the world function σ. This structure (world function σ) does not contain any constraints. The world function σ

$$\sigma: \quad \Omega \times \Omega \to \mathbb{R}, \quad \sigma\left(P, Q\right) = \sigma\left(Q, P\right), \quad \sigma\left(P, P\right) = 0, \quad P, Q \notin \Omega \tag{1.3}$$

does not contain restrictions characteristic for structure $\mathcal{L}$. In particular, the discrete geometry description can be constructed only on the basis of the σ-representation, because one cannot introduce structure $\mathcal{L}$ in the discrete geometry $\mathcal{G}_{\mathrm{d}}$. Description of $\mathcal{G}_{\mathrm{d}}$ is dimensionless, because one cannot introduce metric dimension n_{m}. The metric dimension n_{m} is the maximal number of linear independent vectors, which are attributes of the structure $\mathcal{L}$. Metric dimension n_{m} differs, in general, from the coordinate dimension n_{c} which is the number of coordinates used for labelling of points of the set Ω.

The coordinate dimension n_{c} has no relation to geometry. It may be arbitrary. But in the geometry of Minkowski as well as in the Riemannian one, where there is the structure $\mathcal{L}$, one chooses usually $n_{\mathrm{c}} = n_{\mathrm{m}}$. Then in the discrete geometry $\mathcal{G}_{\mathrm{d}}$ which is defined via $\mathcal{G}_{\mathrm{M}}$ by means of (1.2) one obtains the same n_{c} and the same coordinate system as in $\mathcal{G}_{\mathrm{M}}$, although in $\mathcal{G}_{\mathrm{d}}$ there is no metric dimension n_{m}. (see details in [3])

Obtained from the discrete geometry $\mathcal{G}_{\mathrm{d}}$ in the limit $\lambda_0 \to 0$, the σ-Minkowski geometry $\mathcal{G}_{\mathrm{M}\sigma}$ differs from the geometry of Minkowski $\mathcal{G}_{\mathrm{M}}$, constructed as Riemannian geometry, although both geometries $\mathcal{G}_{\mathrm{M}\sigma}$ and $\mathcal{G}_{\mathrm{M}}$ are described by the same world function

$$\sigma_{\mathrm{M}}(x,x') = \sigma_{\mathrm{M}\sigma}(x,x') = \frac{1}{2} g_{ik}\left(x^i - x'^i\right)\left(x^k - x'^k\right), \quad g_{ik} = \mathrm{diag}\,(1,-1,-1,-1) \tag{1.4}$$

The difference between $\mathcal{G}_{\mathrm{M}\sigma}$ and $\mathcal{G}_{\mathrm{M}}$ lies in the definition of the equality (equivalence) of two vectors.

Two vectors $\mathbf{P}_0\mathbf{P}_1 = \left\{x^0, \mathbf{x}\right\}$ and $\mathbf{Q}_0\mathbf{Q}_1 = \left\{x'^0, \mathbf{x}'\right\}$ are equivalent In $\mathcal{G}_{\mathrm{M}}$, if their coordinates are equal

$$x^0 = x'^0, \quad \mathbf{x} = \mathbf{x}' \tag{1.5}$$

The same vectors are equivalent in $\mathcal{G}_{\mathrm{M}\sigma}$, if

$$(\mathbf{P}_0\mathbf{P}_1 \mathrm{eqv} \mathbf{Q}_0\mathbf{Q}_1): \quad |\mathbf{P}_0\mathbf{P}_1| = .\,|\mathbf{Q}_0\mathbf{Q}_1| \wedge (\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = |\mathbf{P}_0\mathbf{P}_1| \cdot .\,|\mathbf{Q}_0\mathbf{Q}_1| \tag{1.6}$$

where

$$(\mathbf{P}_0\mathbf{P}_1.\mathbf{Q}_0\mathbf{Q}_1) = \sigma(P_0, Q_1) + \sigma(P_1, Q_0) - \sigma(P_0, Q_0) - \sigma(P_0, Q_0) \tag{1.7}$$

$$|\mathbf{P}_0\mathbf{P}_1|^2 = 2\sigma(P_0, P_1) \tag{1.8}$$

Definition (1.5) contains four equations, whereas definition (1.6) contains only two equations. Nevertheless for timelike vectors ($\sigma_{\mathrm{M}}(P_0, P_1) > 0$) solution of equations (1.6) is unique and both definitions coincide. In the case of spacelike vectors ($\sigma_{\mathrm{M}}(P_0, P_1) < 0$) definitions (1.5), (1.6) are different, because there are many vectors $\mathbf{Q}_0\mathbf{Q}_1$, $\mathbf{Q}_0\mathbf{Q}_1'$,$\mathbf{Q}_0\mathbf{Q}_1''$ which are equivalent to vector $\mathbf{P}_0\mathbf{P}_1$, but they are not equivalent between themselves.

What of two geometries $\mathcal{G}_{\mathrm{M}\sigma}$ or $\mathcal{G}_{\mathrm{M}}$ is a real space-time geometry? Considering only particles with timelike world lines (tardions), one cannot answer this question. Contemporary theory does not consider particles with spacelike world lines (tachyons). It is supposed that tachyons do not exist, because one failed to detect them.

From logical viewpoint the space-time geometry $\mathcal{G}_{\mathrm{M}\sigma}$ is to be valid, because it may be formulated without a reference to the way of description (coordinate system). From experimental view point a single tachyon cannot be detected in $\mathcal{G}_{\mathrm{M}\sigma}$ because of infinite wobbling, when two adjacent points of the tachyon world chain are divided by infinite spatial distance and by the temporal distance of the same order. However, this infinite wobbling creates very large pressure in the tachyon gas. The tachyon gas may create large galos around some galaxies. Gravitational field of such galos is observed experimentally, but radiation of the galos is not observed. As a result one consider that galos consist of dark (hidden) matter. Explanation of the dark matter nature is one of difficult problems of contemporary cosmology.

Conventional formulation of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$ is pluralistic in the sense, that $\mathcal{G}_{\mathrm{E}}$ is formulated in terms of several fundamental quantities (point, dimension, straight line, angle etc.). Properties of fundamental quantities are formulated in the form of axioms. Properties of other geometrical objects are deduced from axioms and definitions of these objects in terms of fundamental quantities. However, the proper Euclidean geometry $\mathcal{G}_{\rm E}$ may be formulated as a monistic conception (σ-conception), where there is only one fundamental concept world function $\sigma = \frac{1}{2}d^2$, where d is a distance between any to points of the set, where the geometry is given. Such an approach is known as a metric approach to geometry [4, 5]. However, consequent metric approach was not formulated. At the metric approach all geometric objects and concepts can be expressed via the world function σ.

Metric approach in the Euclidean geometry admits a coordinateless description of geometric objects. For instance, segment $\mathcal{T}_{[P_0P_1]}$ of the straight line between points P_0 and P_1 is described as a set of points R, defined by the relation

$$\mathcal{T}_{[P_0P_1]} = \{R|d(P_0, R) + d(P_1, R) = d(P_0, P_1)\} \tag{1.9}$$

Metric approach admits one to obtain generalized geometries as a result of a deformation of the proper Euclidean geometry $\mathcal{G}_{\rm E}$, when the world function $\sigma_{\rm E}$ of $\mathcal{G}_{\rm E}$ is substituted by another world function σ of a generalized geometry $\mathcal{G}$. This way of the generalized geometry construction is essentially simpler, than conventional method, when the statements of the generalized geometry are deduced from modified axioms, and the modified axioms are to be compatible.

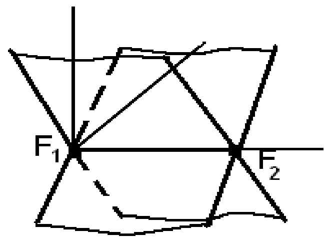
Two regions $\mathcal{R}_1$ and $\mathcal{R}_2$ of the space-time may have different geometries described relatively be world functions σ_1 and σ_2. Let a physical body, having the shape $G_1 = g_1(\sigma_1)$ in the region $\mathcal{R}_1$ evolves as a free moving body and appears in the region $\mathcal{R}_2$ with other geometry. The shape of the body is described now as $G_2 = g_2(\sigma_2)$. How are functions g_1 and g_2 connected? As far as the physical geometry is a monistic construction, which is described completely by the only quantity (world function), the only possibility may take place

$$g_1(\sigma) = g_2(\sigma) = g(\sigma) \tag{1.10}$$

In the pluralistic space-time geometry identification of the straight segment $\mathcal{T}_{[P_0P_1]}$ meets difficulties in the discrete space-time geometry, described by world function (1.2).

The discrete geometry $\mathcal{G}_{\rm d}$ may be constructed only at the metric approach. In $\mathcal{G}_{\rm d}$ all segments are three-dimensional surfaces. Timelike ($\sigma_{\rm d}(P_0, P_1) \gg \lambda_0^2 > 0$) segments $\mathcal{T}_{[P_0P_1]}$ are tubes of radius ~λ_0, whereas spacelike segments $\mathcal{T}_{[P_0P_1]}$ are infinite 3-dimensional surfaces. This result follows from the fact, that, in general, one equation (1.9) generates in the 4-dimensional space a 3-dimensional surface. If $\lambda_0 \to 0$ any timelike segment $\mathcal{T}_{[P_0P_1]}$ turns into one-dimensional segment in $\mathcal{G}_{\rm M\sigma}$, whereas spacelike segment $\mathcal{T}_{[P_0P_1]}$ turns to two 3-dimensional planes. In $\mathcal{G}_{\rm M\sigma}$ the spacelike segment $\mathcal{T}_{[F_1F_2]}$ has the shape, shown in the figure

The fact, that in a discrete geometry $\mathcal{G}_{\rm d}$ the segments $\mathcal{T}_{[P_0P_1]}$ are three-dimensional surfaces, is incompatible with the conventional pluralistic approach to geometry, where all segments of a straight line are one-dimensional. Blumental [5], who tries to construct the distant geometry by means of the metric



approach, could not overcome the preconception about the one-dimensionality of the straight line. He defined the straight line as a result of a continuous mapping of numerical segment [0,1] onto Ω. Continuous mapping cannot be defined in terms of the world function. As a result the approach of Blumental was not a consequent metric approach. He could not construct a discrete geometry $\mathcal{G}_{\mathrm{d}}$, because he failed to overcome the preconception about the one-dimensionality of the straight line segment.

The physical geometry $\mathcal{G}$, i.e. a generalized geometry obtained from $\mathcal{G}_{\mathrm{E}}$ by means of a deformation (replacement of σ_{E} by σ), has been constructed only in the end of the twentieth century [6, 7].To overcome the reconception on one-dimensionality I had wasted thirty years. Arguments in favor of definition of $\mathcal{T}_{[P_0 P_1]}$ in the form of (1.9) in all generalized geometries was the fact that world lines of annular strings are three-dimensional tubes. Besides, in the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ world line of a particle is replaced by a world chain, whose links are segments $\mathcal{T}_{[P_s P_{s+1}]}$. Length $\mu = |\mathbf{P}_s \mathbf{P}_{s+1}|$ satisfies the restriction $|\mu| > \lambda_0$. The world chain of a particle is stochastic. It wobbles with amplitude of the order of λ_0. Statistical description of stochastic world chain coincides with the quantum description in terms of the Schrödinger equation [6], provided $\lambda_0^2 = \hbar/(bc)$, where $\hbar$ is the quantum constant c is the speed of the light and b is some universal constant, connecting the particle mass m with the geometric mass $\mu = |\mathbf{P}_s \mathbf{P}_{s+1}|$ by means of the relation

$$m = b\mu \tag{1.11}$$

This relation introduces the quantum constant as a geometric characteristic of the space-time.

The monistic conception of the space-time geometry is more valid, than the conventional pluralistic conception, because it is a more general conception, which can be presented in the coordinateless form. Construction of the monistic conception is more simple, because it does not need invention of geometric axioms, test of their compatibility and proofs of numerous theorems.

According to the pluralistic conception the space-time geometry is a kind of pseudo-Riemannian geometry, and general relativity is based on this supposition. In reality, the set of Riemannian geometries is only small part of possible space-time geometries, constructed on the basis of metric approach to geometry. Real space-time geometry is not a Riemannian geometry, in general. A use of the real space-time geometry in the construction of the general relativity is conditioned by the circumstance, that formalism of a physical geometry and, in particular, that of a discrete geometry was not known in the twentieth century.

Extended general relativity (EGR) [8] is a theory of the space-time, constructed under the supposition that the space-time geometry is physical geometry. In reality the space-time geometry is a discrete geometry, but in cosmological scale the geometry discreteness is of no importance, whereas the metric approach to geometry is important, because it enlarges the geometry capacities. In framework of EGR such cosmological problems as dark matter and dark energy do not appear. Appearance of these problems in conventional general relativity is connected with unfounded restriction by Riemannian space-time geometries. When this restriction is removed the problem of the dark matter does not arise.

In EGR the homogeneous dust sphere cannot collapse into a black hole [8, 9]. Calculation [9] shows that inside the dust cloud appears the induced antigravitation, which prevents from formation of the black hole. There are different version of the dark energy nature [10, 11], but all these versions try to explain cosmic antigravitation which is a real reason of the accelerated expansion of universe. Conventional general relativity, based on the Riemannian space-time geometry can explain antigravitation only by means of negative mass, by negative pressure or by so-called Λ-term, taken with a proper sign.

One may expect, that proper cosmological model will be able to explain accelerated expansion of universe on the basis of EGR. But necessary calculations have not been produced yet.

We consider here explanation of such a phenomenon as the dark matter. The dark matter has such properties: (1) single particles do interact with the electromagnetic field and single particles of the dark matter cannot be discovered, (2) dark matter may form a huge halo around some galaxies. Gravitational field inside the halo drops rather slowly, and rotational velocities of the stars do not decreased with increase of distance from the center of the galaxy. If the dark matter is a gas, the pressure of this gas mast be very large, in order such a gas can form a huge halo around the galaxy. A tachyon gas has such properties, and it may be treated as a dark matter.

Tachyon is particle, whose velocity is larger, that the speed of the light. It is supposed that such particles are impossible. Such particles have not been discovered. But trying to detect tachyons experimentally, one supposed, that world lines of tachyons are spacelike one-dimensional curves. The tachyon world line is to have such a shape at the conventional pluralistic approach to geometry, where spacelike segment of straight line is a one dimensional curve. At the metric approach to geometry the spacelike segment $\mathcal{T}_{[F_1F_2]}$ is defined by the relation (1.9).

The world chain

$$\mathcal{C} = \bigcup_s \mathcal{T}_{[P_s P_{s+1}]}, \quad |\mathbf{P}_s\mathbf{P}_{s+1}|^2 = 2\sigma\left(P_s, P_{s+1}\right) = \mu^2 = \text{const} < 0 \tag{1.12}$$

consisting of links $\mathcal{T}_{[P_s P_{s+1}]}$, defined by (1.9), wobbles with infinite amplitude, and this wobbling is a reason why a single tachyon cannot detected. This wobbling is conditioned by the fact, that any segment $\mathcal{T}_{[P_s P_{s+1}]}$ is a three-dimensional surface. The wobbling amplitude is infinite in the geometry $\mathcal{G}_{\mathrm{M}\sigma}$ and in the discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$. However, gas of many tachyons may be detected by its gravitational influence.

We are interested in pressure inside tachyon gas, because it describes parameters of halo of tachyon gas. The pressure P is defined by the relation

$$P = \frac{1}{3}\rho\left(\left\langle \mathbf{u}^2\right\rangle - \left\langle \mathbf{u}\right\rangle^2\right) \tag{1.13}$$

where ρ is the mass density of tachyon gas, $\langle \mathbf{u}\rangle$ is the mean velocity of tachyons in the gas. The quantity $\left\langle \mathbf{u}^2\right\rangle - \left\langle \mathbf{u}\right\rangle^2$ is the velocity dispersion in the tachyon gas. To obtain the mean quantities, one needs to calculate the velocity distribution in the gas. In usual gases this distribution is formed as a result of molecular collisions. In the tachyon gas the velocity distribution is obtained as a result of indeterministic motion of single tachyons. Dispersion of the velocity distribution appears so much, that the tachyon collisions (if they take place) cannot change this dispersion essentially.

2 Dynamics of tachyons and pressure of the tachyon gas

Let us consider the free motion of a tachyon. It is described by coordinateless dynamic equation (1.12) equipped by additional constraint that adjacent vectors $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$ are equivalent ($\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}$). it means that the length of vectors are equal and vectors are in parallel. It means, that $\mathbf{P}_s\mathbf{P}_{s+1}$ and $\mathbf{P}_{s+1}\mathbf{P}_{s+2}$, are equivalent for all sThe relation of equivalence ($\mathbf{P}_s\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}$), which is defined by relations (1.5) - (1.8). These equation are written in the form

$$\sigma\left(P_s, P_{s+1}\right) = \sigma\left(P_{s+1}, P_{s+2}\right), \quad \sigma\left(P_s, P_{s+2}\right) = 4\sigma\left(P_s, P_{s+1}\right) \tag{2.1}$$

Equations (2.1), are dynamic equations for a free tachyon, written in the coordinateless form. Note, that these equations describe prescription for construction of the straight line be means of only compasses. Here this prescription is applied in $\mathcal{G}_{\mathrm{M}\sigma}$ or in the discrete geometry $\mathcal{G}_{\mathrm{d}}$.

Solution of two equations (2.1) in the 4-dimensional space is not unique, in general. Let us set $s = 0$ and chose the coordinate system in such a way, that

$$P_0 = \{x_0, \mathbf{x}\}, \quad P_1 = \{x_0 + p_0, \mathbf{x} + \mathbf{p}\}, \quad P_2 = \{x_0 + 2p_0 + \alpha_0, \mathbf{x} + 2\mathbf{p} + \boldsymbol{\alpha}\} \tag{2.2}$$

where the quantities p_0,$\mathbf{p}$ are given, and 4-vector $\alpha = \{\alpha_0, \boldsymbol{\alpha}\}$ is to be determined as a solution of equations (2.1). The 4-vector $\alpha = \{\alpha_0, \boldsymbol{\alpha}\}$ is a discrete analog of the acceleration vector. The world function in (2.1) is the world function of $\mathcal{G}_{\mathrm{d}}$ (1.2), where instead of the world function σ_{M} one uses

$$\sigma_{\mathrm{M}}(x, x') = \frac{1}{2}\left(\left(c^2 - 2V(\mathbf{y})\right)(x_0 - x_0')^2 - (\mathbf{x} - \mathbf{x}')^2\right), \quad \mathbf{y} = \frac{\mathbf{x} + \mathbf{x}'}{2} \tag{2.3}$$

Such a world function σ_{M} appears in the extended general relativity [8, 9] with slight gravitational field described by the gravitational potential $V(\mathbf{x})$.

For tachyons ($\mu^2 < 0$) dynamic equations (2.1) are written in the form

$$\left(c^2 - 2V\right)(p_0 + \alpha_0)^2 - (\mathbf{p} + \boldsymbol{\alpha})^2 = \left(c^2 - 2V\right)p_0^2 - \mathbf{p}^2 \tag{2.4}$$

$$\left(c^2 - 2V\right)(2p_0 + \alpha_0)^2 - (2\mathbf{p} + \boldsymbol{\alpha})^2 = 4\left(\left(c^2 - 2V\right)p_0^2 - \mathbf{p}^2\right) - 3\lambda_0^2, \tag{2.5}$$

After transformation these equations take the form

$$\alpha_0 = \frac{2\boldsymbol{\alpha}\mathbf{p} + 3\varepsilon\lambda_0^2}{2p_0\left(c^2 - 2V\right)} \tag{2.6}$$

$$\frac{\left(v^2 - \left(c^2 - 2V\right)\right)}{\left(c^2 - 2V\right)}\left(\alpha_{\|} - \frac{\frac{3}{2}\lambda_0^2 p}{\left(p^2 - p_0^2\left(c^2 - 2V\right)\right)}\right)^2 - \boldsymbol{\alpha}_{\perp}^2 = r^2, \quad v = \frac{p}{p_0} \tag{2.7}$$

where the quantity r is defined by the relation

$$r^2 = 3\lambda_0^2 - \frac{9}{4}\frac{\lambda_0^4}{p_0^2\left(v^2 - \left(c^2 - 2V\right)\right)}, \quad v = \frac{p}{p_0} \tag{2.8}$$

$$\boldsymbol{\alpha}_{\|} = \mathbf{p}\frac{(\boldsymbol{\alpha}\mathbf{p})}{\mathbf{p}^2}, \quad \boldsymbol{\alpha}_{\perp} = \boldsymbol{\alpha} - \boldsymbol{\alpha}_{\|}, \quad \alpha_{\|}^2 = \frac{(\boldsymbol{\alpha}\mathbf{p})^2}{\mathbf{p}^2}, \quad \alpha_{\|} = \frac{\boldsymbol{\alpha}\mathbf{p}}{p}, \quad \mathbf{p}^2 = p^2 \tag{2.9}$$

Here $\boldsymbol{\alpha}_{\|}$ is the component of 3-vector $\boldsymbol{\alpha}$ which is in parallel with the vector $\mathbf{p}$, whereas $\boldsymbol{\alpha}_{\perp}$ is the components of 3-vector $\boldsymbol{\alpha}$, which are perpendicular to the vector $\mathbf{p}$.

Vector $\mathbf{v} = \mathbf{p}/p_0$ may be interpreted as 3-velocity of a particle described by world chain (1.12). In the case of continuous geometry ($\lambda_0 \to 0$) the quantity $\mathbf{v}$ is the usual 3-velocity.

Solution of equation (2.7) is not unique. Solution of (2.7), gives (see details in [3]

$$\alpha_{\|} = \frac{3\lambda_0^2}{2p_0\left(v^2 - \left(c^2 - 2V\right)\right)}v + \frac{r\sqrt{c^2 - 2V}}{\sqrt{v^2 - \left(c^2 - 2V\right)}}\cosh\theta \tag{2.10}$$

$$\alpha_{\perp 1} = r\sinh\theta\cos\phi, \quad \alpha_{\perp 2} = r\sinh\theta\sin\phi \tag{2.11}$$

Here θ, ϕ are arbitrary real numbers, α_0 is defined by (2.6) The wobbling amplitude (the value of $|\boldsymbol{\alpha}|$) is infinite because of functions cosh and sinh. The wobbling amplitude is infinite even in the case of space-time geometry close to $\mathcal{G}_{\mathrm{M}\sigma}$, when $\lambda_0 = 0$ and $r = 0$.

Components of the tachyon velocity $\mathbf{u}$ are defined by relations

$$\mathbf{u} = \frac{\mathbf{p} + \boldsymbol{\alpha}}{p_0 + \alpha_0} \tag{2.12}$$

Averaging components of $\mathbf{u}$, one assumes, that all solutions (directions) are equiprobable. The quantities p_0, p are fixed at the averaging. Then one uses the formula

$$\langle \mathbf{u} \rangle = \lim_{\Theta \to \infty} \frac{1}{N} \int_{-\Theta}^{\Theta} \cosh \theta d\theta \int_0^{2\pi} \mathbf{u} d\phi, \quad N = 4\pi \sinh \Theta \tag{2.13}$$

where symbol $\langle ... \rangle$ means averaging. One obtains as a result of averaging (see details in [3])

$$\langle \mathbf{u}_\perp \rangle = 0, \quad \langle u_\parallel \rangle = \frac{\left(c^2 - 2V\right)}{v} < c \tag{2.14}$$

$$\left\langle u_\parallel^2 \right\rangle = \langle u_\parallel \rangle^2 = \langle u \rangle^2 = \frac{\left(c^2 - 2V\right)^2}{v^2} < c^2 \tag{2.15}$$

$$\left\langle \mathbf{u}_\perp^2 \right\rangle = \frac{\left(c^2 - 2V\right)\left(v^2 - c^2 + 2V\right)}{v^2} < c^2, \quad \left\langle \mathbf{u}^2 \right\rangle = c^2 - 2V \tag{2.16}$$

According to (2.14) - (2.16) one obtains

$$\left\langle \mathbf{u}^2 \right\rangle - \langle \mathbf{u} \rangle^2 = \left\langle \mathbf{u}_\perp^2 \right\rangle = \left(c^2 - 2V\right)\left(1 - \frac{c^2 - 2V}{v^2}\right) < c^2 \tag{2.17}$$

In the limit $v = p/p_0 \to \infty$

$$\left\langle \mathbf{u}^2 \right\rangle - \langle \mathbf{u} \rangle^2 = \left(c^2 - 2V\right) \tag{2.18}$$

In the gravitational field of a galaxy the tachyon gas may be at rest, if the balance condition is fulfilled

$$\nabla P = \rho \nabla V \tag{2.19}$$

where the pressure P is given by relation (1.13). According (2.17) this condition is written in the form

$$\frac{1}{3}\left(c^2 - 2V(\mathbf{x})\right) \nabla \rho = \frac{5}{3} \rho \nabla V(\mathbf{x}) \tag{2.20}$$

Equation (2.20) is integrated in the form

$$\rho = \frac{\rho_0 c^5}{\sqrt{\left|c^2 - 2V(\mathbf{x})\right|^5}} \tag{2.21}$$

Here $\rho_0 = \text{const}$. If the gravitational field is not strong and $V(r) \ll c^2$, one obtains

$$V(r) = \frac{GM}{r} + \frac{4\pi G}{3}\rho_0 r^2 \quad \rho(r) = \frac{\rho_0 c^5}{\sqrt{\left|c^2 - \frac{2GM}{r} - \frac{8\pi}{3} G \rho_0 r^2\right|^5}}$$

Here G is the gravitational constant and M is the mass of the galaxy (without tachyon gas). Thus, one can see that tachyon gas can form galos with almost constant mass density.

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