Continuous Spacetime From Discrete Holographic Models

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Introduction

Our current theories of nature, as well as physics departments across the world, are organized by length scales, or equivalently (since our world is quantum mechanical), by energy scales. In this essay I'll attempt to apply this organizing principle to the set of questions posed in this essay competition: Is the universe continuous or discrete? Can we have models of reality which resemble universal models of computation? Are our theories of fundamental physics, which invariably use the language of continuous variables, only a disguise for a deeper level of reality which is fundamentally discrete?

Many attempts have been made to demonstrate fundamental discreteness at short distances, with the assumption that this modifies only aspects of spacetime invisible to us, without contradicting anything we currently know about nature. If spacetime is fundamentally discrete, with very fine grained structure replacing our continuous spacetime, it seems obvious that on very large distance scales we recover our usual description of space and time, with all the continuous structures we know and love. But, do we really?

I'll start this essay by pointing out the challenge this research direction is faced with. Ultimately, much of the difficulty can be traced to the fundamental differences between space and time in special relativity. I'll argue that Lorentz invariance, which is an irreplaceable part of our theories of nature, is very likely irreparably damaged by any such violent truncation of spacetime at short distances.

This leads the way to the second part of the essay, in which I will discuss another type of discreteness, of a more novel and profound type. I'll demonstrate that fundamental discreteness, distinct from merely a short distance cutoff, becomes a possibility when we introduce a new element, that of *holography*. In the context of black hole quantum mechanics, this idea is helpful in reconciling the discreteness of the black hole spectrum with the continuous spacetime implied by the principle of equivalence.

This holographic discreteness does not manifest itself as a truncation of short distance physics. Rather, it is encoded in subtle and interesting correlations in long distance physics, in a manner which is not yet completely understood. Taking our clues from this example, the main conclusion of this essay is that holography may hold the key to construction of discrete models of our universe.

Lorentz Invariance

Space versus Spacetime

We start our discussion by considering theories in flat spacetime, with the line element:

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

This is fairly similar to the Euclidean three dimensional space in which we live, and a lot of our geometrical intuition carries through unchanged to the more complicated example of four dimensional *spacetime*. The crucial difference is the indefinite metric: in spacetime some distances square to a negative number, and are then of distinct nature from what we normally think of as distances between points in space. In what follows we will see that the difference between space(like) and time(like) underlies many counter intuitive ideas and results.

The symmetries of this line element are the Lorentz transformations. They consist of rotations around the three coordinate axes, and three boosts along the three independent directions. Together with spacetime translations they form the Poincare group, though here we only need to discuss the (sometimes called homogeneous) Lorentz transformations.

The difference between space and time manifests itself in the difference between boosts and rotations. The crucial difference is that rotations are periodic: rotate by an angle 2π, in some direction and around some axis, and you find yourself at the same point again. On the other hand there is no periodic structure to boosts, which simply increase the velocity in one direction. In the mathematical lingo we say that Lorentz transformations are "non-compact", as the boost can be any unbounded real number, and all those choices are different. The consequences of this fact will be important in what follows.

Effective Field Theories

Having discussed the basic structure of Lorentz transformations, we are now ready to discuss some of their consequences for physics. Ultimately we are interested in theories with quantized gravity, which is an essential part of the real world, and will be an important part of our story. However, it is useful to start by discussing theories where gravity is not quantized, and spacetime is a fixed arena where physics takes place.

The reason we can get away with it is simple: quantum gravity effects are expected to take place, by and large, on a fantastically short distance scale, the Planck length $l_p \sim 10^{-35}$ meters. On distance scales larger than that, which include all distance scales we can experimentally probe, we expect the theory of our universe to be well-described by some *effective field theory*. It turns out that this information alone is sufficient to give us valuable hints about the physics of the Planck scale. As we are short on direct experimental information about that physical regime, it is prudent to take any hint we can get very seriously.

In effective quantum field theories, the locations in both space and time are labels which characterize potential observations, or physical effects. These effects are encoded by *local operators*: operators in some Hilbert space which have spacetime labels, generically denoted as $\hat{O}(x,t)$. The operators are polynomials in the fundamental fields which obey all symmetries and gauge redundancies of the system. For example, we can take the matter content of the standard model - quarks and leptons, and all the force carriers, and build all possible operators which are consistent with the gauge invariance of the standard model $SU(3) \times SU(2) \times U(1)$, and all the symmetries of the standard model. Including, crucially, Lorentz invariance. This then classifies all possible effects consistent with the structure of the standard model of elementary particle physics.

We can further classify operators according to their importance at low energies. This classification can be done systematically using a tool called the renormalization group, which is a generalization of ordinary dimensional analysis to include quantum effects. Genereally, there are three types of operators, or physical effects:

- *Irrelevant operators*: Those are effects whose importance decreases at low energies. They are therefore unlikely to influence ordinary physics, unless we have access to higher energies (e.g. by building large accelerators) or we develop measuring devices sensitive even to tiny effects.

- *Marginal Operators*: Those are effects whose order of magnitude is roughly the same, as we flow from the fundamental scale to everyday energies.

- *Relevant Operaors*: Those are effects whose importance increases at low energies. Even tiny effects of that form at very short distances accumulate to huge effects at observable energies. Said differently - these are the effects which give us most information about short distance physics.

When discussing experimental probes of our world, or suggestions for theories that extend our current knowledge, this catalogue of possibilities is invaluable. The strength of this classification is that it is *model independent*. A specific dynamical theory, at all energy scales, is needed to calculate the strength of all potential effects. However, the above classification amount to a universal estimate of their strength.

Lorentz Violations

We are now ready to discuss possible violation of Lorentz symmetry at short distances, using the tools of effective field theory to organize the discussion.

First, a point of clarification. We discuss here potential violations of Lorentz invariance at high energies, or short distances. Of course, the world around us is not Lorentz invariant - Lorentz invariance is a symmetry of empty space, and our world is not empty. It has building and planets, cosmic microwave background and many other structures which break Lorentz Invariance *spontaneously*. This spontaneous breaking of the symmetry is relevant to the way we view the world in our everyday life, but is not relevant for high energy physics. Collisions in particle accelerators take plave on short enough distance and time scvales, as to be insensitive to the location of the accelerator on earth. Such collisions reveal Lorentz invariance to very high accuracy, and that symmetry at accessible energy scales is the starting point of our discussion.

Moving to even higher energies, beyond our current experimental capabilities, we are faced with the question: if Lorentz invariance is not a symmetry of our fundamental, short distance theory, can it emerge as an approximate symmetry at longer distances? Using the tools of effective field theories, we can discuss the issue without committing to a specific model of high energy physics.

There are known examples of such "accidental symmetries. Perhaps the most well-known one is baryon number in the standard model. We live in a world where there are more baryons (protons and neutrons) than anti-baryons, so we know for sure that our fundamental theory distinguishes baryons from anti-baryons, baryon number cannot be a symmetry. On the other hand, despite years of experiments, we have not been able to observe directly a process in which baryon number is not conserved. This presents us with a conundrum.

This problem is nicely resolved in the standard model. It turns out that when you write all possible operators, all possible physical effects, consistent with the symmetries and gauge redundancies of the standard model, it so happens that the only effects violating baryon number are *irrelevant*. Meaning, they are more important at high energies (say in the hot and dense early universe) than at our mundane low energy universe. happily, this is precisely what we observe.

We see from this example that the emergence of symmetries at low energies is a very sensitive question, depending on many details. The precise matter content and symmetries of the standard model are required for this miracle. Indeed, in many extensions of the standard model the miracle of accidental baryon number conservation at low energies is spoiled.

Similar story holds for Lorentz symmetry in some systems. There are condensed matter systems in which Lorentz invariance can emerge at long distances. In all such cases it can be understood as an "accidental symmetry in the sense we just described. The system in question has the matter content and symmetries which prevent any relevant and marginal effects which violate the symmetry.

This sets the stage to discussing the question of Lorentz invariance in our world, and in particular in particle experiments. Whatever theory ultimately describes our world, it has to contain at the very least the standard model of particle physics. The possible physical effects in the standard model, were it to violate Lorentz symmetry, were studied and classified (with certain assumptions), for example in [1]. The result is clear and unambiguous: with the matter content and symmetries of the standard model, there are numerous operators which are either marginal or relevant. Each one of these operators corresponds to a potential physical effect, which is expected to be visible in many experiments if Lorentz invariance were to be violated at the Planck scale, but is not seen.

Thus, based on the arguments in this section, we expect that in any theory complex enough to contain the standard model of particle physics, Lorentz invariance is a symmetry up to, and beyond, the length scale characteristic of quantum gravity. Perhaps there are clever ways to evade that conclusion, but in the rest of this essay I take this conclusion seriously, and concentrate on its implications for ideas of discrete structure of spacetime.

The Challenge For Short Distance Discreteness

On the face of it, the view of spacetime as fundamentally discrete at short distances is an eminently reasonable one. All continuous media in our everyday experience reveal themselves to be granular when probed at short distances. Water appears as a fluid to our eyes, but we know this description only emerges at long distances, it is an illusion forced on us by observing electromagnetic waves of certain wavelengths only. When probed with more sensitive tools, water reveals its true nature as being made out of tiny discrete ingredients, water molecules. Surely then, spacetime could be discrete on a fantastically short distance scales, such as the Planck scale. How can we possible able to tell the difference with our blunt tools (which are, in this case, gigantic particle accelerators)?

The previous section ended with the conclusion that such intuition, reasonable as it sounds at first, has something to prove, some threshold to pass. Models which impose short distance discreteness on spacetime are likely conflict with Lorentz invariance, even if the scale of this discreteness is exceedingly small. Quantum effects tend to magnify Planck scale violations of Lorentz invariance, and thus such violation is likely incompatible with any of the many classical tests of special relativity. I will make the assumption then that any model describing reality, including all the complications of particle physics, is Lorentz invariant at the Planck scale.

To see the tension between this requirement and fundamental discreteness, imagine at first the most naive possible discrete model, the assumption that our space is arranged into a cubic lattice of sides whose length is precisely l_p. It is immediately clear that this statement depends on your reference frame, your friend in possession of the latest spaceship model will zoom by this lattice and will see it Lorentz contracted. The principle of relativity is clearly at odds with this very naive picture.

We can easily see that such difficulty arises in any model in which spacetime is a fixed classical background, albeit a discrete one. Consider the set of all possible locations that could be measured in our model. For simplicity let us restrict ourselves to locations within some huge spacetime interval, say we look at all possible locations in the milky way in the last billion years. If spacetime is discrete the number of different outcomes whenever location is measured is finite. Huge number, to be sure, but not infinite. On the other hand, the set of all those numbers is something all inertial observers should be able to agree on. In other words, it forms a representation of the Lorentz group.

And therein lies the problem. One of the main differences of Lorentz invariance from other symmetry principles in physics is that the Lorentz group is *non-compact*. We have seen that this is simply a restatement of the fact that Lorentzian spacetime has indefinite signatures, or that time is fundamentally different than space. The mathematical consequence of this is that all representations of the Lorentz group are infinite dimensional[1]. We have therefore arrived to a contradiction between our two assumptions: the principle of relativity, and the assumption that the set of all location measurements within a finite spacetime volume is finite.

This simple argument applies to any theories in which spacetime is a fixed background, for example any quantum field theory. We see that quantum field theories are not discrete in any meaningful way. They have too many degrees of freedom, and the attempt to fit them into a discrete structure causes phenomenological and theoretical difficulties. In investigating models with fundamental discreteness wed have to move beyond the paradigm of quantum field theory, and consider quantization of spacetime itself.

Quantized Spacetime?

In quantum field theory without quantized gravity, the location is not a quantum variable or an operator, it is a label which is ultimately a classical object. We have seen that this results in contradiction between special relativity and the idea of discrete spacetime. Perhaps, the argument goes, this is the source of the problem. When we promote the location to a quantum mechanical object, can discrete structures be made consistent with Lorentz invariance?

To see how this intuitive argument might fit in the discussion here, it instructive to consider a concrete example. It is sufficient for us to consider two of the spatial coordinates X, Y and the time coordinate T of a Lorentzian flat spacetime. Suppose, in analogy to angular momentum in non-relativistic quantum mechanics, these coordinates are promoted to operators satisfying the commutation relations:

$$[\hat{X}, \hat{Y}] = l_p \hat{T} \quad [\hat{Y}, \hat{T}] = l_p \hat{X} \quad [\hat{T}, \hat{X}] = l_p \hat{Y} \tag{2}$$

where the length scale associated with the non-commutativity of the coordinates is chosen to be the Planck length, l_p, which is the scale where quantum gravitational corrections are expected to be important.

The commutation relations above are called an $SL(2)$ algebra, which is a non-compact version of the familiar $SU(2)$ algebra of angular momentum. From our experience with ordinary quantum mechanics, we might guess what happens then: whenever we measure a location variable, we would get a discrete result, some quantized multiple of the basic unit of angular momentum. On the other hand, these commutations relations do not break Lorentz invariance [2]. Under Lorentz transformations the states of the systems transform into each other, while the result of any potential measurement remains discrete. It would seem then that we found a way to reconcile Lorentz invariance with discreteness of spacetime.

However, once again the fundamental difference between space and time makes the situation more subtle. For an $SU(2)$ algebra, the unitary representations are discrete, and

[1] Even when discussing finite spacetime volume, as the non-compactness has to do with large momenta, or short distances.

thus angular momentum is quantized. This follows from the compactness of the $SU(2)$ group, in other words from the fact that rotations are periodic. On the other hand, the algebra $SL(2)$ is non-compact, this can be traced to the fact that boost parameters have no underlying periodicity. This simple geometrical fact manifests itself in the structure of the $SL(2)$ representations, which can be either continuous or discrete. Thus, the "angular momentum' following from the $SL(2)$ algebra need not be quantized[2].

We see that in this framework, the question of Lorentz invariance is merely pushed back one step: is the *state* describing our world approximately Lorentz invariant? Presumably states describing discrete spacetimes can be constructed using the discrete representations of the $SL(2)$ algebra only, but most of those states do break Lorentz invariance. If Lorentz invariance is broken at short distances, and if this framework can be reduced to an effective field theory at long distances (as it should), it is difficult to see how observable violations of special relativity are avoided, as the arguments put forward in previous sections are universal.

Certainly this line of reasoning does not rise to the level of a "no-go" theorem, but it hopefully raises some doubts. There seems to be a general difficulty in claiming that spacetime is discrete at short distances. The non-compactness of the Lorentz groups presents a general obstacle to all such attempts, if they are to be Lorentz invariant at the Planck scale. Instead of finding ways to circumvent this difficulty, I'd like to turn to a discussion to what I view to be a crucial element in any attempt to construct a discrete model of reality, that of *holography*.

Holography and Discreteness

I conclude this essay by describing a research direction which aims at resolving the tension between the continuous and the discrete. The context of black hole quantum mechanics is useful in elucidating this issue - in this context we can see clearly both the arguments for fundamental discreteness of quantum gravity, and the need to reconcile those clues with the continuous structures underlying our notion of spacetime. This is a large and complex research area, and the presentation will necessarily be sketchy. I'll confine myself to presenting evidence that *holography* and the associated non-locality is an essential element of the story.

Discrete versus Contiunous in Black Hole Physics

Black holes famously obey the laws of thermodynamics [3]. To an observer staying outside the black hole, the horizon looks like a hot surface at thermal equilibrium, which possesses temperature, entropy, and other thermodynamics quantities. Mysetriously, the Bekenstein-Hawking entropy of a black hole scales with the surface area of the horizon,

[2]The $SL(2)$ algebra does not have finite dimensional unitary representations, which rules out strictly finite models. Perhaps some quantum deformation of this algebra is more suited for that purpose.

instead the more intuitive scaling as the volume enclosed within the horizon. Extrapolated from the context of black holes, this suggests the idea of the *holographic principle*: quantum gravitational dynamics in some region is most naturally described in terms of degrees of freedom living on the boundary of the region, not in its interior [4, 5].

But, more relevant to the current discussion is the question of the *finiteness* of the black hole entropy. In the framework of general relativity, one may think about the entropy of a black hole as resulting from our inability to access information hidden behind the horizon. States outside and inside the horizon are entangled, and since we can only make measurements outside the horizon, we describe our partial knowledge of the system by a density matrix. The entropy of this mixed state is the entanglement entropy across the horizon, and it is tempting to interpret it as the deep reason behind the thermodynamic description of black holes.

Furthermore, in [6] the authors have put forward an argument that discreteness of the black hole spectrum is an essential ingredient in a resolution of the black hole information paradox. In rough terms, if the spectrum of energies of the black hole is discrete, one should be able to identify the microstate underlying the black hole spacetime uniquely by a very precise measurement of its energy, and thus trace the information precisely as it is recovered from the black hole during its evaporation.

However, the discreteness of the black hole spectrum is at odds with the continuous spacetime picture of the black hole horizon. Attempts to quantize small fluctuations around the black hole spacetime invariably result in continuous spectrum, and in information loss [7]. Similarly, the entanglement entropy of any local quantum field theory is infinite, resulting from fluctuations of near-horizon modes with arbitrarily short wavelengths.

The deep reason behind these difficulties is the principle of equivalence: for a locally infalling observer, the black hole horizon ought to be locally indistinguishable from flat space. One can attempt to impose short distance cutoff, to make the entropy finite (see for example [8] and references therein). One can similarly assume the horizon of the black hole has a discrete structure explaining the discreteness of energy levels. Unfortunately, it is hard to reconcile such short distance discreteness with the view of the locally inertial observer, for which the horizon is just a patch of flat spacetime. For that locally inertial observer, discreetness will seemingly be at conflict with local Lorentz invariance, as described above.

Holographic Discreteness

We see the general tension discussed in this essay emerging in the current context as well. How is the tension resolved in the context of black holes? We have no general answer, but we have hints from studying instances of black holes embedded in complete non-perturbative theories of quantum gravity. For example, a complete definition of quantum gravity with asymptotically Anti-de-Sitter (AdS) boundary conditions is given by Maldacena's gauge/gravity duality [9]. I'll end this essay by giving some indication how things work in this context.

Consider the so-called small black holes in asymptotically AdS space, which like their asymptotically flat cousins undergo Hawking evaporation. These are in principle described by the boundary theory: for our purposes it is sufficient to think of it as an ordinary quantum mechanical theory, with large number of degrees of freedom, a number which is inversely proportional to the Planck length, call it N. This is the description most naturally encoding the measurements of an observer far away from the black hole.

While the complete picture of black hole formation and eventual evaporation is mysterious in this description, some information is available through simple scaling arguments [6, 10]. The black hole has energy of order N, and thus is made of order N "bits", the energy spacing between nearby levels is therefore of order e^{-N}. We see that the spectrum is discrete, as it should be, but becomes continuous in the infinite N limit. In that limit we recover the problems associated with semi-classical quantization around a fixed spacetime, including infinite entropy and information loss.

For our purposes it is important to note that when thinking of N as large but finite number, this framework encodes naturally a discrete description of spacetime, manifested in discrete black hole spectra. We can therefore address the question raised in this essay: how can we think about this discreteness? In other words, what is the set of measurements sensitive to such discreteness? Is the discreetness manifested as a short distance cutoff?

From the above description it is clear that in order to see the fundamental discreetness, one only has to resolve the energy differences between different microstates, one way or another. There are many ways to do that, but, and this is the central point – there is no indication that one needs to resolve short distances in spacetime in order to see the underlying discreteness. To give just one example, precise enough measurement of the *total* energy of the system, without localizing it anywhere, is sufficient in order to see the discrete nature of spacetime.

We conclude therefore that the discreteness achieved in this context is unlikely to be related to short distance cutoff in spacetime. This holographic type of discreteness is subtle and interesting, and deserves further study.

Conclusions

At our present level of knowledge, the set of questions described in this essay cannot be answered conclusively. Nevertheless, to the author it seems clear that the discreteness associated with quantum gravitational effects has little to do with any truncation of spacetime at short distances. Rather it is manifested in global correlations between the degrees of freedom making up spacetime. This is the best way to avoid the tension between Lorentz invariance and the underlying discreteness of quantum gravity.

Much more remains to be discovered about black hole evaporation in quantum gravity, and about the subtle discreteness that holography imposes on spacetime. We hope for rapid progress in the research direction described, it is our belief that it holds the key to construction of discrete holographic models of spacetime.

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