Relative information at the foundation of physics

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I observe that Shannon's notion of relative information between two physical systems can effectively function as a foundation for statistical mechanics and quantum mechanics, without referring to any subjectivism or idealism. It can also represent the key missing element in the foundation of the naturalistic picture of the world, providing the conceptual tool for dealing with its apparent limitations. I comment on the relation between these ideas and Democritus.

I. IS THERE A SUBJECTIVE ELEMENT IN STATISTICAL MECHANICS?

Thermodynamical quantities such as entropy and temperature depend on the macroscopical variables chosen to describe a system with many degrees of freedom. They depend on a coarse-graining. For instance, entropy can be defined (in the microcanonical language) in terms of the the number of microstates compatible with what we know about the system. With this definition, entropy changes if we know more. This fact seems to insert a puzzling subjective element in physics. There is a tension with the fact that termodynamical laws appear pretty much true quite independently on any choice by us. Is the Sun "hot" just because we "choose" a certain coarse graining for describing it? Does entropy increases because of our choices?

The way out of the puzzle is simple. Entropy is neither something inherent to the microstate of a system, nor something depending on our abstract subjective "knowledge" about it. Rather, it is a property of certain (macroscopic) variables. For instance, the full state of a gas in a box is described by the position and velocity of its molecules. No entropy so far. But volume, total energy and (time averaged) pressure on the box boundaries are well defined functions of this state, and Entropy is a function of *these*. This is the first step.

Now consider a context where the gas interacts with a *second* physical system coupled to the gas only via the gas volume, total energy and pressure (for instance, by a mercury thermometer and a spring holding a piston). Then the physical interactions between the gas *and this system* are *objectively* described by thermodynamics.

In other words, it is not an arbitrary or subjective choice of a coarse-graining that makes thermodynamics physically relevant: it is the concrete way another physical system is coupled to the gas. If the coupling is such that it depends only on certain macroscopic variables of the gas, then the physical interactions of the gas and this system are *objectively* well described by thermodynamics.

This key observation clarifies the role that information plays in physics. Entropy is information: in the microcanonical language, for example, entropy is determined by number of micro states compatible with a given macro state. The number of states in which something can be, compatible with what we know is also precisely the definition of "information" given by Shannon in his celebrated 1948 work that has started the development of information theory [1]. But "information", that is, the number of alternatives, is not significative in physics insofar as it depends on idealistic subjective knowledge: it is relevant in physics when it refers to an interaction between two systems where the effects of the interaction on the second depend only on few variables of the first, and are independent on the rest of the variables. Under these circumstances, the number of states of the first system which are not distinguished by these variables is the number of Shannon "alternatives" relevant for the definition of thermodynamical entropy. Here "information", counts the number of states of a system which behave equally in the interaction with a second system.

Therefore the information relevant in physics is always the *relative* information between *two* systems. There is no subjective element in it: it is fully determined by the state *and* the interaction Hamiltonian which dictates which variables are the relevant ones in the interaction.

Pictorially: it is not the microstate of the Sun which is hot, it is the manner the Sun affects the Earth which is *objectively* hot.

II. RELATIVE IRREVERSIBILITY

Let us reconsider the quintessential irreversible phenomenon: a cup that falls to the floor and breaks. On one account this is obviously an irreversible phenomenon, since the structure of the cup is lost in the process, but is it so on any possible account? I can interpret the same event as just one of the many possible dynamical evolutions of a bunch of molecules, that evolve from one position to another one. What is it that makes the starting position more "special" that the final one? Clearly there is something that does so, but it is not in the microstate of the molecules, it is the manner we describe it, or, at the light of the previous section, in the manner we interact with the cup. It is because of our specific macroscopic account of the cup, dictated by the variables with which we interact with it, that the initial state is special and therefore entropy increases.

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To illustrate this, consider a box full of balls, characterized by two properties, say color and electrical charge. Say they have two possible colors: half are white and half black; and two possible value of the charge: say half are neutral and half charged. Consider a microstate C where all the white balls are on the left of the a box and all the black balls on the right, while charge is randomly distributed. And a different microstate Ch, where all the charged balls are on the left and all the neutral balls on the right, while color is randomly distributed. To normal eyes, C looks like a low entropy state and Ch as a high entropy one. But to a person who is color blind but has an electrometer it us Ch that looks low entropy and C that appears to be high entropy.

What if the breaking cup could in fact be observed by somebody else, coupling differently to it, as a process where entropy decreases? There is nothing a priori forbidding this.

If these considerations are correct, then the irreversibility of the worlds is to be understood as a property of the couplings between systems, more than a property of the evolution of isolated systems.

III. THE LIMITS OF MICROPHYSCS WITHOUT INFORMATION

The idea that the world can be essentially described as a vast see of interacting atoms and nothing else, can be traced back to the ancient atomism of Democritus. The naturalistic and materialistic world view of Democritus, however, was soon criticized by Plato and Aristotle on the ground that it does not account for the forms we see in the world. What makes a certain ensemble into a given object we recognize? Plato and Aristotle (in different manners) wanted to add "forms" to the naturalistic view of Democritus. For Plato, a horse, for instance, is not just an aggregate of matter: it is an imprecise realization of the abstract form ("idea") of a horse, and for Aristotle the same horse is the union of its substance and its form. But if the form is something above the substance, what is it?

What is it that makes a random disposition of molecules into a cup? What are the properties of the Democritean atoms that generate collective variables? And how?

In fact, Democritus's idea was more subtle than the fact that everything is just atoms. Democritus says that three features are relevant about the atoms: the shape of each individual atom, the order in which they are disposed, and their orientation in the structure. And Democritus uses then a powerful metaphor: like twenty letters of an alphabet can be combined in innumerable manners to give rise to comedies or tragedies, similarly the atoms can be combined in innumerable manners to give rise to the innumerable phenomena of the world.

But what is the relevance of the way in which atoms combine, in a world in which there is nothing else than atoms? If they are like letters of an alphabet, to whom do they tell stories?

I think that the key of the answer brings us back to the observation in the first section: physical systems interact with one another and affect one another. In the course of these interactions, the way one of them happens to be, leaves traces on the way another is, so that correlations are established between systems. Let us make this idea precise, recurring again to Shannon's theory of information. Following Shannon, we can say that a system O has information about a system S if there is a physical constraint such that the number of total states of the two systems is smaller than the product of the number of states of each. For instance, if the system S can be in two states, say a and b and the system O can be in two states, say A and B and there is a physical constraint that forbids the states (a, B) and (b, A), thus allowing only the two states (a, A) and (b, B), then we say that O has (one bit of) information about S. In words, if we see the state of O, then we know the state of S. Physical interactions determine continually those constraints among systems: if a tree happen to fall on my head, then I cannot be standing smiling anymore: I have some information about the tree.

Thus, the systems formed by atoms have necessarily information about one another in the sense of Shannon. The negative information (in this sense) that a system have about another is precisely the relative entropy of the second, which is relevant for the interactions with the first, and is the conventional thermodynamical entropy.

IV. QUANTUM THEORY

The discovery of quantum theory in the XX century has sharpened the role of information in our understanding of the world. If we measure some variables of a system, with a given precision, we can represent our resulting information about the system by giving the region R of the phase space of the system compatible with our measurements. The units in which phase space volume is measured are units of action (length^2 x mass x time^{-1}) per degree of freedom. In principle, in classical mechanics we can refine such measurement arbitrarily well, and therefore there is alway a continuous amount of missing information about a system, whatever the precision of the measurement.

Not so after the discovery of quantum theory. If we measure the energy of a harmonic oscillator and we obtain the result that this is between E_1 and E_2, then there is only a *finite* number of possible values that the energy can have. This is given by the area of the region of phase space included between the two surfaces E_1 and E_2, divided by the Planck constant.

This is a general result: for all quantum systems, the orthogonal states are in finite number per each finite region of phase space. The Planck constant determines a minimal phase space volume. Since phase space volume of a region of phase space expresses the (missing) information we have about a system, it follows that quantum mechanics implies that information is not anymore continuous when we do not disregard quantum theory. It is discrete, and the Planck constant defines a minimal unit of information.

This leads to a first principle at the basis of quantum theory: *The information contained in any finite region of the phase space of any system is finite.*

This consideration does not exhaust the physics of quantum theory, since this would be true for any intrinsically discrete classical system as well. What characterizes quantum theory is the fact that information can become "irrelevant", and be renewed. By this I mean the following. If we have measured a system, the information we have about a system allows us to predict its future. In quantum theory, we can always add *new* information to the state of a system, even after we have reached maximal information about it. By doing so, part of the old information becomes irrelevant. The typical case is a sequence of measurements of spins of a two-state system along different axes. Each measurement brings novel information and makes the previous one irrelevant.

This leads to the second principle at the basis of quantum theory: *It is always possible to acquire new information about a system.*

The combination of these two principles yields essentially the entire mathematical structure of quantum theory, up to some technical aspects, as was shown in [2]. Thus, relative information that systems have about one another is a key language for grounding quantum theory as well.

If we do not neglect quantum gravity, a system must include the gravitational field, which is to say spacetime. Therefore systems are naturally identified with spacetime regions. The interactions between spacetime regions are exchanges of informations across spacial regions. These are quantized and discrete. The quantum discreetness, united to the fact that the geometry of spacetime is dynamical, and therefore quantized, leads immediately to the discretization of space, an idea that can be traced back to the thirties [3, 4] and has been concretized more recently in particular in loop quantum gravity. Here the discreetness translates into the discreetness of the area of two-dimensional surfaces [5, 6]. The discreteness of the area is an immediate reflex of the discreetness of the information that can be transmitted across these surfaces.

V. REALITY AND INFORMATION

This ensemble of considerations, I believe, conspire towards a picture where the fog begins a bit to dissipate over the intriguing role of information at the foundation of physics.

Information that physical systems have about one another, in the sense of Shannon, is ubiquitous in the universe, and has the consequence that on top of the microstate of a system we have also the informational state that a second system O has about any system S.

The universe is not just simply the position of all its Democritean atoms. It is also the net of information that all systems have about one another. Objects are not just aggregate of atoms. They are particular configurations of atoms singled out because of the manner a given other system interacts with them.

Amon all systems, living systems are those that selection has lead to reproduce continuously their own structure by, in particular, making use of the information they have about the exterior world. This is why we can understand them in terms of finality and intentionality, because they are the ones that have permaned thanks precisely to the finality in their structure. Thus, it is not finality that drives structure, but the other way around, selected structures define finality. Since the interaction with the world is described by information, it is by dealing with information that these systems effectively persist. This is why we have DNA code, immune systems, sensory organs, neural systems, memory, complex brains, language, books, MAC's and wikipedia. To maximize the management of information.

The statue that Aristotle wants to be made of more than atoms, is indeed made by more than atoms: it is something that pertains to the interaction between the stone and brain of Aristotle, or ours. It is something that pertains to the stone, the goddess represented, Phidias, a woman he met, our education, and else. The atoms of that statue talk to us precisely in the same manner in which a white ball in my hand says that the ball in your hand is also white, if the two are correlated. By carrying information.

This is why, I think, from the basis of genetics, to the foundation of quantum mechanics, to the basis of thermodynamics, all the way to sociology and to quantum gravity, it appears that the notion of information has a pervasive and unifying role. The world is not just a blind wind of atoms, or generally covariant quantum fields. It is also the infinite game of mirrors reflecting one another formed by the correlations among the structures formed by the elementary objects. To go back to Democritus metaphor: atoms are like an alphabet, but an immense alphabet so rich to be capable of reading itself and thinking itself. With Democritus worlds:

"The Universe is change, life is opinion that adapt itself".

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