Je suis, nous sommes Wigner!

A perspectival exploration of the Frauchiger–Renner paradox

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Our story begins with a rather unseemly argument between four colleagues herein identified as the two Wigners, $\overline{W}$ and W, and their Friends, $\overline{F}$ and F. The four are standing in a large lab we'll call M, debating the merits of various possible interpretations of the results of a quantum experiment they've just performed. The debate's main contention, at this late point, would appear to revolve around everyone's differing opinion concerning how $\overline{F}$, in a perfectly isolated lab called $\overline{L}$, should think about W's measurement, from M, of F in another perfectly isolated lab called L ....

$\overline{F}$ insists that "W's measurement of L is either on the composite system $\overline{L}$L, and only after $\overline{W}$'s measurement on my lab $\overline{L}$ … or it's a measurement on L alone, assuming this is possible relative to $\overline{L}$, which is where the paradox lies … at least from my perspective, whatever that is, although I think I'm definitely leaning more towards a relational[1] and/or relative-state formulation."

$\overline{W}$, stridently: "Well you would think that cos I just Hadamarded your $\overline{F}$ brain!"

F in a more conciliatory tone: "Or … perhaps $\overline{W}$, you didn't act directly on the isolated entity $\overline{F}$ but instead you Hadamarded your own correlation with $\overline{L}$?"

W appears curiously incredulous: "But, but … if collapse *isn't* an objectively physical event then …?!?"

$\overline{W}$: "Oh For Frakks Sake! Gods spare me from mad dog Everettians, frakking QBists, and clueless empiricists!"

$\overline{F}$: "HA! That's rich coming from an unreconstructed Bohmian!" Mild jostling ensues, and so the argument proceeds.

Some hours before this increasingly indecent imbroglio, the four colleagues happily entered the main lab M to take part in an experimental test of the venerable Frauchiger–Renner paradox using the recently invented Quantum Isolation Field generator fitted to two self-contained laboratory modules $\overline{L}$ and L. All four read and agreed on the experimental procedure outlined in Box 1 of page 3 of the F-R paper[2] (see technical endnote Frauchiger-Renner Experimental Procedure p. 10).

$\overline{W}$ claps loudly: "Ok, everyone knows what to do? Let's get this show on the road!"

$\overline{F}$ and F enter their respective labs $\overline{L}$ and L, start their QIF drives up, and enter into perfect quantum isolation. At this point our narrative splits into three perfectly isolated threads physically constrained to the labs M, $\overline{L}$, and L.

Narrative 1—Observers $\overline{\mathrm{W}}$’s and W’s perspectives from M

For both Wigners in the main lab M, the two perfectly isolated labs, $\overline{\mathrm{L}}$ and L, are now black boxes such that there is no actual fact of the matter as to either of the labs’ contents at any time until their perfect isolation is partially breached in $\overline{\mathrm{W}}$’s first measurement (at step 5 of the experimental procedure, time n:20). This includes knowing, or indeed caring, whether the occupants $\overline{\mathrm{F}}$ and F are following the experimental procedures, or for that matter whether they are even alive or dead. From the perspective of the two Wigners, the evolution of the experiment is thus a series of timed assumptions until n:20 when $\overline{\mathrm{W}}$ actually measures $\overline{\mathrm{L}}$. And so we start!

n:00 – $\overline{\mathrm{W}}$ and W both look at the clock and note the time as n:00. At this point they can only assume that the quantum ‘randomness generator’ has been triggered in $\overline{\mathrm{L}}$ and that the outcome for the unobserved system is best described by a wave function for heads (h) or tails (t) in a superposition of both values given by $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}}+\sqrt{\frac{2}{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}}\right\}$. Assuming that $\overline{\mathrm{F}}$ then prepares and sends the particle with spin S to F in lab L, the Wigner’s also agree to assume that L becomes partially entangled with $\overline{\mathrm{L}}$ by the end of this timed sequence to form the composite system $\overline{\mathrm{L}}$L relative to M.

n:01 – W and $\overline{\mathrm{W}}$ thus both note in their respective Lab Journals that “I assume that from n:00 $\overline{\mathrm{L}}$ is in a superposition of heads and tails relative to M”.

n:10 – At this point, $\overline{\mathrm{W}}$ and W both agree to assume that F has measured the spin of S. From the perspective of lab M, as there is no fact of the matter as to F’s measurement result, the state of the composite $\overline{\mathrm{L}}$L system then becomes a superposition of the two assumed measurements defined as $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}}|\downarrow\rangle_{\mathrm{L}}+\frac{1}{\sqrt{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}}|\downarrow\rangle_{\mathrm{L}}+\frac{1}{\sqrt{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}}|\uparrow\rangle_{\mathrm{L}}\right\}$.

n:20 – Finally, the first actual measurement occurs in M when $\overline{\mathrm{W}}$ measures $\overline{\mathrm{L}}$ with respect to the basis $|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}}=\frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\overline{\mathrm{L}}}-|\mathrm{t}\rangle_{\overline{\mathrm{L}}})$, $|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}=\frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\overline{\mathrm{L}}}+|\mathrm{t}\rangle_{\overline{\mathrm{L}}})$.

But first, $\overline{\mathrm{W}}$ quickly leafs through the F-R paper to double check the measurement: “Alrighty then, I just want to confirm exactly what I’m measuring here.... Ah, according to equation (7) on page 4 we Wigners should be able to calculate a 1 in 12 probability for both $\overline{\mathrm{w}}$ and w = ok. So following the F-R protocol, I’m guessing my measurement of the composite system $\overline{\mathrm{L}}$L relative to our lab M should be on $\left\{\frac{2}{\sqrt{6}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}|\downarrow\rangle_{\mathrm{L}}+\frac{1}{\sqrt{6}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}|\uparrow\rangle_{\mathrm{L}}-\frac{1}{\sqrt{6}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}}|\uparrow\rangle_{\mathrm{L}}\right\}$ which means there’s a 5 in 6 chance that $\overline{\mathrm{w}}=\overline{\mathrm{fail}}$ and 1 in 6 that it’s $\overline{\mathrm{ok}}$. Looks good so far to me!” $\overline{\mathrm{W}}$ writes this into their copy of the lab notes then pushes the big red Measurement button, the lab’s emergency lighting pulsates as a klaxon sounds and M’s computer announces “Correlation tunnel initiated, measurement executed, tunnel closed” following which $\overline{\mathrm{W}}$ observes $\overline{\mathrm{w}}=\overline{\mathrm{ok}}$.

n:22 – And thus $\overline{\mathrm{W}}$ announces to W that “Given that I’m measuring the composite system $\overline{\mathrm{L}}$L, I am certain that $\overline{\mathrm{w}}=\overline{\mathrm{ok}}$ and therefore also that F knew S = ↑ at time n:11.”

n:26 – W then in turn correctly notes in their Lab Journal that “I am certain that $\overline{\mathrm{W}}$ knows that $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ at time n:22.” At this point W has a shiver of insecurity and wonders “why am I even here? Couldn’t $\overline{\mathrm{W}}$ have done both measurements just wearing a $\overline{\mathrm{W}}$ and then a W hat? What actual purpose does it serve having two of us doing one Wigner’s work?” Grateful for the paid work in these lean times, and certainly not wanting to be made redundant, W keeps this thought to themselves and proceeds regardless.

n:30 – Noting the time, W prepares to measure L from the basis $|\mathrm{ok}\rangle_{\mathrm{L}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} - |\uparrow\rangle_{\mathrm{L}})$, $|\mathrm{fail}\rangle_{\mathrm{L}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} + |\uparrow\rangle_{\mathrm{L}})$. “Ok so, $\overline{\mathrm{W}}$ help me here, I’m a tad confused. Am I measuring the composite system $\overline{\mathrm{L}}\mathrm{L}$ in which case my outcome is dependent on your measurement outcome? Or do I disregard your outcome and just measure the whole composite system … which is the same thing … or do I disregard the supposed partial entanglement between the two labs and just go for L in its apparently pure state?”

$\overline{\mathrm{W}}$: “You’re on your own there Wigner! All I can tell you is if you come up with a 1 in 12 chance for both $\overline{\mathrm{ok}}$ and ok then you’re probably on the right track … as it is this $\overline{\mathrm{W}}$igner is done!”

W: “OK, so if I just use the whole composite system irrespective of $\overline{\mathrm{W}}$’s subsequent interaction with it, then the initial state $\left\{\frac{2}{\sqrt{6}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}|\downarrow\rangle_{\mathrm{L}} + \frac{1}{\sqrt{6}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}|\uparrow\rangle_{\mathrm{L}} - \frac{1}{\sqrt{6}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}}|\uparrow\rangle_{\mathrm{L}}\right\}$ becomes $\left\{\frac{3}{\sqrt{12}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}|\mathrm{fail}\rangle_{\mathrm{L}} + \frac{1}{\sqrt{12}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}|\mathrm{ok}\rangle_{\mathrm{L}} - \frac{1}{\sqrt{12}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}}|\mathrm{fail}\rangle_{\mathrm{L}} + \frac{1}{\sqrt{12}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}}|\mathrm{ok}\rangle_{\mathrm{L}}\right\}$ and it seems to me that there is in fact a 1 in 12 chance that I will measure $\mathrm{w} = \mathrm{ok}$. But given that $\overline{\mathrm{W}}$ has already announced that $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$, then I can just drop the $|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}}$’s which would give me $\left\{|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}}\left(\frac{1}{\sqrt{2}}|\mathrm{ok}\rangle_{\mathrm{L}} - \frac{1}{\sqrt{2}}|\mathrm{fail}\rangle_{\mathrm{L}}\right)\right\}$. Thus I calculate a 50 50 probability of announcing $\mathrm{w} = \mathrm{ok}$ given that $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ at n:21, which means that F knew $\mathrm{S} = \uparrow$ at n:11, and that $\overline{\mathrm{F}}$ knew that $\mathrm{r} = \mathrm{tails}$ at n:01.”

“Unless …” W thumbs through the F-R paper to page 4, “… I assume that the state of L after F’s measurement of S at N:10 is given by $\left\{\frac{1}{\sqrt{2}}\left(|-\tfrac{1}{2}\rangle_{\mathrm{L}} + |+\tfrac{1}{2}\rangle_{\mathrm{L}}\right)\right\}$ … which is orthogonal to $|\mathrm{ok}\rangle_{\mathrm{L}}$ and thus … following F-R equation (4) $\mathrm{w} = \mathrm{fail}$ and only fail … but I don’t quite understand why I would do that?”

$\overline{\mathrm{W}}$: “No no, that’s only from $\overline{\mathrm{F}}$’s perspective just after F’s measurement of S. But it doesn’t matter anyway because at n:20 I just Hadamarded $\overline{\mathrm{F}}$’s brain!”[3]

W starts to hyperventilate: “I have no idea what that could mean in any empirical sense whatsoever!”

$\overline{\mathrm{W}}$: “Oh calm down! It’s actually very simple, $\overline{\mathrm{F}}$ and F are just literally fleshy qubits that we’re manipulating for a quantum computing outcome, and you can run the process[4] yourself to see what I mean[5]. In fact, we could have saved a yuuuuge amount of money by not hiring them, or buying those pointless QIF modules, and just using the funds to build two simple quantum computers to perform the exact same functional tasks …” $\overline{\mathrm{W}}$ looks over at W quizzically, “in point of fact ...”

W excitedly flipping some qubits: “Ohhh, I see now! It’s just a computing problem and they’re just actual qubits … what a relief!!”[6]

n:31 – W busily notes in the Lab Journal: “At n:31 I am fairly certain that after $\overline{\mathrm{W}}$’s measurement outcome of $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$, there is a 1 in 2 probability that w = ok”. W then pushes the big red Measurement button, the lab’s emergency lights pulsate as a klaxon sounds and M’s computer announces “Correlation tunnel initiated, measurement executed, tunnel closed” following which W announces “w = ok!” and the experiment is at an end.

n:40 – The two Wigner’s await the arrival of the two Friend’s in the main lab M to compare everyone’s notes and sort this paradoxical logic out once and for all!

Narrative 2—Observer $\overline{\mathrm{F}}$’s perspective

n:00 – After starting up $\overline{\mathrm{L}}$’s Isolation Drive and entering into perfect isolation, $\overline{\mathrm{F}}$ then invokes the quantum ‘randomness generator’.

n:01 – The generator pings and $\overline{\mathrm{L}}$’s computer announces “r = tails”. $\overline{\mathrm{F}}$ duly notes down in the Lab Journal: “At n:01 r = tails and the spin S will be in state $|\rightarrow\rangle_s$ at time n:10.”

n:02 – Following on from this initial measurement of r, $\overline{\mathrm{F}}$ then prepares a particle with spin S = $|\rightarrow\rangle_s$, places the particle in the quantum teleport and hits the big yellow Teleport button. The lab’s emergency lights pulsate as a klaxon alert sounds and $\overline{\mathrm{L}}$’s computer announces “Outbound correlation tunnel initiated, teleport confirmed, tunnel closed”.

$\overline{\mathrm{F}}$ flops down into the control seat with a copy of the F-R paper, shuffling the pages back and forth between Box 1 on page 2, equation (4) on page 4, Tables 1 and 2 on page 7, and Table 3 on page 8: “Thanks $\overline{\mathrm{L}}$ … ok now that’s all done, what’s next? Ah yes, for this all to work as a paradox I need to clearly state in the lab notes that ‘I am certain that W will observe w = fail at time n:31’ ... so then, why would I do that exactly?”

“Between n:02-10 I can assume F has received the particle which means $\overline{\mathrm{L}}$ is now partially entangled with L. And I can also announce that I am not actually in a superposition contrary to Table 1’s time evolution and whatever the others might be thinking at this point in time. But I do wonder in what sense can L now be considered a composite system $\overline{\mathrm{L}}$L relative to $\overline{\mathrm{L}}$?”

“So okay—assuming objective collapse is still a thing beyond my perfect isolation—then after my measurement of r = tails, $\overline{\mathrm{L}}$ is in state $|\mathrm{t}\rangle_{\overline{\mathrm{L}}}$ and I can then also assume that M and L are also in states $|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{M}}$ and $|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{L}}$ respectively … at least relative to my frame of reference … and for any possible future interactions I may have with those systems once our isolation is broken, and this simply because we are assumed to all live in a single, macroscopically classical world.”

“Once F has measured S at n:10, assuming that is happening or has happened, then just as for the Wigners in M there will be for me in $\overline{\mathrm{L}}$ no observable fact of the matter regarding this assumed measurement outcome. Given this, am I justified in assuming that L relative to $\bar{\mathrm{L}}$ is now in a superposition of $\left\{\frac{1}{\sqrt{2}}|\mathrm{t}\rangle_{\bar{\mathrm{L}}\mathrm{L}}|\downarrow\rangle_{\mathrm{L}}+\frac{1}{\sqrt{2}}|\mathrm{t}\rangle_{\bar{\mathrm{L}}\mathrm{L}}|\uparrow\rangle_{\mathrm{L}}\right\}$ … even though I can have no idea whether F knows r = tails or r = heads or tails? I guess if r = tails is now a universally objective fact due to my collapsing its wave function, then both the Wigners' and F's ignorance of that fact is just simple ignorance rather than any quantum indeterminacy … but then what's the difference with my own ignorance of F's presumably objective measurement outcome with S? And why is this tangled Wigner's Friend logic starting to sound so horridly Bohmian!"

"Anywho … at n:11, following the F-R argument from page 4, this current configuration for L would indeed appear to be orthogonal to $|\overline{\mathrm{ok}}\rangle_{\bar{\mathrm{L}}}$ and that therefore per equation (4) w = fail if r = tails, in which case I could write that 'I am certain that W will observe w = fail at time n:31, at least from the perspective of the objective truth of the collapse postulate'. Except … would this still be true after $\bar{\mathrm{W}}$'s measurement of my lab at n:20 given that $\bar{\mathrm{L}}$ is partially correlated with L from n:02 onwards?"

n:20 – In an abrupt interruption the lab's emergency lights pulsate as a klaxon sounds and $\bar{\mathrm{L}}$'s computer announces "Inbound correlation tunnel initiated, contact executed, tunnel closed".

$\bar{\mathrm{F}}$: "HA! Hey $\bar{\mathrm{L}}$, we've just been Hadamarded!! And now we're partially entangled with M. So … where were we again? Ah yes, objective collapse theories and $\bar{\mathrm{W}}$'s measurement of $\bar{\mathrm{L}}$ in the basis $|\overline{\mathrm{ok}}\rangle_{\bar{\mathrm{L}}}=\frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\bar{\mathrm{L}}}-|\mathrm{t}\rangle_{\bar{\mathrm{L}}})$, $|\overline{\mathrm{fail}}\rangle_{\bar{\mathrm{L}}}=\frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\bar{\mathrm{L}}}+|\mathrm{t}\rangle_{\bar{\mathrm{L}}})$ at n:20. Now from my perspective, M at n:01 is in the initial state $\{|\mathrm{t}\rangle_{\bar{\mathrm{L}}\mathrm{M}}\}$ with respect to $\bar{\mathrm{L}}$ which then becomes $\left\{\frac{1}{\sqrt{2}}|\mathrm{t}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\downarrow\rangle_{\mathrm{L}}+\frac{1}{\sqrt{2}}|\mathrm{t}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\uparrow\rangle_{\mathrm{L}}\right\}$ at n:20 when $\bar{\mathrm{W}}$'s interaction with $\bar{\mathrm{L}}$ transforms that to give $\left\{\frac{1}{\sqrt{4}}|\overline{\mathrm{fail}}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\downarrow\rangle_{\mathrm{L}}+\frac{1}{\sqrt{4}}|\overline{\mathrm{fail}}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\uparrow\rangle_{\mathrm{L}}-\frac{1}{\sqrt{4}}|\overline{\mathrm{ok}}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\downarrow\rangle_{\mathrm{L}}-\frac{1}{\sqrt{4}}|\overline{\mathrm{ok}}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\uparrow\rangle_{\mathrm{L}}\right\}$."

$\bar{\mathrm{F}}$: "This state would give a probability of 1 in 2 for $\bar{\mathrm{w}}=\overline{\mathrm{ok}}$ regardless of F's measurement outcome for S. At n:21 I could then note that "I calculate a probability of 1 in 2 for $\bar{\mathrm{w}}=\overline{\mathrm{ok}}$. Following this, at n:30, I can assume that W measures L with respect to the basis $|\mathrm{ok}\rangle_{\mathrm{L}}=\frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}}-|\uparrow\rangle_{\mathrm{L}})$, $|\mathrm{fail}\rangle_{\mathrm{L}}=\frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}}+|\uparrow\rangle_{\mathrm{L}})$. The state for the whole LM system relative to $\bar{\mathrm{L}}$, assuming $\bar{\mathrm{W}}$ is maximally entangled with W within M, is then $\left\{\frac{2}{\sqrt{8}}|\overline{\mathrm{fail}}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\mathrm{fail}\rangle_{\mathrm{LM}}-\frac{2}{\sqrt{8}}|\overline{\mathrm{ok}}\rangle_{\bar{\mathrm{L}}\mathrm{M}}|\mathrm{fail}\rangle_{\mathrm{LM}}\right\}$. At which point I would note that "there is no chance that at the end of the experiment both $\bar{\mathrm{w}}$ and w = ok if r = tails. Therefore, the F-R paradox = true from an objective collapse perspective!"

$\bar{\mathrm{F}}$ flips the pages of the F-R paper back and forth: "Hmmmm … now where have we gone so very wrong? Any ideas $\bar{\mathrm{L}}$?"

$\bar{\mathrm{L}}$'s computer says: "Sorry $\bar{\mathrm{F}}$ but quantum foundations do not compute."

$\bar{\mathrm{F}}$: "Really? Still? Who'd a thunk it! So … I've been going back over Everett's long and short theses, and probably reading far too much Husserl,[7] but … and just humour me here … I want to try out a relative-state approach, minus the worlds bit as, strictly speaking, there is only one world, that of one's own empirical experience, and all the other 'branches' are merely state vectors relative to that observer dependent reality. And you know, I think maybe Everett was onto something with his first alternative to the multiple observer paradox, which is, if I may paraphrase:”

> *Alternative 1: To postulate the existence of only one observer ~~in the~~ [per observable] universe. This is the ~~solipsist~~ [phenomenological] position, in which each of us must hold the view that he [sic] alone is the only valid observer, with the rest of the universe and its inhabitants obeying at all times Process 2 [continuous, deterministic wave function evolution] except when under his [sic] observation.*[8]

“What do you reckon $\overline{\mathrm{L}}$?”

$\overline{\mathrm{L}}$’s computer says: “Sorry $\overline{\mathrm{F}}$ but quantum foundations do not compute.”

$\overline{\mathrm{F}}$: “I hear ya! But it does occur to me that this whole F-R paradox thing is observer dependent in a fundamental sense, in that there are three perfectly isolated systems to consider, these being $\overline{\mathrm{L}}$, L and M. Each system is a physically constrained observable universe in itself, it’s just that for the two Wigner’s their observable universe M+ is very, very much larger than ours and F’s. And it also seems to me that there is an observer bias in prioritising Wigner’s M perspective over those of us Friends. If QM is observer dependent relative to each observer’s frame of reference then … we are all Wigner!”

$\overline{\mathrm{F}}$: “From this more or less Everettian relative-state perspective then, what happens when I measure r = tails if there is no spontaneous or otherwise collapse of the wave function and no hidden variables? Can I consider my measurement of r in $\overline{\mathrm{L}}$ to have caused both M and L to enter into their relative vector superpositions with respect to $\overline{\mathrm{L}}$? The two relative-states would be $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{M}} + \sqrt{\frac{2}{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{M}}\right\}$ and $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{L}} + \sqrt{\frac{2}{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{L}}\right\}$ respectively.”

$\overline{\mathrm{F}}$: “Empirically speaking, from my perspective lab $\overline{\mathrm{L}}$ is self-evidently not in a superposition, just as for the two Wigners M isn’t in one either, and presumably also for F in L. So what exactly do these quantum states describe if not my potential correlation with whatever physicality potentially still exists beyond this observer’s perfect physical isolation? Is it the relative correlations themselves that are objectively real? In this case, $|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{M}}$ and $|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{L}}$ are orthogonal relative-state vectors that I can never directly interact with, as my own relative-state vector is defined by r = tails. But those orthogonal vectors may nonetheless provide interference effects that could change the outcome of any future interactions with M and L, at least up until the quantum isolation is broken and I decohere back into M.”

$\overline{\mathrm{F}}$: “So following this logic, once F measures S in L at n:10, I might then assume that L relative to $\overline{\mathrm{L}}$ is now in a superposition of $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{L}}|\downarrow\rangle_{\mathrm{L}} + \frac{1}{\sqrt{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{L}}|\downarrow\rangle_{\mathrm{L}} + \frac{1}{\sqrt{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{L}}|\uparrow\rangle_{\mathrm{L}}\right\}$. Here, the full state reflects F’s potential state relative to my empirical reality which is r = tails. The potential for the orthogonal element $|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{L}}$ causing interference effects would assume that relative-state vectors can interfere across ‘branches’ before the two systems decohere.

And given the potential for weak measurements to preserve orthogonal elements in a composite system with highly peculiar quantum interference outcomes I don't see why not!"[9]

$\overline{\mathrm{F}}$ glances at the clock counting down the first pass: "Alrighty then let's get through this! At n:20 I can assume that $\overline{\mathrm{W}}$ measured my lab $\overline{\mathrm{L}}$ in the basis $|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}} = \frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\overline{\mathrm{L}}} - |\mathrm{t}\rangle_{\overline{\mathrm{L}}})$, $|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}} = \frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\overline{\mathrm{L}}} + |\mathrm{t}\rangle_{\overline{\mathrm{L}}})$. This partial entanglement with M has transformed the M superposition relative to $\overline{\mathrm{L}}$ and its entanglement with L. Thus $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{M}} + \sqrt{\frac{2}{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{M}}\right\}$ becomes $\left\{\frac{1}{\sqrt{3}}|\mathrm{h}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\downarrow\rangle_{\mathrm{L}} + \frac{1}{\sqrt{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\downarrow\rangle_{\mathrm{L}} + \frac{1}{\sqrt{3}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\uparrow\rangle_{\mathrm{L}}\right\}$ then $\left\{\frac{2}{\sqrt{6}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\downarrow\rangle_{\mathrm{L}} + \frac{1}{\sqrt{6}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\uparrow\rangle_{\mathrm{L}} - \frac{1}{\sqrt{6}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\uparrow\rangle_{\mathrm{L}}\right\}$!

$\overline{\mathrm{F}}$: "This entanglement of M and L relative to $\overline{\mathrm{L}}$, gives a probability of 1 in 6 for $\overline{\mathrm{w}} = \mathrm{ok}$ provided F's measurement of $\mathrm{S} = \uparrow$. And at n:21 I can duly note that 'I calculate a probability of 1 in 6 for $\overline{\mathrm{w}} = \mathrm{ok}$ provided F's measurement of $\mathrm{S} = \uparrow$'."

$\overline{\mathrm{F}}$: "Likewise at n:30 I assume W measures L with respect to the basis $|\mathrm{ok}\rangle_{\mathrm{L}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} - |\uparrow\rangle_{\mathrm{L}})$, $|\mathrm{fail}\rangle_{\mathrm{L}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} + |\uparrow\rangle_{\mathrm{L}})$. Again the superposition for the combined LM system relative to $\overline{\mathrm{L}}$, assuming $\overline{\mathrm{W}}$ is maximally entangled with W within M, is $\left\{\frac{3}{\sqrt{12}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\mathrm{fail}\rangle_{\mathrm{LM}} + \frac{1}{\sqrt{12}}|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\mathrm{ok}\rangle_{\mathrm{LM}} - \frac{1}{\sqrt{12}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\mathrm{fail}\rangle_{\mathrm{LM}} + \frac{1}{\sqrt{12}}|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}\mathrm{M}}|\mathrm{ok}\rangle_{\mathrm{LM}}\right\}$. At this point, assuming the relative interference state still holds, I can note that "there is a 1 in 12 probability that at the end of the experiment both $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ and $\mathrm{w} = \mathrm{ok}$, provided that F measures $\mathrm{S} = \uparrow$."

n:40 – F finishes furiously writing these relative-state notes down in the Lab Journal just as the lab's emergency lights pulsate, a klaxon sounds and $\overline{\mathrm{L}}$'s computer says "The experiment is halted, disengaging Quantum Isolation Drive, have a nice day!" And so the experiment ends with $\overline{\mathrm{F}}$ joining everyone else in one of the non-orthogonal M branches to compare notes.

Narrative 3—Observer F's perspective

F enters L, happily punches the big red Launch button on the main console, the labs emergency lights pulsate and a klaxon sounds as L's computer says "Hello F, engaging Quantum Isolation Drive, welcome to perfect isolation."

F has been dreaming of this day their whole life long—to be literally and perfectly physically severed from the universe of others, de-correlated and cast adrift in their very own Bayesian bubble: "Oh Fuchs, if only you were alive to experience this perfection for yourself!"

F peruses the F-R experimental procedure: "Not a lot for me to do here is there?" And placing their Shakti Mat on the floor F settles into Anapana meditation feeling the touch of breath on the upper lip, knowing the in–breath … knowing the out–breath …"

n:10 – The lab's emergency lights pulsate as a klaxon sounds and L's computer announces "Inbound correlation tunnel initiated, teleport executed, tunnel closed". F exclaims "How exciting, a correlation with $\overline{\mathrm{F}}$'s bubble!"

n:11 – F measures S with outcome $z = +½$.

n:12 – F then notes in the Lab Journal that “I am certain that I believe that $\overline{\mathrm{F}}$ cannot know the outcome of my spin measurement of $S = \uparrow$. And given that measurement—and the assumption that the procedure was followed correctly—I am also certain that I believe that $\overline{\mathrm{F}}$ knew that $r =$ tails at time n:01.”

n:13 – F consults the F-R paper, Table 3, page 8: “Therefore … apparently I should now be ‘certain that F is certain that W will observe $w =$ fail at time n:31’ … ummm yeah riiight! Don’t get me wrong but while $\overline{\mathrm{F}}$ is perfectly free to assign whatever probabilities make sense from the perspective of their own bubble, seriously what does that have to do with me?”

F consults a QBist self-help manual[10]: “So what would Mermin do? And what exactly can I be certain of in this case? Given that I personally experienced the empirical reality in L that $S = \uparrow$, then from the moment that outcome was determined at n:11, I could justify the belief that sometime between n:02–09 $\overline{\mathrm{L}}$ probably prepared the particle with spin $S = \rightarrow$.” F duly notes in the Lab Journal that “at n:13 I believe that my personal knowledge of $\overline{\mathrm{L}}$ is $\{|\mathrm{t}\rangle_{\overline{\mathrm{L}}} |\rightarrow\rangle_s\}$.”

“As for predicting any future outcomes … well I assume that at n:20 $\overline{\mathrm{W}}$ measures $\overline{\mathrm{L}}$ but that outcome’s irrelevant to me in L, in fact it might as well not even happen! Then at n:30 W will perform their measurement of L, from M, with respect to their basis $|\mathrm{ok}\rangle_{\mathrm{M}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} - |\uparrow\rangle_{\mathrm{L}})$, $|\mathrm{fail}\rangle_{\mathrm{M}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} + |\uparrow\rangle_{\mathrm{L}})$. Given that W will interact with the state L that I currently believe to be $\{|\mathrm{t}\rangle_{\overline{\mathrm{L}}} |\uparrow\rangle_s\}$, then from my bubble in L, the probability that $w =$ ok in M will be given by $\left\{\frac{1}{\sqrt{2}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}}|\mathrm{fail}\rangle_{\mathrm{M}} - \frac{1}{\sqrt{2}}|\mathrm{t}\rangle_{\overline{\mathrm{L}}}|\mathrm{ok}\rangle_{\mathrm{M}}\right\}$. Beyond that, W’s interaction is of no more interest to me.” Thus F notes in the Lab Journal that “At n:31 there is a 1 in 2 probability that $w =$ ok”.

Following this F sits back down on their Shakti Mat and enters a serene state of meditation, wilfully oblivious to all distractions until n:40 when the lab’s emergency lights pulsate, a klaxon sounds and L’s computer says “The experiment is halted, disengaging Quantum Isolation Drive, have a nice day!” F gathers their things and somewhat reluctantly exits L to rejoin the team in M to compare notes.

The Group Narrative

Our largely un-correlated narratives now join back together in the main lab M, where all four colleagues are seated in a circle reading through the collated Lab Journal notes and, so far, politely discussing the outcomes.

$\overline{\mathrm{F}}$, holding the F-R paper in one hand and Lab Journal in the other: “To start with, the F-R reasoning in Table 3 on page 8 devolves from W knowing at time n:26 that $\overline{\mathrm{W}}$ knows at n:22 that F knows at n:12 that $\overline{\mathrm{F}}$ knows that $r =$ tails at time n:01. So far so good and this is of course supported in the lab notes. What I’m not so sure about is …” $\overline{\mathrm{F}}$ flips back a few pages “… this bit here on page 4 where F-R then make the claim that, based on my statement at n:01 that 'the spin S will be in state $|\rightarrow\rangle_S$ at time n:10,' it is then reasonable to assume that I could conclude from this that the later state of the lab L at n:10-20 is $\left\{\sqrt{\frac{1}{2}}(|\downarrow\rangle_{\mathrm{L}} + |\uparrow\rangle_{\mathrm{L}})\right\}$, which will then be orthogonal to $|\mathrm{ok}\rangle_{\mathrm{L}}$ for W in M when they finally get around to measuring it at n:30. And apparently based on *this assumption alone* it follows by inference that I might then note at n:02 that 'I am certain that W will observe $w = \text{fail}$ at time n:31', followed by F inferring the same at n:14, $\overline{\mathrm{W}}$ likewise at n:24, and W at n:28 just before they measure $w = \text{ok}$. This just seems like a long bow to draw, although as you will note from my lab notes in $\overline{\mathrm{L}}$ that's precisely what I did conclude from an objective collapse perspective."

$\overline{\mathrm{F}}$: "However, from an Everettian perspective…"

$\overline{\mathrm{W}}$: "Oh here we go!"

F: "Now now $\overline{\mathrm{W}}$, let $\overline{\mathrm{F}}$ speak."

$\overline{\mathrm{F}}$: "… I just think it's not at all clear what kind of physical reality we can assign to a quantum state…"

$\overline{\mathrm{W}}$: "How about we just use the term physical reality to mean physically real things? Yeah? Ok? And then use the quantum formalism to approximate to a very fine degree indeed how that underlying reality functions? What more do you need?!?"

F: "What I think we've got here, is a failure to communicate. And you know, whichever way you wish to interpret your experience I think we can at least agree to disagree in good faith. What about you W, how do you feel about this, in yourself?"

W: "I don't know, why can't we just stick with the data and not argue about the metaphysics?"

$\overline{\mathrm{F}}$: "Please, if I might continue! W's measurement of L is …"

And so we leave our four intrepid colleagues where we began, entangled in a tangle of non-sequiturs that is as old as quantum physics itself. But if I might also now take your leave with this final thought, dear Reader, what do you make of this our Narrator's narrative? Our shared, overarching perspective that flits from one place to another across isolated worlds and yet only ever from our own individual isolation? Is there not something uniquely familiar and yet also universal about this observational experience? For are you not, as are we not all of us, Wigner?

Frauchiger–Renner Experimental Procedure:

1. Agent $\overline{\mathrm{F}}$ enters lab $\overline{\mathrm{L}}$ and F enters lab L, both then engage their respective Quantum Isolation Drives. Agents $\overline{\mathrm{W}}$ and W stay behind in the main lab M so as to perform the measurements on the perfectly isolated $\overline{\mathrm{L}}$ and L respectively.
2. At time n:00 $\overline{\mathrm{F}}$ invokes a quantum randomness generator that outputs $\mathrm{r} = \text{heads (h)}$ or $\mathrm{r} = \text{tails (t)}$ with probabilities $1/3$ and $2/3$, respectively.
3. $\overline{\mathrm{F}}$ then sets the spin S of a particle to $|\downarrow\rangle_S$ if $\mathrm{r} = \text{heads}$ and to $|\rightarrow\rangle_S$ if $\mathrm{r} = \text{tails}$, and sends it to F.
4. At n:10 F measures the spin S in the $|\uparrow\rangle_S, |\downarrow\rangle_S$ basis recording the outcome as either $\mathrm{S} = \uparrow$ or $\downarrow$.
5. At n:20 $\overline{\mathrm{W}}$ measures lab $\overline{\mathrm{L}}$ with respect to the basis $|\overline{\mathrm{ok}}\rangle_{\overline{\mathrm{L}}} = \frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\overline{\mathrm{L}}} - |\mathrm{t}\rangle_{\overline{\mathrm{L}}})$, $|\overline{\mathrm{fail}}\rangle_{\overline{\mathrm{L}}} = \frac{1}{\sqrt{2}}(|\mathrm{h}\rangle_{\overline{\mathrm{L}}} + |\mathrm{t}\rangle_{\overline{\mathrm{L}}})$ and obtains the result either $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ or $\overline{\mathrm{w}} = \overline{\mathrm{fail}}$.
6. At n:30 W measures lab L with respect to the basis $|\mathrm{ok}\rangle_{\mathrm{L}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} - |\uparrow\rangle_{\mathrm{L}})$, $|\mathrm{fail}\rangle_{\mathrm{L}} = \frac{1}{\sqrt{2}}(|\downarrow\rangle_{\mathrm{L}} + |\uparrow\rangle_{\mathrm{L}})$ and obtains the result either $\mathrm{w} = \mathrm{ok}$ or $\mathrm{w} = \mathrm{fail}$.
7. At n:40 if $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ and $\mathrm{w} = \mathrm{ok}$ then the experiment is halted. Otherwise repeat from 2.

The Lab Journals

n:01

$\overline{\mathrm{W}}$: I assume that from n:00 $\overline{\mathrm{L}}$ is in a superposition of heads and tails relative to M.

W: I assume that from n:00 $\overline{\mathrm{L}}$ is in a superposition of heads and tails relative to M.

$\overline{\mathrm{F}}$: At n:01 r = tails and the spin S will be in state $|\rightarrow\rangle$ at time n:10.

n:02

$\overline{\mathrm{F}}$: I am not actually in a superposition contrary to Table 1's time evolution.

$\overline{\mathrm{F}}$: From a relative-state perspective both L and M are in a superposition of heads and tails relative to $\overline{\mathrm{L}}$.

n:11

~~$\overline{\mathrm{F}}$: I am certain that W will observe w = fail at time n:31, at least from the perspective of the objective truth of the collapse postulate.~~

n:12

F: I am certain that I believe that $\overline{\mathrm{F}}$ cannot know the outcome of my spin measurement of S = ↿. And given that measurement—and the assumption that the procedure was followed correctly—I am also certain that I believe that $\overline{\mathrm{F}}$ knew that r = tails at time n:01.

n:13

F: At n:13 I believe that my personal knowledge of $\overline{\mathrm{L}}$ is $\{|\mathrm{t}\rangle_{\overline{\mathrm{L}}} |\rightarrow\rangle_{s}\}$.

n:20

$\overline{\mathrm{W}}$: I calculate a 5 in 6 chance that $\overline{\mathrm{w}} = \overline{\mathrm{fail}}$ and 1 in 6 that it's $\overline{\mathrm{ok}}$.

F: At n:31 there is a 1 in 2 probability that w = ok.

n:21

$\overline{\mathrm{W}}$: $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$.

~~$\overline{\mathrm{F}}$: I calculate a probability of 1 in 2 for $\overline{\mathrm{w}}$ = ok.~~

$\overline{\mathrm{F}}$: I calculate a probability of 1 in 6 for $\overline{\mathrm{w}}$ = ok provided F's measurement of S = ↿.

n:22

$\overline{\mathrm{W}}$: Given that I'm measuring the composite system $\overline{\mathrm{L}}$L, I am certain that $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ and therefore also that F knew S = ↿ at time n:11.

n:26

W: I am certain that $\overline{\mathrm{W}}$ knows that $\overline{\mathrm{w}}$ = ok at time n:22.

n:31

W: At n:31 I am fairly certain that after $\overline{\mathrm{W}}$'s measurement outcome of $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$, there is a 1 in 2 probability that w = ok.

~~$\overline{\mathrm{F}}$: there is no chance that at the end of the experiment both $\overline{\mathrm{w}}$ and w = ok if r = tails. Therefore, the F-R paradox = true from an objective collapse perspective!~~

$\overline{\mathrm{F}}$: there is a 1 in 12 probability that at the end of the experiment both $\overline{\mathrm{w}} = \overline{\mathrm{ok}}$ and w = ok, provided that F measures S = ↿.

n:40

W: "w = ok"

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