The secret love affair between It and Bit

Trouble in Olympus

Hera came to Zeus, furious as never before."Where are my peacocks?"

Zeus shrugged, "Our physical reality is down. I forgot to update it with information. No physical reality, no peacocks."

"What does information have to do with physical reality?"

"Reality gets its form from somewhere."

Hera objected, "What was wrong with the old Platonic Forms?"

Athena told her, "Na, Kant had the same idea. Then Lobachevsky's results knocked the stuffing out of it."

Hera pouted. "I insist on getting a new reality. Or at least to repair this one well enough to get me my peacocks. As I recall, we got the last one from Great Granddad. Is he still able...?"

Zeus told her, "Chaos? He didn't exactly plan it. He doesn't plan anything. He had an affair with Chronos and... well, somehow, some regularities came out of it, like a great attractor in a fractal. That was physical reality. Athena, where's the old boy now? "

"Last I knew, he was amusing himself with oracles... Themis on one side, and the ones in Turing machines, those idealized computers, on the other."

"He'll need a refresher course to be able to repair our physical reality. But without humans...."

Athena turned to her secretary, "I have an idea. Iris, take a message to my half-brother: 'Head for the underworld, cross the Styx and turn right to the Elysian Fields. There get shades to help you compile a manual about the relationship between information and reality.'"

When Iris conveyed the message, Hermes asked, "There are a lot of shades."

Iris said "You're a god; you'll know their names. But the names are unimportant unless you already know them, or want to google them to see their connection to their statements. They're just shades making remarks that are related to their former lives. Concentrate on their ideas, not their avatars."

What to avoid

Hermes got to the top of a hill in the Elysian Fields. He bellowed, "I need a definition of... mmph ..."

The shade of Søren Kierkegaard took his hand away from Hermes' mouth. "You were about to say the R-word, weren't you? Are you suicidal? That's a group of paid-by-the-word philosophers down there. Better ask the shades of physicists."

"Ah, they have an accepted definition, do they?"

"Hm...not completely. But at least their arguments are straightforward." The shade of Kierkegaard looked down the hill. "Oh, blimey. You've already attracted the attention of a few philosophers. Look, if they get involved, talk about information in the same way as one talks about physical reality. It's only fair. As in relativity, where an observer can be any particle, not just a human. Just don't get downright Platonist about it; call it 'epistemological objectivity' or something."

The dispute

Hermes briefly explained his mission to the next group of shades he encountered. Niels Bohr (that is, his shade) said, "Physical reality is observation."

Wheeler appeared and said, "Right, Niels. Each measurement is the answer to certain yes/no questions. But that's information. Hence reality comes from information. *It from bit*. [26]"

Heisenberg objected: "John, information can't predict all the answers of the questions that reality throws it."

Wheeler responded, "Werner, the questions have to make sense. Your own uncertainty principle shows that a question asking for having no spread on the probability distribution of, say, momentum and a finite spread for position is just a nonsensical question. You might as well ask how you write zero in Roman numerals."

Dirac added, "But one *can* ask for the collection of possible values. Eigenvalues is what I like to call them. So that is what information gives you. But that information does not tell me *which* eigenvalue will happen. Reality gives information structure. So reality is primary."

Wheeler retorted, "I didn't say that information had *temporal* primacy. True, *information* cannot predict reality, but *reality* cannot completely predict reality either."

Boltzmann, Shannon and von Neumann huddled together and agreed to put aside their minor differences about entropy [16], and hence of information; Boltzmann spoke for the group, "Information's an abstract concept, but we've always applied it only to physical situations." Rolf William Landauer added, "You need energy to give information." Jacob Beckenstein chimed in with, "There's only so much information you can get into a given region of space." Asher Peres said, "Information cannot be transferred between two entangled particles without a signal between them, like light."

They all chorused, "So information is physical. Information cannot be primary."

Rosalind Franklin said, "But a gene *holds* information; a photon *carries* information. Where's the word *is*?"

Wheeler said, "It's too early to define..."

Leibniz objected, "Usage gives meaning? C'mon, John, formalize..."

Hume mused, "Try this: physical reality is perceptions."

Galileo said, "Not enough, David; reality is more than disconnected measurements. It's also their structure."

Bohr said, "How about: reality is anything that produces a change so that the beginning and end states can be distinguished."

Berkeley asked, "By whom?"

Feynman retorted, "By *what*. Humans are only an example. Yes, this is a participatory universe, as Wheeler likes to say, but it is a participation by everything."

Thomas Young suggested, "This obviously includes energy." James Maxwell added, "...and fields."

Plato asked, "What about the mathematical tools used?"

Dedekind said, "No. Look at infinity; no one calls it real."

"Embedded dimensions," Einstein remarked, "like in the rubber-sheet analogies, are just easier to calculate with."

Hermes asked, "And entities which are predicted?"

"Depends," Paul Dirac said. "Virtual particles are, well, virtual. Not real."

"But the universe beyond the observable universe," Hubble objected, "...or the centers of black holes," continued Karl Schwarzschild, "are considered real, based on indirect evidence."

"But whether parallel universes are real," bitterly said Hugh Everett, "is still debated."[12]

Karl Pearson asked, "And those tiny things? You calculate them using something called a wavefunction. Is that just statistics?"

"I doubt it," said John Steward Bell. "Theorems– from me [8], Kocher-Specker [1], and Pusey-Barrett-Randolph [23]—as well as the occasional experiment [21] hint that they're real. Reality is fuzzy." He motioned to David Bohm. "You can ignore his grumbling; he's still hoping that someone will work through a loophole."[20]

Hermes pouted. " Despite Photoshop, mirages, smart lawyers, optical illusions, psychedelic drugs, clever magicians, and so forth, you still just group a few allegedly physical events together, formalize with mathematics, generalize, then do all that in reverse, and declare it reality."

Karl Popper said, "It also has to fit like a jig-saw; pieces that don't fit aren't real."

Emmy Noether said "Then efficiency, or laziness, produces regularities."

"Like, say, a brain in a vat?" Plato asked.

Popper shrugged. "If you wish. The modern form of that question is: 'are we data in a computer?' "

"I read in a newspaper about a German team that simulated our universe on a computer." Hermes hazarded. "Sounds like *it from bit* to me."

"Newspapers oversimplify. More precisely, they [4] first made some assumptions about such a computer, and then what testable consequences would appear in our universe if we were a simulation. It would be a classical computer; precisely its limits would be indirectly responsible for some of the effects. One might extend that with David Deutsch's results [11] which hint that the simulation might be done on a quantum computer [22]. But in any case, then there are rules guiding the vat, or computer, or nature, or physical reality, or whatever. Rephrasing the question doesn't change much, except maybe to make it easier to answer. The best phrasing today is asking about the relationship between information, whatever that is, and physical reality, whatever that is. "

"Before all this quantum stuff, the idea of physical reality as a computer seemed reasonable," Konrad Zuse explained. "There was physical reality described in mathematics, which could be reduced to logic, which could be reduced to binary, which was information for a computer."

"But wasn't the mathematics *about* something?" Hermes asked.

"There was the rub. Perhaps there was something basic...." Zuse answered.

Willard Quine interjected, "...and these were formalized as urelements, elements which belong to sets, but are not themselves composed of elements, despite not being the empty set."

Ernst Zermelo stepped in, "But in set theory [13], you assume the empty set and infinity exist, and..."

Abraham Fraenkel continued, "...and include the idea of the Power set, the set of all subsets of a given set,..."

John von Neumann interjected, "...with the union of the Power sets of everything that came before, starting from nothing...."

Felix Hausdorff finished, "... with an additional axiom about something called inaccessible... no, let's make that measurable cardinals,[7] the mathematics used in quantum theory can be modeled by this or a similar set theory."

Georg Cantor remarked, "Form versus content. The hierarchy of sets is form which provides its own content. No urelements necessary, albeit possible. So whether you think urelements exist is a belief thing."

Saunders MacLane started, "From the perspective of Category Theory..."

Hermes asked, "That's with objects and arrows, right?"

Samuel Eilenberg took up his colleague's idea, "That's it. Sometimes it is convenient to replace the objects by identity arrows; arrows which bring you back to where you started. It works. Form gives content."

Descartes said, "What about....well, our feelings of reality...."
William of Ockham replied, "René, you like the so-called Mind-Body problem. Go ask a good neurosurgeon. Either a complete description is somehow separate from the 'real thing', or, if they can't be distinguished from each other, not. When they make synthetic diamonds to be indistinguishable from real diamonds, watch the market drop."
"What would these urelements be?" Hermes wanted to know.
"Right now, they seem to be space and time," started Einstein, "or rather spacetime..."
" ...or maybe momenta and positions" said Schrödinger,
Bryce DeWitt added, "Or something else. Entanglement can seem to make a mockery of our usual concepts of spacetime. Julian Barbour [3] tries to eliminate time. The holographic theory gives an alternate version of space. Brian Greene [17] speculates that a theory of quantum gravity may eliminate spacetime."
"It doesn't much matter what the universe of the theory is; everything is relations," said Einstein."One uses collections of relations: metrics, geometries, and so forth."
"Do you have to describe it with set theory?" Richard Dedekind asked.
"No, but it's convenient," Marshall Stone replied.
"Why not? Theories that have the same model are equivalent." Paul Cohen assured him.
Socrates asked, "Reality is emergent from mathematics?"
"Maybe," Gottlob Frege mused. "But I want to go back to structure versus content. I've heard that physical reality gives 'meaning' to information. What, then, is 'meaning'?"
Abraham Robinson said, "I believe my field in mathematics can help with that."
Shades asked, "What's your field? Lie Groups? Information Science? Statistics? Functional Analysis? Differential equations? Harmonic Analysis? Non-commutative geometries?"
Robinson replied, "Model Theory."
Silence.
Finally someone asked, "I thought a model *was* a theory."
"The words 'model' and 'theory' have very special meanings in my field."
The listeners all knew of special terms that were peculiar to their individual disciplines that contradicted the uses in other disciplines.
"I can't give you all the details at once, so I will simplify a bit to give you the taste."
"No formulas, please," implored Hume.

Model Theory in a nutshell

"Fair enough. The main ingredients are: theories, models which include an interpretation map, and truth maps. A model is made up of collections, aka classes. The first class of entities is called the universe. Urelements if you want. Maybe, spacetime points, paths, strings, branes, spin networks, an idealist's perceptions, or even relations from other models, such as geometries. Whatever."
Cantor said, "We use all sorts of infinities in our theories. [7], [13] Does the model have to be as big?"
"No. For example, you can talk about as many points as are in a line in your theory, but that can be satisfied by having, say, only as many entities as can be counted in the model. Ask Skolem and Löwenstein over there about that. [13]"
Then come subclasses of the universe, collections of subclasses, and so forth. Relations could include fields. Collections of subclasses might be laws. A theory is a collection of strings of symbols formed

according to a grammar. By themselves they have no meaning. The interpretation map associates the symbols and strings with entities from a model. In first-order theories, the variables and constant-symbols correspond to members of the universe. In higher-order theories, certain symbols can also be interpreted by classes. The truth map is specific to its model; it measures how well the theory matches with the model. Quantifiers in the statements give detail about the classes that satisfy their variables."

"Satisfy?" Hermes stopped him.

"Patience. If model-entities are substituted for the corresponding symbols (as designated by the interpretation) in a sentence, and the result is one of the classes in the model, then the model satisfies that sentence, and the sentence is called true under that model. If every sentence is satisfied by the same model, then that model satisfies that theory."

"There can be several levels, so that a model can have a theory, but in the theory you can have a model construction. In other words, a model A for a statement 'B models C'."

Hume started to philosophize, but Robinson turned on him. "You again! We *could* hypothesize an ultimate model, modeling everything, but not fully describable by its theory."

"But..."

"The medieval monks said that a description of God was actually describing some aspect of one of God's creatures. It was called the Reflection Principle; the analogy in set theory [13] says that, although the universe of its model is not describable in the theory, there are sets in the theory that are similar in some ways to the universe. So, the ultimate model may or may not exist, and you can believe what you want. Go chew on that.... somewhere else."

"Get to the Physics," shouted someone.

<u>Old Physics in new clothes</u>

"The problem is that the standard mathematical theory lumps the various levels of models, truth, interpretation maps, theory all together into an indistinguishable lump," Robinson continued. "If one however restates the mathematics in the terms, one will be able to distinguish the information, which is the structure, from the physical reality, which are the models and relationships between them. It is obvious that one tends to think of classical logic as disconnected from physical processes because of the lack of things like time in their formulation."

"But that's where temporal logic [9] comes in," protested Arthur Prior.

"True. Logic then left the realm of being only about thought, just as mathematics is used for the type of physics which is stated in objective terms. There are lots of other examples of logic coming out of the closet to form a basis for physics." [5].

"You're using the active tense. This implies it's still dependent upon a human observer," broke in Hume.

" I'm trying to use the passive, but that gets tiring. The language is built that way. 'One looks' can mean any interaction; the 'one' doesn't have to be human. Epistemology and ontology do have valid differences, but a lot of results about one can be applied to the other if one is careful when one inserts sentences using space and time."

"I still think..."

Robinson talked a bit louder to drown out Hume. "To describe a cycle is always tricky: you have to start somewhere, and there will be people who conclude that the cycle starts there. But I'll risk it, and start with the existence of sentences, of structure. Any theory has lots of models. In order to try to pinpoint one, nature plays a sort of game [19] with alternating the quantifiers 'there exists...' and 'for all....' until you get down to 'there exists exactly one individual x such that (something about x)' for an individual x, or 'for all x there exists a y such that (something about x and y). If it gets down to 'there exists exactly one...', then we either find or, if it doesn't exist, create a corresponding symbol in the theory."

This creation is called skolemizing," grinned Skolem. "If it didn't exist, you add it to the universe of your model to expand your model."[7]

"Or you can look at it the way I do in forcing [19]," added Paul Cohen. "Expand the domain of your truth values to new sentences, thereby expanding your model."

"However," Robinson continued, "sometimes the game resembles more a dishonest shell game whereby the pea will only appear in a cup after you have chosen the shell. You know however that the pea is there, but, in order to be sure of finding the pea, you would have to look at all of them at once, something the con-artist does not allow. In other words, sometimes these existential sentences get down to the existence of a class, and can't go further. There are lots of relations that the model must abide by, relativity, and so forth, but there are still lots of leeway for different models that are not submodels of each other. This leeway is translated into truth values."

Robinson looked around. "Anyone object to a little analysis of existence quantifiers? No? We can then rephrase 'there exists an individual **x** so that **x** has a property **P**' as 'there exists a class **S** so that for every element **x** of **S, x** has a property **P**'. We can then extend this to find a maximal **S** in the model, looking for the satisfaction of 'there exists a class **S** so that **x** has a property **P** if and only if **x** is an element of **S**.'"

Robinson accompanied this by diagrams. Nonetheless Hume objected. "You promised not to use formulas!"

"Sorry. I try to avoid them, but sometimes they're like abbreviations. Anyway, this **S** becomes the universe of a submodel. But there will be lots of submodels to this one. From these truth values, you can figure out the respective probabilities that a value will result if a measurement, a transfer of energy, were to occur. "

Pearson asked, "Why, if we are interested in probabilities, we keep using these truth values?"

Dirac answered, "Some nuances would be lost, such as how they combine. Non-zero truth values can add up to zero, and quantum probability doesn't work quite like classical probability."[18]

"So, a further constraint on the truth values is that the associated probabilities must follow a tweaked probability logic" [6] said Jan Łukasiewicz.

Robinson continued. "A sentence can of course have several variables it asserts the existence of. So we get truth values on a circle for every variable; together, we represent them as a single arrow in a sphere created by all these circles, each orthonormal.... like perpendicular when you do this in space....to all the others. These truth values are then shifted around, ordered according to implication of the corresponding sentences, and so forth. This is the topic of quantum logic [8]. The arrow represents a class, or for an information scientist, a qubit."

Franklin strolled up and reflected, "Just think, one little bit of DNA contains, in a way, a tinker, tailor, soldier and spy, all these futures superimposed on one another."

Feynman pointed out, "Those spheres of fuzzy truth values are like snapshots, but the snapshots are joined together in the time evolution to make a bit of cinema, in which arrows are still the stars. It's useful, for example, to explain the interactions between electrons and photons with the addition of these arrows."[14]

Robinson said, "There are special 'operators', which shift the evaluation of the sentences from one truth value to another, rotating the arrows for each sentence, such that a new model is formed. These are called Hermitian operators. No, Hermes, not named after you. Up until now, we have described a ball, but it has to have axes. Remember we mentioned the submodels? The sentences associated with the axes are the minimum ones, the ones that also satisfy the sentence 'there exists exactly one **x** such that...'.The Hermitian operator finds the axes and the possible values for a given measurement simultaneously; this operator acting on the unit arrows of the axes gives the value associated with those axes. The probability to go from one state to the other one is given by a relationship called the inner product. Then the axes are given as certain states which will show up on the measuring device, and the state for one variable is expressed in terms of the inner product of that state with the respective axes in each circle."

William Hamilton said, "If there is a closed system, that is, no interaction between different systems, then the system evolves in time by itself: this Hermitian operator is called a Hamiltonian," he smiled, "shifts the truth values ...".

Schrödinger continued, "...as described by my equation. For example, as a photon moves along, the probabilities of the magnetic and electric fields shift in time according to the energy of the photon. "

Robinson picked up the dialog to bring it to the next stage. "An interaction between two physical systems combines your systems into a single closed system, which is greater than the sum of its parts.

Karl Pearson, "Sounds like combining a couple of probability distributions, with the caveats you mentioned."

"Sort of, but the analogous operation to multiplying all the probabilities is called a tensor product, and then it is completed," Gregorio Ricci-Curbastro explained.

"Completed?" Hermes asked.

"Geometrically, think of adding a new axis to a circle. If you complete this arrangement, you will get a sphere. You get points that were not in the circle, and not on the new axis. When you complete the result of taking the tensor product of two systems, you are combining sentences which then will together generate sentences which were not in the original arrangement, and which could not be obtained by taking the tensor product of any two individual sentences. In other words, you get emergent entities. The emergent states are called entanglements," Robinson said. Then he looked guiltily around. "Um, I warned you that it would be a bit simplified..."

David Hilbert laughed. "We don't expect you to rig up the entire space, Abraham. Just keep going, we'll forgive you."

Robinson smiled and continued. "Anyway, this new completed combination makes up a new unified system, with a new set of values for the model, and a new theory. This system which will have its own Hamiltonian, and so develop along with it, shifting arrows as time goes on for its group of potential values."

"Possible, potential.... it seems rather too abstract to be real," Bohm objected.

"Still grumbling?" asked Robinson. "You don't object to the reality of potential energy, do you? But OK, we'll bring it down to what you think of reality. Measurements. Measurement only looks at some of the axes and their closures. A subsystem of the whole system. A sort of cross section. If you include all the cross sections, that is all the different interactions -- your measuring instrument, the air in the room, and maybe parallel universes, you would have one huge ball; the arrows are still of a unit size. However, one cannot look at all of the ball at once, so one looks at only some cross sections, according to what is to be observed – so that the shifting arrows may seem to be changing size, a bit like perspective in art. The restriction of the Hamiltonian to the subsystem is another Hermitian operator, which again changes the arrows, the classes. That is, it changes the corresponding universe. The universe changing means that the possible values change, and of course the corresponding truth values. Measurement is identified with energy changes, which come in quantized packets, so the corresponding truth values of measurements come in quantized rotations of these arrows, which are identified with bits."

Hermes requested, "Maybe you could clarify that a little?"

"I'll try," said Robinson. "A truth value which does not lie on an axis corresponds to a class of more than one value. Without shifting the axes, the truth value would not correspond to sufficient energy to be measured. When the arrow lands on an axis, the corresponding sentence is being interpreted as 'true' while the other axes have truth value 'false'. When the operator produces a class of only one value, then a measurement is possible. True, the predictive value is not classical. But there are bits, even if only after the fact. "[2]. Anyway, coming back to the submodels: there are relationships among them. These submodels with these relations make up another model, called a Kripke frame [24]. Corresponding to these shifts are shifts of the theories..."

Carlos Alchourrón started in a lawyer's monologue, "There are people who are investigating shifts in axiom systems, starting from human belief, but ending up with a formalism that can be applied to other contexts[10], [15]. For example, often, if not always, the quantifier which stands for 'I believe that' can be otherwise interpreted as 'it is provable that', [18] and 'proof' of certain temporal propositions, with a few clauses about the light cones thrown in, can be causal chains. From there you can interpret the Kripke frame as physical reality."

Kurt Gödel said, "You know, Carlos, that would lead to some interesting twists. There are enough numbers so that every sentences can correspond to a unique number code; that is, so that the quantity and the code have the same interpretation. But the code is also interpreted as a number-sign. Therefore, if a sentence **S** and its code **s** are each interpreted by the same quantity **A** in the model, then we identify **S** with the number corresponding to **s**. We ambiguously use the word 'number' for both the number and its symbol."

Alfred Tarski continued, "Our diagonal theorem [24] could be used to guarantee that there are sentences that are true if and only if they are not caused. This is why the models, those circles, exist. That's why the game that nature plays stops short of causality when a sentence 'there exists' is true but 'there exists exactly one...' is not true. Otherwise put, there are also the sentences at any point in spacetime which are not decidable under the model used at that moment. But then the Kripke frame is forced by time to go along some branch, so the model is forced to expand in the way we discussed earlier."

"This sounds," said Bayes, "like my updating." [25]

Robinson objected, "Except that Bayesian updating, like Gödel's and Tarski's theorems, are usually interpreted as being applicable only to purely epistemological questions. They can, however, also be interpreted (with great care!) when referring to physical theories."

Robinson paused, seeing that philosophers were rushing over to drown him in treatises over this last statement. He quickly added, "The combination of causation, and the selection of undecidable elements, is what the expansion of spacetime is all about. Otherwise put, nature is continually trying, in vain, to complete itself." Then he grabbed Hermes and ran.

Conclusion

Safely out of range of the marauding philosophers, Hermes asked, "Tell me, what did you recount all that for? What do I tell Zeus?"

Robinson panted, "Your relative, Chaos, has become domesticated. Chronos, or whoever he's with now, gives him restricted possibilities, sometimes so restricted that it's not even a choice. But when there is a choice, then Chaos has a free hand inside those choices."

"But where is he?"

"All around you. He's easiest to spot in the little cracks, of course."

"OK, but I am still confused. I thought it was clear cut that models were reality, and the truth values, selecting theories out of all the sentences, formed the information."

"Not a bad summary."

"But it turns out that models can be really just parts of theories, and each theory can engender lots of models. So where is the dividing line?"

"What, you thought there was a clear dividing line? That's like thinking that there is a clear dividing line between mass and spacetime."

Wheeler had caught up to them; he quipped: "Reality tells information where to exist, and information tells reality what to do."

"So information and physical reality are inextricably entwined?" Hermes asked.

Robinson replied, "Entwined, yes. It from bit and qubit?" Sure; look at the creation of models, the computer simulations, and so forth. Bit from it? Sure; a model has a theory. However, I wouldn't use the word *inextricably*. Use the microscope of model theory. Are bit and it the same? Are Romeo and Juliet the same?"

Hermes said, "But... how do I use that to re-boot reality ....?"

Iris appeared and said, "Hi, cousin. Prometheus found the loose connection in the information cable, so physical reality is up and running again, complete with Hera's peacocks."

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