The Common Mechanics of Quantum Computing and the I Ching

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"Your task is not to foresee the future, but to enable it." ~ Antoine de Saint-Exupéry

Introduction

Imagine a roulette wheel with infinite slots, all of which save one are hidden from our view. Through a window, we see either a "1" or a "0" when the wheel is at rest relative to us observers. By the probabilistic interpretation of quantum mechanics, the numbers are in superposition; there is zero probability that any number physically exists until we observe it.

What if we assume instead that every slot – like the infinite rooms of Hilbert's hotel – is occupied alternately with each number? We get exactly the same probability, P(1/2), for a 0 or 1to appear – in this *ordered* distribution – as when we assume that the numbers are either *randomly* distributed in the slots or assumed in superposition. The Bell-Aspect result[1] tells us that only our real observed value, 0 or 1, is "local" at any moment of observation and the other value is nonlocal and nonphysical. A hypothetical locally real result that would simultaneously satisfy orthogonal physical properties, such as position and momentum – i.e., the wheel at relative rest, and the spinning wheel – is said to be dependent on our choice to interfere with the system's energy by "stopping" the wheel (coming to relative rest) or "spinning" it (accelerating in relation). In that sense, says quantum mechanics, the observer creates physical reality by adding energy to the system.

There was a novelty song on the radio (in the early 1960s, I think) titled "Time after Time." The lyrics were the same as the title, repeated over and over. A friend suggested to me that the title is actually "After Time After." Why not?—for a repeating binary series, does the initial condition make a difference?

Measured quantum states are characterized by a property called "spin." A particle in its lower energy excited state—the triplet state—is said to possess spin 1. The singlet state—the state of higher energy—has spin 0. As the names suggest, there are three possible triplet states and only one singlet state, given in terms of angular momentum:

To calculate spin angular momentum, one independently measures results on orthogonal axes (position/momentum). When reduced to a fictional "collapse of the wave function," the state vector compromising results between two orthogonal properties is unitary, $\langle\Psi|\Psi\rangle = 1$. Yet when we have an ordered continuum of 010101 … or 101010 … the measured results imply the linear maps:

$$\Phi 0 \longrightarrow \Phi\, 101\ldots$$

$$\Phi 1 \longrightarrow \Phi\, 010\ldots$$

In other words, the middle value that was unimportant in the nonlocal, probabilistic measure schema is the *only* significant input value in this deterministic argument. The arrangements, 010 and 101 resemble an I Ching[2] oracle where tossing three coins produces two heads and one tail in one case and two tails/one head in the other. Of course, all possible combinations are actually eight—000, 111, 010, 101, 110, 001, 100, 011— and an I Ching reading takes six tosses of the coins (or yarrow stalks, or 0s and 1s) to make a complete "hexagram" composed of two "trigrams" one atop the other (fig. 3).

Whether quantum bits or I Ching results, however, information that originates from a nonorientable horizontal range of continuous variables is read from a vertically oriented line of discrete values (fig. 1):

The "No Dead Cats" Hypothesis

The state vector of an event is never *prepared in the lower energy state.* (Ever hear of starting the Schrödinger experiment with a dead cat?[3]) All initial state vectors are in the highest state of energy, accounting for the experimenter's energy input—whether the experimenter is an actual human being or any other physical operator. The singlet state is trivial, infinitely regressive all the way to the cosmological initial condition. In the Schrödinger cat example,

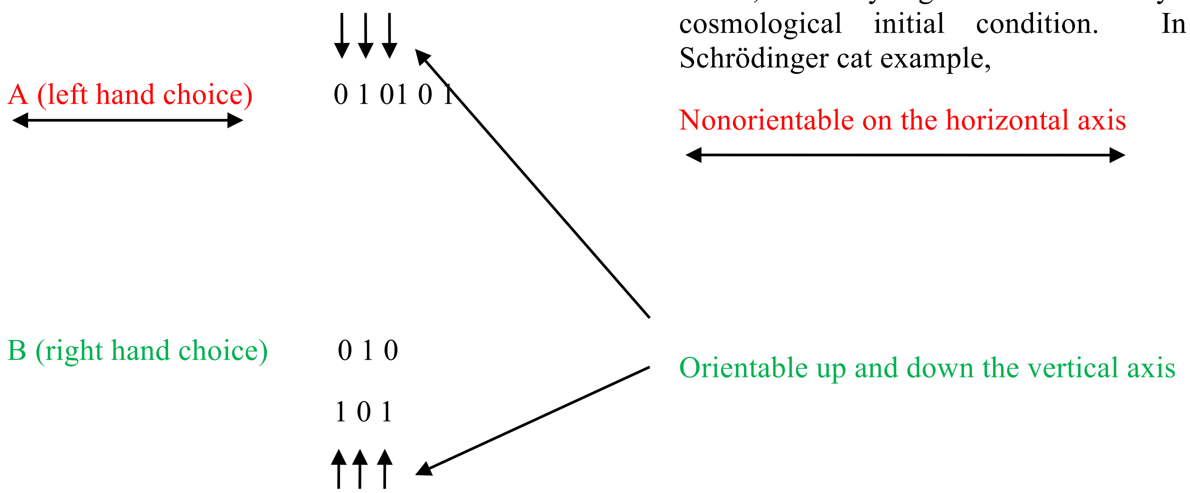

Figure 1. Discrete Bits from a Continuum of Its

We find that orientability of the state vector is a continuous function requiring unitary left hand and right hand choices, from a continuous range of pseudo-random variables on the horizontal axis. The right hand choice produces one well ordered bit [0,1) and one partially ordered bit [1,0). Let us use the well ordered bit [0,1) to symbolize the path from a higher energy state (spin 0) to the lower and [1,0) from the lower energy state (spin 1) to the higher.

Trouble is, in a *physical* model, by the second law of thermodynamics – systems left to themselves only orient [0,1). If we never stopped the wheel to observe a number, the system's momentum would be continuous and conserved as spin 0, i.e., with no change in energy.

the hammer that breaks the deadly vial of poison when a particle is emitted, is at its highest energy potential.

A 2-slit electron experiment finds the self interacting electron in its highest energy state at the wall, doubled in wavelength until the wavefunction ostensibly "collapses" with P(1/2) to either slit # 1 (D1) or slit # 2 (D2).

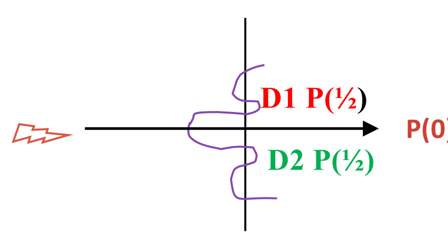

Figure 2. The Two Slit Experiment.

We can treat this numerically by assigning probability values to the question ("Did an electron pass this way?") asked of each detector # 1 and # 2.

+ ½ = yes - ½ = no

+ ¼ = ~ yes - ¼ = ~ no

The combined probability of yes and not-yes and no and not-no is never unitary. In other words, where each answer is valued probabilistically at P(1/2), perfectly random 50% probability for each slit, one expects – by the collapse interpretation – to encounter a click or no-click state at one of the detectors and assign either ½ - ½ = 0 (no click) or ½ + ½ = 1 (click). However, ~ yes and ~ no, ¼ + ¼ = ½ or ¼ - ¼ = 0 is the *actual* measurement result, for each slit, respectively, accounting for the whole system.

The collapse interpretation assumes, reasonably, that there is no negative probability. For if ½ = yes and – ½ = no, the sum at the no-click detector is not ½ + ½ = 1; it is - ½ - ½ = - 1. Then the sum of probabilities for a particle passing through both detectors simultaneously is – 1 + 1 = 0. Because we know that this is *not* the real physical result (the wave function had to send an electron through both slits simultaneously for us to observe a wave interference fringe) – we *assign* the probability of unity *a posteriori* to one slit or another and discard the negative probability.

The wave function collapse is illusion. "Zero" represents the doubled wave amplitude – {0|0} – the trivial singlet state identical to the complex plane origin of the Hilbert space – and the singlet state is only realized in the initial condition; it cannot be a real measurement result. Real physical results map:

$$\mathbf{\Phi\ (0)\ 1,\ 0 \longrightarrow \Phi'\ 1,\ 0,\ 1}$$

$$\mathbf{\Phi\ (0)\ 0,\ 1 \longrightarrow \Phi''\ 0,\ 1,\ 0}$$

The variables are continuous on the horizontal axis, and physically meaningful only on the vertical, where the physically real action is; i.e., where they can *change* continuously. The I Ching (*The Book of Changes*) "reads" that way, too:

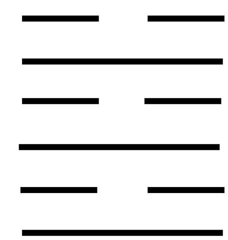

Figure 3. I Ching Hexagram (no. 63 of 64).

The lines could have been all solid, all broken and any combination in between, a total of 64 possible "hexagrams" and 64 X 2 = 128 "trigrams" above and below. Also, any or all of the lines could be "changing," i.e., oriented lines of two tails/one head or two heads/one tail. The 2-dimension representation of a hexagram therefore implies 24 degrees of freedom (4 X 6 above and below, backward and forward) to account for the changing results and their orientations. Like the I Ching hexagram, quantum mechanical results are written in 2 dimensions, with *n*-finite degrees of freedom in the Hilbert space.

I Ching and quantum mechanical structures share the common property of horizontal continuity and vertical discreteness. In other words, the flatness of the horizontal field can be understood only in the metric (measured) properties of the orthogonal axis describing values that are changing. Informed readers will recognize this as teleparallelism, promoted by Einstein to explain the dynamic relation between a continuous field of Minkowski spacetime and the metric tensor.[4]

Bounding the Continuum

While Einstein's continuous function model of partial differential equations requires assigning of boundary conditions – the Hilbert space model of standard quantum mechanics gives definite solutions; like an I Ching hexagram, a quantum result is a 2-dimension probabilistic answer in an *n*-dimension Hilbert space – which is why to make quantum mechanics physically comprehensible, one normalizes measurement results. Quantum computing oracles, exactly like the I-Ching oracle, apply to an interval that exists only *at the moment* the question is asked. What both the Hilbert space quantum mechanical model and the I Ching cannot do, however, is to reproduce the continuous function of field dynamics, Einstein's choice of Minkowski space which gives *meaning* to the metric tensor because it includes a time continuum.

Early on, quantum theorists decided that the 2-dimension discrete complex plane mathematics of non-ordered terms in the Hilbert space works so well, that no further meaning is required – continuous and reversible classical time is ignored and replaced with discrete state vector evolution.

This operational view lacks the first principles of a mathematically complete theory. Compare to the first principle of gravity—relativity—which is supported by the Galilean transform (and later by the Lorentzian), and has been known for hundreds of years. Gravity does not fit into quantum mechanics because one cannot derive a continuum of information from a bit of information, in any non-arbitrary way. Isn't this what Wheeler is telling us? – "The situation cannot declare itself until you've asked your question. But the asking of one question precludes the asking of another."[5] "It" – the answer to a question – whether one addresses one's inquiry to the I Ching oracle, a quantum computer, a favorite deity, or the universe itself – is only "it" for that moment, some fixed interval of changing spacetime. And changing spacetime curvature in classical physics is identical to gravity. Do answers change covariant to changes in spacetime curvature?

If Bit is a local book and It a global alphabet – an ordered continuum of It can theoretically exist on its own, while changing Bits cannot exist independently of It—which might suggest to us that things that are true for long intervals of space or time are not necessarily true for short intervals. Yet *some* obviously are! (E.g., planetary orbits and galactic rotations). So even though scale plays an apparent role (via the Planck constant) in locally definite measures – we ask, is scale a barrier to the indefinite global coherence of the wave function? Can we derive accurate 1 to 1 correspondence between a totally ordered global It and a partially ordered local Bit? There is no present algorithm that smoothly unites nonlocal future physical states with the local configuration space of quantum mechanics and quantum computing. If there were, some NP-complete problems such as protein folding would be a snap. I.e., we know how properly folded proteins are supposed to look—we just don't know how to continuously predict the folding path from the (computationally) 1-dimension string of amino acids, to a 3-dimension folded protein.

The long and short of NP-Completeness

Scott Aaronson asks a seminally important question – "Can NP-complete problems be solved in polynomial time using the resources of the physical universe?"[6] – arguing that NP-complete problems themselves possibly constrain physical theories. We find, however, that *infinite self similarity in any finite time interval* promises self organized, and therefore self limiting, maps of short (local) intervals to long (global) intervals. (Cf. Perelman's proof strategy for Thurston's geometrization conjecture.[7])

A local measurement "stops" the wheel; said in another way, a sensitive dependence on local initial conditions implies an infinite regress of global interval endpoints defined by the middle value input state. What constitutes the middle value depends on where the program halts – so if we wished to know, for example, the endpoints of a well ordered sequence of primes, we'd input an arbitrary median value of the sequence. We can freely choose by convention whether the subsequent answer to the left or right represents a drop in energy state—spin 1, or no change, spin 0 — ("… the asking of one question precludes the asking of another.") Suppose we input "17" as the median of the sequence and the program outputs a spin 1 change in energy which we have decided represents a left hand value, with the answer "3." The endpoints of the sequence 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37 are 3 and 37. So for every cardinal-odd-number well ordered sequence of primes, from the median at which we wish to interfere with the "spinning wheel," we can determine the endpoints non-probabilistically. What if the sequence has an even number of primes? – the right hand value will show no change (spin 0) in the energy state and will output 31, indicating that the sequence has no prime median and is evenly divided between 17 and its successor with end points 3 and 31. Up to the highest number of primes known, we have a way to precisely map pairs of median integers between short and long (well ordered) sequences of primes.

The change we record in integral spin values requires no nonphysical assumptions of nonlocality or superposition. We simply use the continuous "resources of the physical universe" and its law of thermodynamics. We have the advantage in the above example, of comparing a computation with a table of known values (as was done, e.g., with slide rule and logarithm tables before the digital computer age). Does nature possess such a universal table, to which we can compare our measurements in a non-probabilistic way? Let's see.

Positron, Positron, Who's Got the Positron?

In 1940, Richard Feynman got a phone call from his professor, John Archibald Wheeler: "'Feynman, I know why all electrons have the same charge and the same mass' 'Why?' 'Because, they are all the same electron!' And, then he explained on the telephone, 'suppose that the world lines which we were ordinarily considering before in time and space—instead of only going up in time were a tremendous knot, and then, when we cut through the knot, by the plane corresponding to a fixed time, we would see many, many world lines and that would represent many electrons, except for one thing. If in one section this is an ordinary electron world line, in the section in which it reversed itself and is coming back from the future we have the wrong sign to the proper time—to the proper four velocities—and that's equivalent to changing the sign of the charge, and, therefore, that part of a path would act like a positron.' 'But, Professor', I said, 'there aren't as many positrons as electrons.' 'Well, maybe they are hidden in the protons or something', he said."[8]

Let us consider that a two-point boundary value demarcates the infinities of energy exchange, i.e., boson interactions, from finite energy potential, i.e., fermion rest energy. Then the classical orientation-entangled processes sharing the same rate of change are at rest relative to one another, at any nonzero spacetime separation.[9] This fact underscores the classical assumption that *individual events are irreversible, while ensemble physical processes are not.* A perfect union of boson and fermion statistics would admit the Wheeler-Feynman conjecture as physically real. Is it demonstrably so? We will suggest that it is, based on the same principle that supports the physical reality of a continuous spacetime, and which obviates nonlocality. Perhaps the positrons are hidden, like Poe's purloined letter, in plain sight.

Breaking the Tie that Binds Again

The Pauli Exclusion Principle points to a clear analytical distinction between boson (integer spin) and fermion (fractional spin) statistics. Any number of bosons may occupy the same state simultaneously, which leads to Bose-Einstein condensation, while a fermionic phase of condensed matter decoheres when sufficient energy is introduced to break electron pair binding. That energy threshold is extremely low.

Now that Deborah S. Jin[10] has proven the experimental viability of a fermionic phase of superfluidity, we propose that the Pauli Exclusion Principle assures a fairer test of local realism for the reasons:

1) The bound state of electron pairs (Cooper pairs) is stable at sufficiently low energy. Therefore, a low-energy input of unbound pairs at the threshold of superfluidity is expected to momentarily break symmetry as the pairs bind—i.e., the binding energy should be released from the fermionic condensate as particle spin values of either singlet (spin 0) or triplet (spin 1) states.

2) This expectation results from conservation laws—for if energy released in the interaction equals energy input, momentum transfer (as opposed to annihilation) conserves total angular momentum. This is equivalent to annihilation without radiation; i.e., energy released as angular momentum neutralizes the energy content of the pair absorbed into the superfluid state.
Because the system outputs an integer spin value from input of non-integer values (fermion spin 1/2), we infer that if the output does not change randomly (i.e., a result of spin 1 or spin 0 is constant for a fixed orientation of the apparatus), the system state was predictable—determined—before measurement and not observer-created. Both position and momentum are therefore intrinsically encoded in the integer spin, as a precise measure of angular momentum, as clearly as that measured for celestial motion of bodies. Conservation of angular momentum for repeated events at a fixed orientation is then equivalent to classical orientation entanglement and obviates quantum entanglement as an element of reality—because at any input vector an observer will record a constant integer spin for repeated measures of a unique orientation, meeting the EPR requirement that "… every element of the physical reality must have a counterpart in the physical theory." We are assured that this is true, if change of orientation of the measurement apparatus determines state (spin 0 or 1), which is fully relativistic, i.e., there is no privileged frame. The system also obeys classical time reverse symmetry, because the apparatus is expected to reproduce the identical result at the identical orientation at any later time. *So we should be able to catalog a set of unique results for a continuous range of discrete orientations.* Since input orientation equals position and output result equals momentum, there can be no ambiguity—local realism holds for these simultaneously measured values. The entire universe is *not other than* locally real.

See a World in a Grain of Sand …

A rarely spoken assumption of both quantum mechanical formalism – and the I Ching – is that time itself has no physical reality beyond a probabilistic moment. Yet classical mechanics relies explicitly on an assumption of continuous functions with time-reverse symmetry. The quantum state of the condensate remains undisturbed by the infinite orientability of an observer testing the energy output of particle momenta *in situ*.

The fermionic phase of condensate (FEC) exactly reproduces the requirements of a primordial unit ball of spacetime, because unlike the Bose-Einstein Condensate (BEC) phase, its fermionic nature assures discrete components (by the Pauli exclusion principle) yet like the BEC, its spectrum of continuous energy exchange corresponds to a pure spacetime state; i.e., classical interaction of mass and spacetime is retarded to a massless state of zero angular momentum, which is a property of discrete particles and not a property of continuous spacetime. With the change of state introduced by e^+e^- , the transfer of angular momentum to singlet and triplet states of particle spin, 0 or 1, is the energy that sustains particle-spacetime interaction at the low energy classical limit. So in fact, the measure of angular momentum in one system is self-similar to the conservation of angular momentum in *every* system at that unique orientation regardless of space or time separation between the event and the moment of measurement, equivalent to classical orientation entanglement at multiple scales. That is, the variable rate of physical processes random for systems of discrete interacting particles is constant for the continuous geometry of spacetime. As Wheeler noted, a positron e^+ behaves like an electron e^- moving backward in time. So if e^-e^+ interaction defines a moment—an event—an ensemble of events defines a self-limiting process of e^-e^+annihilation. We know the process is self limiting, because quantum mechanical processes are *not* self limiting. Every e^+ created, for experimental purposes or naturally, is compelled to pair with its original historical partner, because a finite history in which every pair is annihilated demands that particle pairs are conserved as rest pairs. In other words, rest energy—defined as a (relative) fixed rate of change between two operators—is unique for some measured orientation. We could not know this, however, unless a backward in time e^+ limited the momentum of its mirror partner e^-. When we test the low energy momentum of e^-e^+ , we are seeing a local version of the state of the universe in the immediate (time reversible) past; higher energy takes us back to earlier historical states. No physical principle prevents Einstein's general theory—describing the world as finite and unbounded, assumed to be finitely bounded in time and unbounded in space—from being finite in space and unbounded in time. The non-arbitrary 2-point boundary condition of

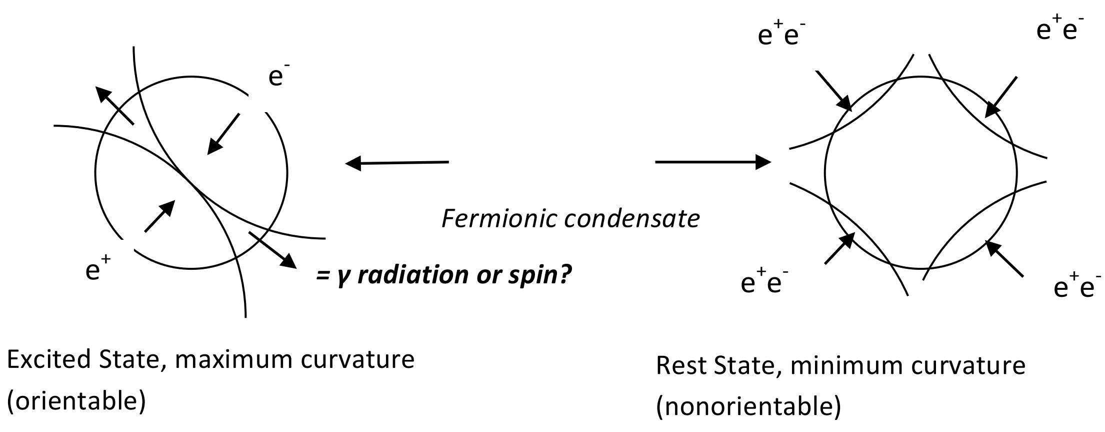

Figure 4. Rest energy in terms of spacetime curvature.

spacetime makes all our physical measures locally real, on the relativistic cusp of past and future. Orientability—a property of continuous topology—informs that time reverses, without hurting our sensibilities over why broken teacups don't reassemble themselves and hop back up onto the table. Point events are not compelled to reverse trajectory—if the metric of time is infinitely orientable—so continuous order is not, therefore, determined by the orientation of individual events; time is a geometric flow of reversible trajectory. [11]

Hold Infinity in the Palm of Your Hand …

This model explains the cosmological energy boundary, because the higher the rest energy, the more extended the time metric. Were it possible to reproduce the original singular state of the universe, we would find it to be 2-valued and nonorientable with the pairs at relative rest and the energy unchanging. So the fermionic condensate faithfully replicates the initial condition of primordial space, a simply connected network without boundary.

The remarkable thing about this relative state is that *the universe is still in a 2-valued state of relative rest*: Therefore, all anticorrelated particles in the present are anticorrelated in the past and in the future– every antiparticle ω^+ is primordial.

In terms of physical information, then, we have suggested that the ordered continuum is "It" and that partially ordered measures of information events (the "books" we create from the symbols) are the "bits" of how we understand "It" when we reorder information to suit physical conditions—"All science is the search for unity in hidden likenesses," according to the esteemed scientist and humanist Jacob Bronowski.[12] Observed likenesses, i.e., relations, between It-continua and discrete Bit-networks, were so pervasive in late 20th century science and continuing into our own century that Lee Smolin calls it a "relational revolution."[13] Time is real and laws evolve, says Smolin, an idea expressed in computational evolutionary terms by Gregory Chaitin.[14] Darwin's idea of common ancestry[15] assumes an undifferentiated Origin, a touchstone of unitary qualities regressing from species boundaries, to cells, to coding in cells, and finally to information alone. This algorithmic regression to the mean makes Chaitin's point: the world is made of information that organizes in continuously changing ways. And Smolin reinforces, "We live in a world that is always changing, full of matter that is always moving."[16] Exactly that – says *The Book of Changes*, the *I Ching*.

Is information identical to time? [17] If time is a geometric flow, the orientation of temporal history at the asking of a question ("… the situation declares itself …") continues on its unchanging path to the answer. Though quantum entanglement assumes that no other metric is locally real in that unique interval, if the answer of the future that meets the question in the present is sensitively dependent on the initial condition – the geometric orientation – every metric is physically real, and available to the observer who freely chooses a different orientation of spacetime in that same interval.

. . . And Eternity in an Hour.[18]

Stripped of all mathematical jargon and mysticism, quantum computing algorithms that rely on quantum entanglement are no different in principle from the I Ching or any other oracle such as Astrology or Tarot. Even the most primitive of oracle predictive techniques is judged against collected lore stored in the heads of shamans or in some book or books of "hidden" knowledge. We tend to think that only numerical implementation is a precise fit to "reality," and more "scientific" because it is constrained by the rules of arithmetic—we neglect the fact that *we* created the (self consistent) rules of arithmetic, as surely as generations of shamans and intelligentsia created the self consistent rules of their own predictive systems.

We don't judge folklore as universally true; actually, pagan oracle systems rarely even *claim* to be universally true. The results of casting dice, cards, bones, coins, yarrow stalks, or of observing the positions of celestial bodies … only claim to represent a true reply to a specific question in a specific moment (As C. G. Jung put it, "Whatever is born or done in a moment of time has the properties of this moment in time"[19]). The occult soothsaying arts are in fact *rational* in a way that conventional ideas of quantum computing are *not*. Those oracle results, after all, are said to correspond to a body of archetypes, a book, a code, a local interpretation of universally cyclical events accumulated over generations of repeated observations. Even our non-mystical classical computing algorithms resemble Paul Erdös' imagined "Book of Proofs,"[20] a book of all true mathematical statements, both known and unknown, yet never unknowable. Could a quantum computer say the same? Would quantum error code correction be even *as* good as the collected wisdom of the I Ching—partial results capturing a probabilistic measure in one moment of time? As Chaitin has shown that even classical arithmetic has a degree of built in computational uncertainty[21], how exponentially more uncertain the results of computing dependent on conventional quantum mechanics that assumes quantum entanglement and Bell's theorem?

If there is escape from this conundrum, it is through a Book we cannot hope to read unless we first learn how to create, comprehend and complete it. Quantum computing that relies on numerical implementation, of a quantum entanglement program whose principles are unknown and may even be unknowable, not only does not reach that level, a rational research program at the end of the day echoes the ancient alchemist's aphorism: "As above, so below."

[*Acknowledgement*: Thanks to Jonathan J. Dickau, for kindly reading a draft of this essay and suggesting significant qualitative improvement. Any possible errors of fact are my own.]

For Sara, Keri and all my grandchildren

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Technical Endnote: Well Ordered, Totally Ordered, Partially Ordered or Random?

Mathematicians have many ways to speak of "order." We have tried to make clear in these pages that we have taken our terms from mathematical set theory and applied them to the theory of computation.

When we speak of computing "using the resources of the universe," (Aaronson's question) we have assumed a parameter outside of classical computability using the rules of arithmetic—numbers aren't physical, though counting is. I.e., physical quanta are countable by definition.

"Countable" doesn't necessarily imply well ordered, though—cardinality of sets {a,b,c…} might be represented by labels that are well ordered (1, 2, 3 …) independent of the orderedness or randomness of the set. For example, when we speak of the set of all books, the information contained in Tolstoy's *War and Peace* differs from Vonnegut's *Slaughterhouse Five*, yet both have the cardinality of the Continuum (uncountably infinite)—even though the set N of all books is countably finite. As strange as it may seem to speak of a finite set of infinite things, the case is true; a comprehensible explanation may be found in Hermann Weyl's 1918 classic, *The Continuum: A critical examination of the foundation of analysis*.

In our essay, we have proposed that global information is totally ordered and finite, while our ways of computing it are partially ordered and infinite. Gregory Chaitin's application of his Algorithmic Information Theory would seem to bear this out in an experimental way; Chaitin's Omega number is produced from an algorithm that outputs different and random results depending on which computer language is "speaking." Metaphorically, one program would output *War and Peace*; the other, *Slaughterhouse Five*, from the same algorithm.

So it goes.

Some Web references to order in set theory:

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