The experimental method and the constitutive limits of the mathematical description of physics

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Nature is believed to be organized by a mathematical fundamental structure. Therefore, the experiments are interpreted through mathematical models. Unfortunately, experiments can only provide macroscopic outputs, even when referred to quantum elementary object. Starting from such observation, I first consider the concept of anomaly as groundbreaking information to falsify a theory. The separability of a system between an experimental equipment and a microscopic object is discussed. Non commutative microscopic observables of elementary entities are postulated from a set of measurements of macroscopic observables interpreted as their eigenvalues. I explain the major role of the Gelfand-Naimark-Segal construction of the representation of classical and quantum abstract C^*-algebras to recognize the impossibility of building a theory with a unified domain for the microscopic and the unavoidable macroscopic observables. I discuss implications of the Gelfand theorems on both macrorealism emergence from coarse grained measurements and decoherence programs. Finally I apply the results to determine the fundamental impossibility to identify a Theory of Everything with the mathematical structure attributed to Nature.

I. INTRODUCTION

Most of the efforts of contemporary fundamental theoretical physics are devoted to define a mathematical theory capable to describe all the properties of elementary constituents of Nature and their interaction, called Grand Unification Theory [1]. The hope that string theory could provide a *unified description of all forces, all particles, space and time, free of arbitrary parameters and infinities* [2] created the ground to introduce the idea of Theory of Everything (ToE) [3]. Such theory is believed to include electroweak [1], strong [4] and gravitational [5, 6] forces in a unique set of partial differential equations whose solutions reflect the known elementary particles and gauge fields [1]. Physicists are driven by the expectation that Nature is governed by a well defined and closed mathematical theory. It is therefore possible that such theory will be expressed in the future and classified as a candidate to be *the* Theory of Everything. It is possible that several theories will account all the observed effects. Someone could consider unsatisfactory that the identification between the mathematical structure of Nature (MSN) and one of the (or the only) possible ToEs could not be recognized for some principle reason. The hope to identify the two follows from the wrong expectation that if a theory T explains and predicts the experiments E then T should be regarded as the representation in mathematical terms of a fundamental property of Nature. On the contrary, as already mentioned by Leggett [7], the realization of a predition $T \to E$ does not mean that $E \to T$. Is it therefore possible to ultimately identify a ToE with the mathematical structure of Nature?

This paper answers negatively to such question. It is shown that the worst case occurs: a ToE can not even be expressed by our mathematical language, for both experimental and intrinsic mathematical reasons. First, the way experiments are organized and interpreted is analysed. Models of the microscopic objects are tested by means of macroscopic experimental apparata. An experimental equipment can only convert the quantum information into macroscopic variables. I will show that it is impossible to separate a physical system between two distinct entities, namely the experimental apparatus and the object of the measurement. Such conclusion leads to look at a mathematical framework which accounts both classical physics and quantum fields at the same time. Consequently the second argument has ground on Gelfand-Naimark-Segal construction [8, 9] of the representation of observables from abstract C^*-algebras, further developed by Haag. [10] The structure of the algebra leads to incompatible representations for microscopic and macroscopic systems respectively. This work is organized as follow: in Section II the concept of anomaly is introduced and its fundamental macroscopic nature is explained. Section III is devoted to show how a physical system it is implicitly separated in two subsystems (the macroscopic experimental equipment and the object of the measurement). The impossibility to achieve such separation in the quantum limit is explained. Section IV describes the general mathematical formalism of abstract C^*-algebras which unifies the mathematical description of both classical mechanics and quantum theories within the same language. The foundational implications of the Gelfand-Naimark theorems are discussed. The incompatible representation of sets of classical and quantum observables is analyzed. In Section V, the implications of the previous discussion are applied to those programs intended to explain the transition from quantum to classical such as macrorealism emergence and decoherence, and to the problem of building a Theory of Everything. The conclusion resumes the principal findings discussed in the essay.

II. ANOMALIES AS SOURCE OF EVOLUTION OF THEORIES IN PHYSICS

Mathematical models are at the base of modern science. Generally a model M has some ranges of validity, like a minimum and a maximum length, energy, time scale where it can successfully be applied. Mathematical models can be separated in two classes. The first class is constituted by the empirical laws of macroscopic phenomena. The second class contains those theories T describing the behaviour (scattering amplitudes, etc.) of microscopic elementary objects (not composed by further constituents governed by different mathematical laws).

This section is devoted to define *anomalies* and to show that, while empirical models and theories T apply to macroscopic and microscopic systems respectively, the falsification of both occurs always through anomalies observed in the *macroscopic domain*. Theories intended to describe elementary objects are tested by macroscopic anomalies.

> A simple example to introduce anomalies is the discovery that the Moon was not a perfect sphere by Galilei. In the XVI century it was commonly accepted that the surface of the Moon was perfectly smooth. Such belief was supported by the direct observation of the light reflected by the Moon with the human eyes. The use the telescope, which increased their resolution, let visible the shadows of the mountains on the surface. The previous *model* M of celestial bodies as perfect spheres encountered an anomaly (the shadows on the surface) which imposed to define a new theoretical framework M'.

Anomalies are recorded at the macroscopic level because the measurements of all the known physical quantities of a system S can be reduced to simultaneous measurements of few macroscopic observables like positions of indicators on analogical displays, and current values represented by numbers on LCD displays.

Macroscopic observables $\{C_i\}$ *are functions of outputs of experimental equipments. Microscopic observables* $\{Q_i\}$ *are defined by arbitrary sets of macroscopic observables* $\{C_i\}$ *which can be associated to eigenvalues of an algebra* A *of non commutative operators.*

An anomaly is defined as a discrepancy between the output of an experiment E and the prediction of a model M obtained in terms of combination of macroscopic quantities $\{C_i\}$.

Current theories provide room for anomalies to manifest. When a theory in the microscopic domain is considered, anomalies and internal consistency may push macroscopic related concepts beyond their limits of application. For example, the risk connected to the abuse of theory-dependent concepts has been shown in the problem of the mass of quarks [11, 12]. Mass is a macroscopic property which proved to be well behaving also for some elementary particles like electrons. Here I resume how the mass of quantum objects like quarks in nuclear physics can not naively be extended, and how the problems arise from the natural language and the previous models.

> *Quark masses* introduced by the non-abelian Lagrangian of quantum chromodynamics (QCD) in analogy to the lepton masses in quantum electrodynamics (QED), together with the asymptotic freedom, are not directly connected to the quark masses obtained from experiments on hadrons. In the region where perturbative QCD are applicable, with $m_q << p$ (where m_q are the current quark masses - those used in the Lagrangian, and p the momentum) the masses cannot be extracted since they are negligible. On the other hand, at small momenta, when $m_q \approx p$ we are unable to extract the masses in a reliable way since the perturbative analysis cannot be applied. Therefore, the quark masses have been extracted by adopting an internally inconsistent procedure. In this procedure, one forgets about the confinement and describes external quarks with the help of on-mass-shell Dirac bispinors, and internal quarks with the standard propagators, in full analogy to leptons. Unfortunately free quarks are not observed, and this contradicts the use of both the on-mass-shell Dirac bispinors and the pole propagator at $p^2 - m^2 = 0$. If one proceeds to use such quantity as standard quark mass for e.g. weak radiative hyperon decays, the calculation fails badly, precisely because of the use of standard concept mass for quarks. It seems therefore impossible to extend the traditional concept of mass *a la* lepton to quarks.

The application of macroscopic categories to such fundamental objects in the microscopic domain leads to an anomaly, leaving the Standard Model unsatisfactory in terms of consistency. It is fundamental to observe that the theory is tested by means of macroscopic quantities. Even if the experiment deals with ultimate objects, the equipment records the position traces of hyperons decay in a chamber. Such traces are then converted in energies, momenta, angular momenta, positions, probability of occurrence, by virtue of a theory. The experimentalist records a collective quantity built from very basic quantities like position and current intensity, and combines those quantities with simple laws in terms of spin and cross sections. An anomaly is recorded as a deviation from such law in the mentioned space of composed parameters, looking at some specific combination.

It is universally accepted that science requires falsification. The falsification manifests as an anomaly, i.e. a deviation from the prediction of a theory T in terms of *macroscopic quantities* which are interpreted as non commutative microscopic observables by means of a theory.

III. UNSEPARABILITY OF PHYSICAL SYSTEMS BETWEEN MACROSCOPIC AND MICROSCOPIC SUBSYSTEMS

This section shows that the separation of a physical system between an experimental equipment and a physical object under investigation is an oversemplification caused by the small coupling energy normally involved in the macroscopic domain and their large space separation. Indeed, in classical physics the experimental apparatus S_1 and the object of a measurement S_2 are clearly distinct. The energy scale E_I of the carriers of their interaction is negligible with respect of the total energy of both E_1 and E_2.

The *determination of the position* X of a macroscopic object S_2 referred to an experimental apparatus S_1 is made possible by photons. The photons of two light sources scatter at the surface of the object S_2 so the reflected photons reach a pair of sensors of the experimental apparatus S_1. The time dependent currents $I_j(t)$, where j is an index and t is time, generated by the sensors are converted in an arbitrary coordinate system $O(x, y)$. The energy E_I of the photons is negligible if compared to the total energy of both the experimental equipment E_1 and the macroscopic object under investigation E_2. The error due to the interaction is small if compared to the error due to the spatial extension of the sensor, the mechanical stability, and the other sources of fluctuation.

For macroscopic bodies, S_2 is identified with the *object of the measurement*, distinct from the experimental apparatus S_1, and it is treated as if it owns properties not perturbed by the observer and the equipment which belongs to S_1. S_1 and S_2 can be treated as distinct objects only in an approximate sense in terms of negligibility of the effects of the carriers of their interaction of energy E_I on S_2, together with their large center of mass space separation compared to their size.

Things are dramatically different when the system S'_2 has a microscopic size (one electron, one atom, an elementary particle). In this case the treatment can not be approximated as the energy scale of the interaction is not negligible. The coupling between the experimental apparatus S'_1 and S'_2 has an energy scale E'_I of the magnitude of E'_2.

Let's consider an *electron localized in an electrostatic trap*, sensed by a weak current i which is close enough to couple the spin of the carriers with the spin of the localized electron. The weak current i can be used to probe the spin state of the localized electron when an external static magnetic field is applied for example along the direction z. The electrons become spin polarized and the component S_z of the spin $\mathbf{S}$ of the electron along such direction is a good quantum number. Depending on the spin state S_z, the weak spin current i may alternatively either flow or be stopped. Such effect goes under the name of Pauli spin-blockade. [13–15] The experimental apparatus S'_1 includes the spin current, which is dipole-dipole magnetically coupled with the localized electron identified with the system S'_2. The spin-spin coupling has an energy

$$H_I = -J\mathbf{S}_c \cdot \mathbf{S} \tag{1}$$

where J is the coupling constant, $\mathbf{S}_c$ is the sum of the spins of the current electrons coupled with the localized spin $\mathbf{S}$, whose magnitude is comparable with the energy associated to the Hamiltonian of the localized electron immersed in the static magnetic field

$$H = -\vec{\mu}_S \cdot \mathbf{B} \tag{2}$$

where $\vec{\mu}_S$ is the electron magnetic moment and $\mathbf{B} = B\hat{\mathbf{z}}$ is the static magnetic field. The eigenvalues of the unperturbed Hamiltonian change significantly when H_I is added. It is not therefore possible to apply the approximation used in the previous example. S'_1 and S'_2 can not be treated as two separate systems.

The two examples lead to a relevant conclusion. Looking at fundamental objects the physical system constituted by the equipment and the microscopic entity under investigation - as widely discussed by Bohr and Heisenberg - have to be treated as a whole. Consequently, a unified theoretical description in the same mathematical terms has to be used for all the subsystems.

IV. IMPLICATIONS OF THE GELFAND-NAIMARK-SEGAL CONSTRUCTION ON STRUCTURES OF QUANTUM AND CLASSICAL VARIABLES

The most general mathematical theory capable to account both classical (macroscopic) and quantum microscopic (observables) is given by the theory principally developed by Gelfand of abstract C^*-algebras. In the following the main results of such theory are reviewed. In particular the central role of the Gelfand-Naimark theorem for classical mechanics (CM, in terms of abelian algebras) and the corresponding theorem for quantum field theory (QFT, in terms of non-abelian algebras) are considered. Next, the fundamental consequences implied by their co-existence are discussed.

A. The Gelfand theory to construct classical mechanics and quantum field theory

The most general mathematical theory capable to be applied to both CM and QFT is the theory of abstract C^*-algebras of observables.[8, 10, 16, 17] Such theory provides the explicit construction of the representation of (local) observables, so it is possible to develop first the axioms of Wightman independently from such representation. Forgetting the axioms devoted to implement causality and the group of Poincare, we concentrate on the structure of observable quantities by the axiom defining the correspondence between observables and algebras.

According to the algebraic approach, the net of the algebra of local observables is associated to open regions in the Minkowski space (space-time), and such correspondence may be interpreted as the only available physical content of a theory. Such correspondence, for example, is at the base of the S-matrix and the cross-sections evaluation in QFT. The main class of objects of the theory are C^*-algebras, which are *-algebras (algebras with usual * properties) and Banach algebras [16, 17], a property which grants they own a norm topology, at the same time. The requirement of being a *-algebra and a Banach algebra are the minimal necessary to grant respectively that states (linear positive forms ω) allow the Gelfand-Naimark-Segal (GNS) construction,[8, 10] and to provide a probabilistic interpretation for such states. In turn GNS is required to provide an explicit representation of the abstract algebra of observables. The norm topology is required to demostrate the Riesz-Markov theorem, which is used to demonstrate the spectral theorem. The latter grants that eigenvalues of observables are real, as imposed by experimental outputs. Consequently, the theory states the following:

Axiom The theory is characterized by the net of C^*-algebras

$$O \rightarrow U(O) \tag{3}$$

where O is a finite region of Minkowski space. The self adjoint elements of $U(O)$ are interpreted as measurable observables in O.

The definition of states as linear positive normalized forms implies the existence of the representation of the classical and quantum observables. The kind of representation depends on the commutativity of the elements of the algebra.

According to the Gelfand-Naimark theorem, an abelian C^*-algebra A with the identity is isometrically *-isomorphic to a C^*-algebra of the continuous functions on a Hausdorff compact, called the Gelfand spectrum, with the *-weak topology. In the general case, however, a second theorem, following from the GNS construction and from the Von Neumann theorem on the representations of a Weyl C^*-algebra, states that a C^*-algebra $U(O)$ is isometrically isomorphic to an algebra of bounded operators on a Hilbert space H. Such two theorems differ only for the hypothesis on the

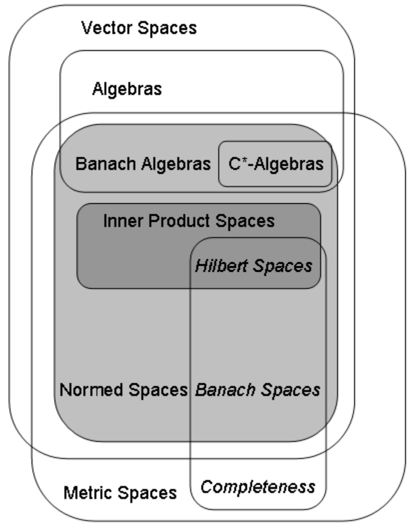

Figure 1: Abstract C^*-algebras are the most general framework to define both classical and quantum observables. The scheme provides an overview of inclusions of several topological classes and their related properties. C^*-algebras are Banach algebras with the *-properties.

structure of the abstract algebra. They lead to two different and incompatible representations. In the next subsection I discuss the implication of such major result.

B. Incompatibility between classical and quantum representations

Differently from the Heisenberg approach, where observables are operators acting on a suitable Hilbert space, the fundamental entities in the general formalism of Gelfand are the algebras constituted by bounded operators. The basic objects are nets of abstract algebras of bounded operators in a region O. Both CM and QFT can be expressed by defining states, observables and dynamics within the same mathematical framework. The definition of states is the same in the two pictures (as positive linear normalized forms). Also dynamics is almost identically defined (dynamics is generated by means of the $*$-homomorphisms on the algebra in itself, which form a 1-parameter continuous group - in the case of quantum mechanics, the *-automorphism are weakly *-continuous). On the contrary, observables have a different representation in the two cases. It is the Gelfand-Naimark-Segal construction which provides a unique representation of the observables in terms of linear operators: in the case of CM the C^*-algebra with the identity is abelian, while in the case of quantum mechanics it is not. The theoretical apparatus provides two explicit representations which are not equivalent. The distinction operated by the GNS construction shows in which sense classical and quantum mechanics are described at two different hierarchy levels. CM is made of commutative observables of *collective variables* (for example the approximation of point-like massive objects in the center of mass.) Instead, quantum theory associates a subset of macroscopic observables to the eigenvalues of the bounded operators of a conjectured non-commutative algebra, intended to specify the characteristics of a conjectured microscopic entity. As noted by Schroedinger, there is no point were quantum mechanics ceases to hold when enlarging the system up to the macroscopic scale.[18] Unfortunately, the observables of a macroscopic object include for example the position of the center of mass $\mathbf{X}(x, y, z)$, which differs from the quantum positions $\mathbf{x}_i$ of the n constituents (atoms, electrons) expressed by their tensor product as

$$\mathbf{X}(x,y,z)_{cl} \neq \mathbf{x}_1 \otimes \mathbf{x}_2 \otimes \mathbf{x}_3 \otimes ...\mathbf{x}_n \quad (4)$$

GNS construction clarifies that the two operators belong to two inequivalent algebras, so the use of $\mathbf{X}$ is an oversemplification. [24] The macroscopic observables obey mathematical laws and they are represented by commutative elements of an algebra, but *a theory which includes macroscopic observables as such and macroscopic observables as eigenvalues of microscopic observables is fundamentally forbidden*.

Macroscopic objects return some fundamental information only when their observables are interpreted as eigenvalues of arbitrary non commutative subsets of observables. To conclude, Gelfand-Naimark-Segal construction implies that classical observables C and quantum observables Q *can not* by factorized by tensor products. Such result is of fundamental importance as it rules out all those arguments based on the naif factorization $\psi_{cl} \otimes \Psi_q$ of classical states ψ_{cl} with quantum states Ψ_q and on mixed observables of the kind $Q \otimes C$.[25]

V. FROM QUANTUM TO CLASSICAL: FUNDAMENTAL IMPOSSIBILITY TO IDENTIFY A TOE WITH THE MATHEMATICAL STRUCTURE OF NATURE

The XX century physics concludes that the knowledge of Nature is intrinsically limited at a fundamental level because of the non-commutative structure of the algebra of observables. The latter, even if intrinsically limited, is precisely the level at which it is meaningful the discussion whether or not it is possible to identify a ToE with the MSN.

As a unified description of quantum and classical objects on the same footing is mathematically forbidden, one may briefly review under such a new perspective those programs of explaining classical world emerging from a quantum substrate. I consider emergent macrorealism from coarse grained measurements [23] and decoherence [20–22] Next, the fundamental macroscopicness of experimental physics may be applied to discuss the ultimate impossibility of the possible versions of ToE programs.

A. Emergent macrorealism from coarse grained measurements

Here I show the problems arising in the attempt to produce macrorealism according to the coarse grained measurements framework developed by Kofler and Bruckner [23]. Their discussion is based on the use of time as an observable parameter of the theory so it is treated as the mathematical parameter used to describe unitary time evolution. Such parameter coincides with clock time T which an approximate macroscopic observable [19] suitable to tag states in the phase space. The implicit assumption that such classical parameter enters in the quantum description makes their theory intrinsically semiclassical, so their conclusions on the emergence of macroscopic world from quantum mechanics is affected by the explicit treatment of quantum mechanics in a macroscopic temporal view. Time enters in the Wightman axioms of QFT as a co-dimension of the manifold introduced to define Poincare invariance. In canonical quantum gravity time is not among the constituents of the theory.[6] The approach of Bruckner and Kofler contradicts the QFT by including a macroscopic time variable.

B. Decoherence program

According to Zurek [20, 21] the information transfer from the quantum state ψ towards the environment Ψ_{env} is permitted by the premeasurement. The premeasurement consists of the coupling between ψ with Ψ_{env}. Depending on the base chosen, there is a symmetry between the two alternative points of view on the measurement process. The quantum apparatus coupled with the quantum object does not destroy the phase of the field, but it progressively shares such information with it. At the same time the quantum apparatus measures the quantum object, while the quantum object measures the apparatus, as granted by interchangeability of their bases. The asymmetry of the measurement process arises because the quantum apparatus is capable to *amplify* an eigenvalue under the condition that *the Hilbert space of the apparatus pointer is large compared with the space spanned by the eigenstates of the measured observable S*.[20]

According to the findings of the previous section, the apparatus pointer should be treated as a macroscopic observables and not to microscopic observables defined on a Hilbert space. Hilbert spaces are only built to define microscopic observables from macroscopic observables. To conclude decoherence provides the relevant result of indicating a path to achieve eigenvalue amplification, but it fails in terms of internal consistency.

Alternatively, emergent macroscopic observables could be constructed from average functions of quantum real eigenvalues preamplified by the same mechanism proposed by Zurek. In this case one observes that in order to grant a bounded spectrum the quantum C^*-algebra is made of the Weyl operators $U_{jk}(\alpha) = e^{i\alpha x_{jk}}$ and $V_{jk}(\beta) = e^{i\beta p_{jk}}$ where $\alpha, \beta \in \Re^n$, instead of Heisenberg observables. They obey to the Weyl commutation rules

$$U_k(\alpha)V_k(\beta) = V_k(\beta)U_k(\alpha)e^{-i\alpha\beta} \tag{5}$$

Such non linear relationship among macroscopic and microscopic variables can be interpreted as a categorical difference, for example between space of microscopic entities and space of macroscopic bodies.

C. Theories of everything

In the last subsection, I list three possible ToE programs. I discuss their compliance with the internal consistency imposed by the existence of the Gelfand-Naimark-Segal construction and the related theorems, and the fundamental macroscopicness of measurements of microscopic objects. The common assumption is that *all the natural phenomena are determined by a unique mathematical closed set of equations called the Mathematical Structure of Nature.* Next, there are three possible levels of strength of the program of building a ToE.

- **Version 1 (Positivist)** It is possible to univocally identify a theory as the explicit expression of the MSN and such theory is the Theory of Everything.
- **Version 2 (Strong)** It is possible to express a theory and to prove that such theory is compatible with the expression of the MSN. Such theory is the Theory of Everything.
- **Version 3 (Weak)** It is possible to define a theory which is the expression of the MSN but their identification it is not possible.

While the first program is a positivist approach ruled out by the requirement of falsificability of a theory, the versions 2 and 3 require a further discussion. According to the framework developed in this essay, microscopic observables used in a fundamental theory are nothing more than suitable combinations of macroscopic observables interpreted as eigenvalues of non commutative sets of operators. The indirect observability of the quantities used in a theory and the impossibility of using macroscopic quantities, which are directly measured but can not belong to the algebra because of the Gelfand theorems, makes impossible to say if the microscopic observable has some properties which depend on the intermediate layer represented by the macroscopic observables. Indeed, the latter are used to build their spectra by arbitrary combination tested by the theory. The fact that the interpretation in simple mathematical objects of the quantum observables provides elegant and efficient mathematical equations does not imply that such mental construct corresponds to something *real*. Therefore, the presence of such macroscopic layer which separates the observer from the microscopic structure, made of average quantities and arbitrary combinations of commutative observables, leads to conclude that neither the Version 2 nor the Version 3 of the ToE programs are a viable way to know more about the MSN. Both the Versions 2 and 3 encounter the lack of a direct observation of microscopic observables, while the information provided directly can only tell about macroscopic world. To resume, the identification of a speculated ToE with the MSN is fundamentally impossible. I would like to conclude with at least some positive program about the fundamental research, which is the following. Whatever the grand unification theory will be, the application of an aesthetic principle will lead us to choose, if possible, its simplest version.

VI. CONCLUSION

In this Essay I have discussed the intrinsic limitation in the mathematical description of physics at a fundamental level. I've first shown that experimental physics provides only macroscopic outputs, even when it deals with microscopic objects, through anomalies which manifest as deviations from an expected trend. Such expectation is based on the interpretation of combinations of macroscopic outputs in terms of suitable physical quantities. Theories building non commutative observables are associated to microscopic objects. Next, the fundamental unseparability of macroscopic and microscopic subsystems of an experiment has been expressed, which implies to treat macroscopic and microscopic observables within the same mathematical framework. Such framework is provided by the theory of abstract C^*-algebras developed by Gelfand. The incompatibility of the explicit representation of such algebras of classical and quantum observables respectively has been clarified, so that the two classes of corresponding observables can not be treated on the same footing. In particular all those arguments based on the tensor product of quantum with classical variables are wrong. Conclusions on the decoherence and macrorealism emergence from coarse grained measurement programs have been drawn. Finally, the analysis of possible ToE versions leads to conclude that it is fundamentally impossible to both identify a theory with the Mathematical Structure of Nature and demonstrate their compatibility.

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[25] The lack of a shared solution to the Schroedinger paradox may be attributed to its formulation. The *Cat* C is a macroscopic body so the paradox arises because the problem implicitly forces the main observable C to belong to an inconsistent set of observables, made of a mix of a non commutative and a commutative algebras.