

FUNDAMENTALNESS OF HOMOCHIRALITY .

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Introduction.

Life exists in merely one chirality - sugars are exclusively right or D - handed, whereas amino acids are left or L - handed. This kind of single handedness is usually biologically defined as "homochirality". Such asymmetry may seem mathematically trivial at the first glance, however, life scientists still do not understand why do so many biological molecules exist in just merely one chirality and how did it emerge on the Earth at all. (Miller,1953; Frank-Soai,1995; Joyce,1987; Steendam,2014; Powner,2009 and Hein,Tse, Blackmond, 2011).

Following Pasteur's pioneering experiments, many organic compounds having identical sets of atoms (but different mirror symmetry) nevertheless are able to demonstrate different biochemistry (different taste and smell).As is known today the functional proteins of all organisms contain only L - amino acids which represent merely structural proteins but not signal proteins.From another side, there are short polypeptides with D - amino acids as well. Such neuropeptides, however, are not structural but are merely signal proteins. It is remarkable that enzymes are completed of more than 100 L - amino acids and living organisms use only D - amino acids for information transfer.

If we make experiments with spontaneous decomposition of the symmetric mixture resulting from the formation of L- and D - crystals , we may also assume an existence of some kind of memory which provides the effect that the average number of homochiral L- and D - crystals will be the same. In other words, chirality always is restored in a large samples. (Miller,1953) Such observation suggests that there exists an analogy between self - organization processes of spontaneous decomposition of the symmetric mixture into L- and D - molecules and chemical self - organization of the Belousov - Zhabotinsky type. Indeed, in chemical self - organization auto waves break up as they encounter an obstacle, giving rise to pairs of L- and D - chemical structures (Ivanitsky , 2010)

Perfect Mathematical Homochirality.

There is remarkable analogy of biological homochirality in theory of the perfect numbers of today's number theory. Perfect numbers were established by Ancient Greeks (Plato, Euclid) and they are represented by following even natural numbers : 6, 28, 496, etc. Each perfect number is a sum of its own divisors. For example , $6 = 1+2+3$, $28 = 1+2+4+7+14$. Perfect numbers are very rare. Mathematicians discovered only 50 perfect numbers now (2018).

Because there are even and odd integers, it is easy to assume an existence of the ODD perfect numbers in good agreement with symmetry mentality. However, it became clear since Euler's discovery of a form of odd perfect numbers :

$$P = q^e \times a'^{2B'} \times \dots \times a^{2Bn},$$

where q and a are distinct odd primes and $q \equiv a \equiv 1 \pmod{4}$, that such sort of "non-aesthetic" or "ugly" numbers cannot exist in mathematics.

Contemporary computer researchers have proved also that there is no Euler's odd perfect numbers smaller than 10^{300} (Brent & Cohen, 1991). Thus, there is certain intuitive belief that odd perfect numbers cannot exist at all. In other words, there is taking "perfect" homochirality seriously of the even perfect numbers.

Indeed, in good agreement with our intuitive aesthetics, simple algebra proves that If all perfect numbers are defined as

$$6 = 2^3 - 2^1 \text{ (because } 2^1(2^2 - 1) = 2^3 - 2^1)$$

$$28 = 2^5 - 2^2$$

$$496 = 2^9 - 2^4, \text{ and}$$

$$2^{86225217} - 2^{43112609} \text{ is perfect number, etc.,}$$

then Odd Perfect number cannot exist in general, because any 2^n (where $n = 1, 2, 3, 4, 5, \dots$) is always Even (correspondingly, Even - Even = Even) (Popov, 1999).

Homochirality of the primes and ABC Hypothesis.

Prime numbers > 2 (3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, ...) exist in merely only one chirality - they are ODD integers. This kind of single handedness produces unsolved theoretical difficulties and similar with homochirality by biologists such sort of asymmetry is not trivial. Mathematicians still do not understand why do prime numbers must exist in just one chirality (or in exclusive oddness) and how did such single handedness emerged in Mathematical world at all ?

Some calculations with natural numbers could be considered as Pasteur - like biological experiments with a spontaneous decomposition of all integers into two classes - class of primes and class of non-prime numbers.

For example, the Fundamental Theorem of Arithmetic suggests that every positive integer, except 1, is a product of primes in one way only. Thus, when we take prime integers n - there is nothing to prove, because primes have divisors between 1 and n . Hence, the Fundamental Theorem of Arithmetic provides natural decomposition of all positive integers into two classes - class of primes and class of numbers which are unique products of the primes.

Another experiment with decomposition is called "the sieve of Eratosthenes". We can write down the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ..., N and strike out successively in the following way.

- 4, 6, 8, 10, ... I.e. 2^2 and then every even number;

- 9, 15, 21, 27, ... I.e. 3^2 and then every multiple of 3;

- 25, 35, 55, ... I.e. 5^2 the square of the next remaining number after 3; etc.

Thus, we may continue such process until all primes could be localized. Hence, Pasteur-like analogy is also arising.

Similar with biological experiments, it is easy to see that mathematical chirality "primes - non-primes" is always restored in the sufficiently large (even infinite) samples.

Well-known ABC Hypothesis represents another consequence of Homochirality of the primes.

Indeed, basic assumption of ABC problem is connected with real decomposition of all radicals into two different classes - class of radicals of the prime numbers and class of radicals of the non-primes :

$rad(5), rad(7),$
 $rad(6), rad(8), rad(10)$
 $rad(11), rad(17),$
 $rad(12), rad(18), rad(24)$
 $rad(19), rad(23), \dots$
 \dots

or, into

Set of primes (because $rad(prime) = prime$) and Set of non-primes.

However, how does our initial mixture of radicals restore its MEMORY in the process of decomposition? Recently S. Mochizuki developed a special "inter - Universal Teichmüller theory" where he showed an existence of some "scrambled endomorphism" which provides non-trivial Diophantine inequality of the radicals of non-primes.

Grothendieck's analogy.

Once mathematician Alexander Grothendieck said that there were only 2 ways of cracking a nutshell. One way was to crack it in one breath by using a nutcracker. Another way was to soak it in a large amount of water, to soak, to soak and to soak, then it cracked by itself... My Homochirality theory is the latter one.

Catastrophes produced homochirality.

Let us suppose that both biological and mathematical homochiralities have to have the same mathematical logic of emerging. Correspondingly, there is some initial event (not only formal mathematical proof) which we would like to call " Catastrophe " that is producing unified homochirality.

Probably, Vladimir Arnold was the first mathematician who considered such sort of theory of catastrophe - inspired homochirality in the 20th century. Arnold showed that initial non-systematic attempts were made already by A. Poincaré and A.A. Andronov (in 1920s). Following Arnold some equations of the type $U_{xxxx} + 2U_{xx} + U^2x + U = 0$ must have periodic solutions and special bifurcations (mathematical catastrophes) connected with a symmetry in their roots : $+x \leadsto -x$ (so-called " hamiltonianness " of the equation of the problem). In other words, symmetry and hamiltonianness are usually produced mathematical catastrophes. Hence, If and only If there is asymmetry (homochirality) some class of symmetric and hamiltonian systems can become more Stable and " Catastrophelessful ". (Arnold, 1983).

Of course, Arnold's result needs new mathematical generalizations, however, it is clear now that in order to explain homochirality of natural and symbolical systems we are needed more deeper Fundamental Mathematical theory.

References.

- S.L.Miller, *Science*, 1953,117-528
M.W.Powner *et al.* *Nature*, 2009,459,239
G.F.Joyce *et al*, *Proc.Natl. Acad.Sci USA*, 1987,94,4398
K.Soai *et al*, *Nature*, 1995,378,767
R.R.E.Steendam *et al*, *Nature. Commun.* 2014,5,5543.
J.E.Hein,E.Tse and D.Blackmond. *Nature.Chem.* 2011,3,704.
G.R.Ivanitsky.*Physics - Uspekhi*, 2010, 53(4).
M.A.Popov. On Plato's periodic perfect numbers. *Bull des Sciences Math*, 1999,123,p. 29-31
V.I.Arnold. Особенности, бифуркации и катастрофы. *Physics - Uspekhi*, 1983,141,4.

