What Would Weyl Do?

B. Pollard

August 31, 2012

Abstract

It is possible that the strict adherence to one particular assumption about the physical world buried within our modern mathematical frameworks might be the limiting factor in the physics community's eager efforts to take a step forward in our understanding of the world around us. In this essay we ask ourselves which foundational concepts H. Weyl might have reconsidered.

1 Introduction

Einstein's theory of General Relativity is often explained to the layman, "Matter tells space how to curve, and curved space tells matter how to move." This is indeed true for the Ricci Tensor, the sum of the diagonal entries of the Riemann Curvature Tensor. The rest of the Riemann Curvature Tensor, namely the trace-free Weyl Curvature Tensor, represents how gravity interacts with space, or doing the naive thing, "Gravity tells space how to curve, and curved space tells gravity how to move." This obviously needs some modification/clarification. Let us recall here the relation of the Weyl Tensor to the electromagnetic tensor, as Weyl proposed first in 1918. According to Einstein the laws of gravitation follow from the components on an invariant quadratic differential form. Electromagnetism is controlled by the coefficients of an invariant linear differential form, these coefficients forming the components of the electromagnetic potential, a 4-vector. Weyl in 1918, following the idea that a purely infinitesimal geometry allows for invariance under the parallel transport of a vector only for a small neighborhood around the initial point, rather than for an arbitrary large closed loop as is present in Einstein's theory of General Relativity. Allowing for arbitrary real gauge transformations, one finds a theory describing a metric tensor that depends on a linear differential form as well as a quadratic differential form. In this theory, the condition for the invariance of the parralell transport of a vector globally in spacetime is precisely the condition that the components of the elctromagnetic potential vanish, and with them the electromagnetic tensor itself, in other words the necessary and sufficient conditions for Einstein's theory of General Relativity is the absence of an electromagnetic field. Weyl had constructed a generalization of Einstein's General Relativity that incorported some non-Riemannian notions of a manifold. Einstein was quick to point out that in Weyl Geometry clocks that travelled different paths in space could have different measures of time. Einstein pointed out that this was inconsistent with the consisten observations of atomic spectra, and hence rendered Weyl's theory unphysical in its infancy.

2 Fudge the Data or Fudge your Math

Today, as in the past, it is observations of physical systems that serve as our raw data. This data enables us to look at the analogues of those observations in our mathematical frameworks and thereby make an educated guess as to the correctness of a particular framework based on its ability to reproduce what is observed in this data. Sometimes the data of our world jogs ahead of our mathematical frameworks and we are left with observations that have no analogue in the framework. At such times it can be that the data is wrong or incomplete, such as the case in which observed deviations in the orbits of planets pointed to the existence of an unobserved massive body in our solar system, namely Jupiter, or it can be that the framework needs revision, as was the case for the unexplained perturbations of the orbits of Mercury masterfully explained by the new framework for gravitation put forth by Einstein, General Relativity (from a talk given by Nima-Arkani Hamed). These examples describe the case of unexplained observation. Imagine if the LHC had been constructed without thought to detecting a particular boson, how long would it have taken physicists to take seriously a new 125 GeV particle, or better yet, imagine the models that would have been constructed to incorporate such an observation into the existing framework, supposedly someone would then discover the symmetry-breaking Higgs mechanism, however it would probably be presented with a much different flavor than modern accounts, where because of the order of prediction it seems like symmetry-breaking in the mathematical theory gave birth to the Higgs itself. This is an example of when we observe within our mathematical frameworks something that is yet unexplained by observation. Dirac and Weyl had an interesting time along these lines, as Dirac first wrote down his equation and correctly interpreted the existence of particles with charge opposite to that of an electron within his framework. Of course as any good physicist would he attempted to explain this feature of his framework with that which had already been observed in nature associating this positively charged field with that of a proton. Weyl was kind enough to observe that these 'holes' in the Dirac-sea are required to have the same mass as their oppositely-charged electron counterparts enabling the successful prediction of the existence of antiparticles, the first of which was soon to be observed, the positron. Weyl had another go at features of mathematical frameworks yet unexplained by nature. He wrote down an analogue of Dirac's equation, the Weyl equation, which seemed to accurately describe a massless neutrino. Pauli, the godfather of the neutrino, pointed out that Weyl's equation did not respect left-right symmetry. Even the founding fathers of quantum mechanics in their heyday could not conceive of a spacetime that preferred left to right, though only two years after Weyl's death CP violation of the weak interaction was first observed. Such stories are ubiquitous in physics. Consider Einstein's dropping of the Cosmological Constant rather than successfully predicting an expanding universe. I am sure such stories are well known to many of the readers, but they are recounted as a good starting point for us to step back from the framework of modern physics and ask ourselves where today we have observations of the universe unexplained as well as features of our mathematics unobserved. An obvious instance of the first is the observation of the accelerating expansion of the universe and the flatness of galactic rotation curves (along with several other observations in the same vein). Explanations of these observations exist within current mathematical frameworks, however their existence is neither explained nor demanded by the mathematics. It is the opinion of the author that the second arena is the one in which progress is to be made. Rather than attempting to modify existing frameworks to explain such observations one can seek to better interpret the mathematics and see if the demands of the theory correspond to the payment of the universe. Somewhere within the foundations of our mathematical frameworks regarding General Relativity and Quantum Mechanics there is a Jupiter-sized planet floating around, awaiting observation.

Now that the attitude of the author and the motivations of this work have been described, our particular example of Weyl Geometry is revisited. This example stems from the earliest histories of the theory of General Relativity and the idea of gauge redundancies. Hermann Weyl in 1918, after studying Einstein's theory of General Relativity, published his first attempt to unite gravitational and electromagnetic phenomena in one framework. In doing so he introduces, for the first time, the concept of invariance under a gauge transformation, or perhaps better stated, the identification in the eyes of physics of a field and its gauge transformed version. We are used to the modern interpretation of gauge redundancies, the simplest example being the invariance of observables under the rotation of a vector in a Hilbert space by multiplication by a unit complex number. In the computing of observables this complex phase is met with its conjugate, rotating the vector back to its original state and leaving the observable unchanged, providing a redundancy between complex vectors and rotated counterparts. The consequences of such gauge redundancies are much deeper than simply leaving observable quantities unchanged, they place stringent requirements on interacting theories, a subject beyond the scope of this work, however it should be mentioned that gauge redunancies point to equivalence classes as the fundamental physical objects, not just individual objects, rather sets of gauge identified objects.

The idea is simple, and the author had not yet had the time, nor the mathematical ability to carry out the calculations outlined here. A generalized gauge transformation is one allowing for both real and imaginary parts in the exponential multiplying the transforming field, where the real part of the exponential corresponds to conformal rescaling and the imaginary part is the familiar rotation of phase. You basically only have two choices about what the field is, either it is a two-component spinor giving rise to a theory of a spin-0 field as in Yang-Mills theory or it is a 4-component spinor giving rise to a spin-2 field for the case of gravity. It has been argued by other authors that the generalization of Einstein's GR to Weyl Geometry brings in extra terms, which dissapear to give convential GR in the appropriate limits, and that the proper interpretation of these terms could lead to an explanation of long-range repulsive forces or short-range attractive forces between objects interacting gravitationally, also beyond the scope of this work. The condition however for globaly conformally flat spacetime is precisely the global vanishing of the electromagnetic tensor. It is the opinion of the author that if these spinor theories are properly formulated in Penrose's Twistor space and the correct physical interpretations of generalized gauge redundancies are considered in two regimes, 1) totally emptpy spacetime, where the Weyl Conformal tensor is truly the only contribution to the total curvature of spacetime, and 2) spacetime with a somewhere non-zero electromagnetic field tensor as well as the presence of spacetime singularities or black holes. It is believed the absence of magnetic monopoles in nature points to an asymmetry in the equations that is filled by a connection to gravity. In other words black holes, which have been shown to be at least theoretically magnetic monopoles, indeed have some type of 'magenetic charge' which on its way out of such highly curved spacetime manifests itself as an apparent short distance attraction and long-distance repulsion. It is believed that the internal clocks of objects in spacetime indeed do depend on their path through curved, non-conformally flat spacetime, however local objects are constrained to have close enough global spacetime trajectories to render such differences unobservable except for the case of observation of distant galaxies. In some way it can be thought of as residue calculus in an almost-complex spacetime, where our few orbits around the supermassive black holes in the center of the milky way are making contributions to the phase angle of all objects within the galaxy, keeping us all in phase with eachother, though not neccesarily with the spiral arms and other parts of the galaxy that orbit with different period than our solar system, and surely not in phase with objects outside of our own local cluster. I believe that with the proper consideration of both cases one will find a more accurate describtion of the foundational relation between electromagnetism and gravity, a relation that Weyl might not have explicitly constructed with his 1918 theory, but as with so many areas of modern physics, a relation with the foundation, cornerstone, walls, roof, everything but the finishing touches due to Weyl.

The author thanks Blaxel for useful discussions in preparing this note, as well as some of Nima-Arkani Hamed's video lectures provided by the Institute for Advanced Study for inspiration as well as the anecdote concerning the discover of Jupiter and the devations of the orbit of Mercury. The author regrets not having time before submisison of this essay to precisely develop these ideas to give them adequate presentation and references.