Is Mathematics Fundamental?

Ajay Pokharel

Kathmandu, Nepal

ajaypokhrel7@gmail.com

Abstract

Humans, from the beginning, have been trying to explain each phenomenon in the universe in the form of theories. We discovered Classical physics, Relativistic physics, Quantum Mechanics and several mysteries of the universe. Moreover, we are searching for something that unites every law of universe i.e. **Theory Of everything (TOE).** But what is it that we have really found between all these years and efforts? Isn't it the mathematics and the patterns that we have actually developed?

Indeed, yes!

In this essay, I am going to discuss how our whole universe (Or multiverse), from tiny atoms to huge galaxies, depends on mathematics and numbers for its foundation and operation. The essay will discuss the strength of mathematics and patterns along with the breakthrough achieved with the help of these patterns. The essay will also try to show the relationship between several fields of science with the mathematics and numbers ultimately concluding that "*Mathematics is fundamental".* The essay focuses on clarifying the physical laws and phenomena in the universe through mathematical perspective.

"Mathematics is the language in which God has written the universe"

-Galileo Galilei

1. **What are Mathematics and Numbers?**

   Simply, mathematics is the study of the patterns whereas numbers are the mathematical object used to count and measure anything. Mathematics is a vast field which cannot be packed between these two words, can we?

   It is, in fact, a tool by which the phenomena in the universe can be described. If observed carefully, then we can see how we use mathematics in quantum mechanics, classical mechanics, space and time, relativity, computing and in fact, almost everything. So, it is not we who define mathematics rather it is mathematics and numbers who define us, in fact not only us but also the things we have never imagined.

   For instance, **Fibonacci Series**, 1, 1, 2, 3, 5, 8, 13, 21….. which defines the pattern that emerges in flowers, living organisms, galaxies and so on. The pattern in these numbers is only a portion of

vast mathematics which tends to define the structure of the universe which make it inevitable that universe is made up of Mathematical components [1].

2. **What does Fundamental mean?**

   Fundamental means phenomenon or law that cannot be further derived from other components. Biology defines cells as the fundamental while physics defines elementary particles or fundamental forces as fundamental. But what exactly is fundamental?

   We have been searching for the fundamental that defines the whole universe or is the basis of the universe. In **1803**, John Dalton defined atom as fundamental and in **1897**, only a few years later, J.J Thomson argued that electron was more fundamental. Now, we have many fundamental particles also called Elementary particles like quarks, gluons, bosons and fundamental forces like weak nuclear force, strong nuclear force, gravitational force and electromagnetic force. We are in a series of these discoveries and hence we still cannot consider what fundamental could be. Is it a law, is it a force or any phenomenon or particles or fields like Higgs Boson or an equation? Who knows, if a physicist will come up with a new model for defining more fundamental?

   Fundamental is a rule by which the universe and its laws are made up. If there is anything fundamental then it must define every component of the universe from top to bottom. However, we have not yet discovered what fundamental is and thus, it seems challenging for the physicists or the mathematicians to discover the fundamental.

3. **What can be the fundamental then?**

   As said above, fundamental must derive every component of the universe directly or indirectly, many philosophers usually hypothesise *nothing* as fundamental. But does it implies that nothing can give rise to everything? Here, arise a question, "How does one define **nothing**?" I define nothing as '0' or zero. Universe follows an increasing (expanding) pattern where it can be a possibility that universe might have begun from zero and patterned to exist like this. I.e. **Universe (0) = Big bang**. [2]

   Similarly, many define **Energy** as fundamental. It is known that for every matter (mass) there exists a positive energy. It means that energy can define many things existing in the universe. We are aware of positive energy and though there are several hypotheses, we actually do not know if negative energy[1] exists if such energy exists then it could have another base. Similarly, energy is derived from Noether's theorem. So, it does not seem that energy is fundamental.

   Also, **Time** is considered as fundamental by some physicists. We know that we are travelling in space-time where time never stops and the arrow of time is always forward. There are several phenomena like entropy, events in space-time, and laws of thermodynamics that depends on time. So, is time Fundamental? As per the latest

[1] Here, someone can argue that negative energy does exist when it is the gravitational potential energy which is always attractive. But by negative energy, I mean that if any particle could exhibit negative energy or not.

experiments done in thermodynamics (Or quantum experiment), the physicist changed (or observed) the arrow of time backwards [3]. If time shows dependency on some quantum phenomenon then how can it be fundamental? Similarly, time can be derived and noted by mathematical equations which hints that time is not completely independent and cannot be completely fundamental.

Many scientists or philosophers in physics consider **Symmetry** as fundamental especially after seeing its uses in conservational laws and on several remarkable fields of physics like General Relativity, quantum mechanics and so on. But it seems to me that the symmetry that summarizes the rules of laws of nature, is also a mathematical formulation and depends on mathematical equations and forms to present itself to nature, which cannot be considered as the fundamental.

So, what actually is fundamental?

Almost every theories, laws or phenomena follow a common thing and that is Mathematics. So, I think that **mathematics could be fundamental** and it could be the basis for every law or phenomenon in the universe directly or indirectly.

My arguments can differ from other people's arguments or thoughts and much worse, my thoughts can be weaker than others' but seeing the patterns in the universe and the applications of mathematics and numbers in the remarkable theories, it is undeniable that mathematics can be the fundamental phenomenon of the universe.

4. How can Mathematics and numbers be fundamental?

Mathematics and numbers are used in almost every theory in physics. Without mathematics, the fields of science like quantum mechanics, astrophysics, engineering and much more are almost impossible. Mathematics defines those smallest phenomena that we cannot see and sense along with those enormous galaxies and infinities that we cannot even imagine with our mind. Without an equation, physics is only a philosophy. Let's see how different fields of science depends on Mathematics for their existence.

i. *Math defines Quantum Mechanics*: Quantum Mechanics is usually defined by several laws and mathematics is used to define those laws. Schrödinger Equation, for instance, is nothing without differential equations. In Quantum Mechanics, we start from some basic laws and move on to derive other laws from those basic laws. For defining the basic laws we use mathematics.

$$\Delta X.\,\Delta P \geq \frac{h}{2}$$

is the famous Heisenberg Uncertainty Principle, one of the basic but most remarkable discoveries in quantum mechanics, which is used in almost all of the Quantum Mechanics. Uncertainty Principle is a mathematical equation. There are many similar examples like wave functions, identical particles and so on which depends on mathematics. Hence, mathematics defines quantum

laws and theorems and thus, without mathematics quantum mechanics is a dead law.

ii. *Math defines classical physics and space-time*: Like quantum mechanics, classical physics is almost impossible without mathematics and numbers. There are many laws in classical physics, Newton's laws of motion, for example, which is inspired by mathematics. Similarly, space-time is an invention of mathematical equations. The two postulates of special relativity: (i*) The laws of physics are the same in all inertial frames of reference*. (ii) *The speed of light in free space has the same value c in all inertial frames of reference,* which are considered as two most breakthrough postulates of physics are some derivation from Lorentz transformations which are also equations. Also, General Relativity and Einstein's field equation are considered the most complex mathematical equations in physics. Most of the fields of classical mechanics like electromagnetism, kinematics, heat and energy and so on are in mathematical form. Thus, without mathematics, we would have never seen these laws which clarify that mathematics defines classical physics.

iii. *Math defines other different fields of science*: There are several other fields of science like Biology, computing, chemistry and many more, which are directly or indirectly related to mathematics. The structure of DNA can also be represented by some mathematical models [4] and the structure of cells and its organelles can also be represented in a mathematical model which shows that biological models are also related to mathematics. Moreover, there are several new discoveries like Quantum Biology[2] that uses mathematics and quantum models for existence.

In several chemical experiments and formulations, mathematics is used to define the bonding structures, entropy change or enthalpy change and many phenomena. Without mathematical equation, some of the chemical tests are not even possible. Thus, chemistry is also related to mathematics.

Similarly, there are several programs and hardware which operate to run a computer. All these huge programs and logic gates depend on only two bits (numbers) i.e. '0' and '1'. A computer, which can solve many difficult problems within no time, only operates on two numbers. Also, many functions inside the computer programs are completely dependent on mathematical operations which shows how computing has numbers and mathematics as a base. **Only two numbers can define whole complexities of a universe, then why can these infinite numbers not define the universe?**

[2] https://en.wikipedia.org/wiki/Quantum_biology

Therefore, we can see that most of the fields of science are directly or indirectly connected to mathematics and they can be represented in pure mathematical form. Doesn't it give a clue that mathematics could be fundamental?

5. **Mathematics shows beyond us: Infinity (∞)**

We cannot clearly explain things that we cannot observe and there are some mysterious numbers or phenomena at the end of the universe that we cannot observe that is infinity. We have not predicted what lies in the limit of mathematics or never will but the fact that mathematics helps us solve the problems related to infinities cannot be ignored. In another way, it helps us to see beyond us, calculate beyond us and predict beyond our abilities. And those infinity's related predictions made by mathematics are mostly correct and valid. In Quantum Mechanics, it is required to normalize any equation from -∞ to +∞ which shows how mathematics makes us able to know the fact that probability of anything (a particle found in the space) from -∞ to +∞ is always 1. Thus, our mind which does not sense infinity is shown by mathematical equations.

Also, we do not know what is in the infinity and through mathematics, we are able to predict the reality. The speed of light, for instance, is considered as the speed limit of the universe by the Special Relativity which shows that the nothing can exceed the speed of light and thus, helps us predict the universal laws in reality. **Mathematics, thus helps us to find the limit and consider the reality as it can define every edge of the universe (-∞ to +∞).**

6. **How numbers define universe?**

Numbers are mysterious and infinite. Without numbers, mathematics is impossible. Numbers also represent the patterns in the universe, for instance, Fibonacci Series. Each number in the universe has its own uniqueness and among them '0' Or zero shows the unique features. Zero is a mysterious number which can solve problems which are not solved by any other numbers like Fermat's last theorem. Also, zero divided by any number is zero but number divided by zero is indefinite (except when used Le Hopital's rules)

$$\frac{0}{X} = 0,$$

$$\frac{X}{0} = \infty \text{(Except for Le Hopital's rule)}$$

$$also,$$

$$0k = 0, 0k + 0X + 0Y + \cdots 0n = 0 \qquad \text{(i)}$$

which apply that this mysterious number has some relation with many unsolved problems in mathematics and thus with the universal phenomenon.

Similarly, the imaginary number '*i*' is also a mysterious number. It is considered as a 2-D number. In Lorentz contraction, if we assume an object (hypothetically) exceeds the light speed (though it violates mass-energy law, we are only assuming) then

$$l = \frac{l'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$l = \frac{l'}{\sqrt{1-\frac{4c^2}{c^2}}}$$ (let's assume that an object can move with speed 2c.)

$$l = \frac{l'}{i\sqrt{3}}$$

$l' = i\sqrt{3}\, l$ *(this is a 2-D expression)*

which shows that the observed length will discover a higher dimension [5]. If this occurs then we can deduce that light speed is the dimensional barrier and anything that exceeds light speed will discover a new dimension. Notice that we are only assuming that it moves in double the speed of light, in reality, nothing can. Thus, numbers are very mysterious and powerful at the same time and they can help to derive the unknown phenomenon in the universe.

Also, there are several *transcendental numbers* like exponential (e) or pi (π) and much more which are mysterious, yet define many important laws and phenomena of the universe. But we still don't know their complete form which also gives us a clue that like these numbers, the universe is also infinite.

Thus, numbers, simple or mysterious, define the universe and derive the laws of the universe which make them as one of the fundamental components of the universe.

7. Can Mathematics and Numbers be derived?

Considering Mathematics as fundamental contradicts the fact that they can be derived.

There are some philosophers of physics that consider the invention of mathematics and calculations as an illusion. What does it mean when a physicist says, "The universe is expanding faster than the speed of light"? Does this mean that our calculation for the expanding universe is an illusion? Since mathematics is used to calculate this, it would mean that the natural facts are illusions which is not possible. So, saying mathematics as an illusion is like saying nature does not exist at all.

But the real answer to this question is beneath another question "How were mathematics and numbers discovered and why?" Mathematics and numbers were discovered to solve many problems related to daily life. Counting and weighing are some of the examples. Mathematics, in fact, was not invented by anyone, rather it existed itself. Mathematics does not need to be invented by anyone because it pre-existed. The discovery of Calculus in the past, which would be further used to derive Newton's laws, was not invented so that it could solve some problems, but the pattern for calculus already existed in the world from the beginning.

Every theory in the universe is already proved but we need someone special to find them. [6]

Since mathematics existed from the existence of the universe, it cannot be further derived. In fact, what would mathematics be derived from? Some smaller components?

And that smaller component also would be mathematics or number, wouldn't it? But if someone says, "mathematics can be derived from nothing." The solution to this argument would be that 'nothing' can be defined as zero which is also mathematics.
Therefore, **it is not possible to have more fundamental than mathematics and numbers**.

8. **Can mathematics predict everything?**

Being fundamental, is mathematics able to predict every phenomenon or event?
Since the beginning of problems in physics or many other fields of science, mathematics is being able to tackle those problems and give sensible solutions. Mathematics has the power to predict many impossible things for humans like the fourth dimension, black holes or wormholes and many things however the challenges like the prediction of future, prediction of a body which exceeds the light speed or what is inside the black hole are considered nearly impossible in this age of science. But can mathematics tackle them?
Let me use mathematics and its logic to tackle *future*. I shall call it "Geometry of Future". I shall approach it by the logic of dimensions.
Geometry of future: We know that the observers in higher dimensions are able to control the events (space-time events) of the lower dimensional body. For instance, a two dimensional (2-D) square events are controlled by a 3-d cube. Similarly, a block can control the events of its shadow. An observer in 3-D knows about the events of its respective part 2-D. Change in the events in higher dimension causes changes in lower dimensions. Means 3-D can predict the future of 2-D or lower dimensions. If we compare this with space-time events then *the future events of an object is a result of some events of its higher dimensional part*. Hence, it can be conjectured that any event or its position in space-time diagram can be predicted if we are able to tackle the higher dimensional part.
This hypothesis can still be wrong as it lacks some strong arguments but we do have many other examples which show us that we have been somehow successful to predict future events. Newton's Second law, in classical physics, helps us to know the position or momentum of an object in the different period of time and similarly, Schrödinger's Equation on time evolution, in quantum mechanics helps to find how a quantum particle (or its energy) exists in the different period of time. Hence, it can be clearly seen that mathematics can predict future. If future then it can predict or solve other impossible problems as well.
Thus, by this example, it seems to me that it is not mathematics which is not able to prove or solve these impossible problems but it's our abilities which lacks proper handling of mathematics and number due to which we are not being able to solve these problems with universe's most fundamental, yet powerful tool.

9. **Can Mathematics represent *Theory of Everything* (TOE)?**

What is the reality? Is everything in the universe connected to each other? Is there anything absolute that can represent everything in the universe?

The Theory of Everything is a term for the ultimate theory of the universe—a set of equations capable of describing all phenomena that have been observed, or that will ever be observed [7].Theory of Everything (TOE) is one of the most discussed and major problems in modern physics. Today the ultimate goal of scientists is to connect every fundamental force and produce one result. But Can Mathematics represent this grand unification?

This has been previously discussed in a paper, "The Mathematical Universe"; where the author of the paper, Max Tegmark, has discussed that every theory is related to every other theory in physical science and the root of the theory is purely mathematical. [8]

Schrödinger's equation is also like a theory of everything to the Quantum Mechanics as it solves many quantum problems. Schrödinger's Equation is also a mathematical equation. Similarly, Maxwell's theory can be taken as an example, which unified the electrical and the magnetic field as electromagnetism which were once believed to be completely different. Thus, we are on a verge of grand unified field theory which would consider and represent every fundamental force or field into one equation by these similar approaches.

It seems that mathematics connects or relates many theories to each other. In our daily experiences, we can clearly see that mathematics can be used to unify different theories. Special relativity and General Relativity can be taken as an example. Similarly, the interpretation of higher dimensional problems done by mathematical formulation is another example which shows that mathematics can unify higher dimensional and lower dimensional theories. Hence, the unified law or TOE might be represented by mathematics and the form of TOE might be mathematical.

Finally, it can be concluded that mathematics and numbers are the fundamental components of the universe. Though, they are not law or theory in themselves but can represent any theories or laws. Mathematics cannot be derived from anything, rather the fact that how mathematics was discovered is still unknown, which in fact hints that mathematics is from the beginning; from nothing. Though it started from nothing, in near future, it will be use to represent the Theory of Everything.

Any other thing that can be said as fundamental laws or theories even use mathematics to present themselves in the universe. Every basic law, fundamental force or elementary particle, all need mathematics to complete themselves. Hence, mathematics is a powerful tool which can be considered as the basic of basics and fundamental of fundamentals.

Now, someone might ask that what part of mathematics is more fundamental. Is it Geometry, algebra or calculus, number theory? It occurs to me that it depends. For a physicist solving Schrodinger's Equation, calculus could be more fundamental, meanwhile for a physicist working on relativity, Geometry could be more fundamental. But the main

point is why should we emphasize which is more fundamental or less fundamental. “Every field of mathematics is inter-related to every other field.” Hence, it is worthless to discuss what part of mathematics is more fundamental, rather both physicists and mathematicians should discuss on only one point and that is how to solve the problems in physics considering mathematics as fundamental. In other words, I mean that without considering the fact about the fundamental features of mathematics and numbers, it is worthless to tackle great challenges in the field of science. Hence, finally the conclusion is that keeping in mind about the impossible challenges given to us by the universe, we should work on improving our mathematical skills and its criterion to achieve what we seek ahead.

Let’s keep in mind that phenomena already exist but it’s us who need to define them and that would be by using fundamental components or in other words, Mathematics.

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