Darwin for physicist: mathematical physics as evolutionary process

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Introduction

This essay is not about the ontological status of mathematics or similar metaphysical ideas. Instead this essay is about how mathematics actually works as a tool for doing physics. The emphasis is on the working of mathematics in constructing physical theories rather than supposed pre-established harmonies between mathematical objects and physical reality.

Main thesis

The key idea is that mathematical objects allow physicists to construct new theories without negating valuable information from older, more limited theories. Consider the inter-theoretic relation between the General Theory of Relativity (GTR) and Newton's theory of gravitation. Despite being conceptually completely different GTR incorporates important ideas from Newton's theory, for example the $1/r^2$ dependence of gravitational force. Now, in the context of Newton's theory, the $1/r^2$ dependence is contingent, whereas in Einstein's theory this functional dependence is necessarily implied by the geometrical set-up of the theory. For this reason we can say that GTR provides us with a better understanding of gravity. But, the 'brut fact' of this functional dependence was 'inherited' by GTR from Newton's theory. In Einstein's own words: "The difficulty was not to find general covariant equations for the metric tensor....but instead the real difficulty was to find general covariant equations which are simple generalisations of Newton's law." (Einstein in letter to Hilbert cited in E. Scheibe : Die Philosophie der Physiker,2007, page 309).In the same way Newton inherited the $1/r^2$ dependence from Kepler's law.

It is precisely this transfer of information about functional dependences from older, limited theories to more comprehensive theories which makes mathematics such an effective tool for physics. This process, the transfer of information between 'generations' of theories is akin to the passing on of genetic information from parents to offspring in phylogenetic biological evolution. Below I will develop this analogy within a specific methodology of physics. But for now let's summarize the main thesis and answer some questions and objections.

Main thesis: The primary role of mathematics in physical theories is the transfer of structural information (usually functional dependences of kinematic and dynamic variables) between theories, not the mathematical structure of any one theory. Moreover, a supposed isomorphism between the mathematical structure of a theory and physical reality is not required to explain the empirical success of physical theories.

Objections and answers

Question 1: Does this not mean that the mathematical structure of one particular theory is irrelevant, arbitrary?

Answer: Of course not. In order to transfer information in form of mathematical objects between theories the theories themselves must be able to 'understand' the information. That means that the mathematical structures of the different theories are constrained by the mathematical form of the information passed between them. Think of the transfer of biological information. DNA-sequences of tri-nucleotides encode for amino-acids. The chemistry of the DNA sequences must 'fit' with the chemistry of the amino-acids.

Observe that Newton/Kepler mainly used elementary geometric structures (conic sections etc.), whereas Einstein used differential geometry in form of tensor analysis, but in each case the $1/r^2$ dependence can be expressed.

Question 2 : Given that any one empirically successful physical theory can be expressed in mathematical language, does this not mean that that there is an isomorphism between the mathematical structure (e.g. mathematical objects like tensor-fields etc.) of the theory and physical facts out there in physical reality? And further, is it not this isomorphism which explains the effectiveness of mathematics in physics?

Answer: No, it simply means that some aspects of physical reality can be modelled with mathematical objects. No complete isomorphism between the reality covered by the theory and mathematical objects of the theory is required for this. To elucidate this point further I want to rephrase this objection (with apologies to EPR):

Question 2A: If, without in anyway disturbing a system, we can predict, with help of a mathematical theory, the outcome of an

experiment on this system, then there exists an isomorphism between mathematical objects of the theory and elements of reality of the system.

Answer: The above statement can only be regarded as a necessary condition for isomorphism. Observe that all current physical theories predict the correct outcome of experiments only for a limited set of inputs. This is known as the domain limitation of physical theories (e.g. Newtonian mechanics valid only for v^2/c^2 small in comparison with 1). A supposed isomorphism could not explain the failure to predict all outcomes correctly.

Now, let's consider this scenario: we are in possession of a mathematical theory predicting correctly all outcomes of all experiments on a given physical system. Does this now imply an isomorphism between mathematical objects of the theory and physical reality?

For this to be true the following condition must be met as well: it must be possible to 'predict' the outcome of any computation of the mathematical theory by performing an appropriate experiment on that system (physical simulation of computation).

None of our current physical theories fulfil this condition. To see this, recall that all these theories are based on real number calculus (standard calculus). But most real number calculations cannot be simulated by physical experiments in a finite time. This and similar considerations are known as the 'excess-baggage'- problem of physical theories.

To summarize, from the fact that a given physical theory (say Quantum Electrodynamics) is empirically successful we cannot conclude that this theory is isomorphic to physical reality.

Question 3: How then can we explain that it is possible to use mathematics to model physical systems?

Answer: This is a very important question and at this point I cannot provide a satisfactory answer. I can only surmise that homogeneity of time and space and stability of matter play an important part in the explanation. But I strongly maintain that an isomorphism in the above sense is not required.

Question 4: How else can we understand the empirical success of a theory, if not in form of an isomorphism, a faithful representation of reality?

Answer: This question is not correctly stated. It is not one physical theory which is successful, but the whole scientific process which leads up to some mathematical structure, which we conveniently call a theory (usually named after a famous scientist). At the heart of this process is the passing on of structural information (usually in form of functional dependences) from one theory to a new theory.

To make this more clear consider non-physical theories, say psychological theories. Freud's psychoanalytical ideas of the sub-conscious were followed by behaviouristic theories. None of the 'results' of the previous were passed on to the latter theory. In effect, with a change in research paradigm all psychological research has to start from scratch. Psychological theories are lacking an underlying methodology which would enable them to communicate and pass on information.

Construction of physical theories as evolutionary process

Physical theories have an underlying methodology, which consists in specifying kinematic and dynamic variables first and then writing down the dynamical equations of the theory. The dynamical equations can be obtained from a Lagrange-function via the variational principle.

It is the form of this Lagrange-function which changes from classical physics, to special relativistic physics, to general relativistic physics and so on. The progress of physics can be traced out by the development of the form of the action integral, i.e. the Lagrange-function.

There is no general scheme to construct Lagrange-functions for increasingly comprehensive theories. Otherwise we would be done with physics. But there are some general constrains on possible Lagrange-functions which can be used to select some functions for further consideration and to eliminate, reject other functions. A minimum requirement is that the resulting action-integral is a scalar quantity. Such Lagrange-functions are called 'valid' Lagrange-functions.

Consider a sequence of valid Lagrange-functions $L_0, L_1, L_2,...$ where L_n depends on exactly n independent dimensionless variables $x_1, x_2,...x_n$. Possible variables are $(v/c)^2, \phi/c^2, m/m(Planck)$, etc. For example $L_2 = L_2(x_1, x_2)$ and $L_3 = L_3(x_1, x_2, x_3)$ and so on.

We call such a sequence an 'evolutionary sequence', just in case, for all n in the sequence, in the limiting process xm →0 we obtain L(n)=L(n-1) + 'constant term'. 'Constant term' means a constant with respect to variation of L(n). For example take L1 as the special relativistic Lagrange-function for a free particle. In the limiting process $x1=(v/c)^2$ →0 we recover the Newtonian Lagrange-function for a free particle + the negative of the rest energy of the particle.

Within this scheme we can represent possible evolutionary path of theories (see diagram).
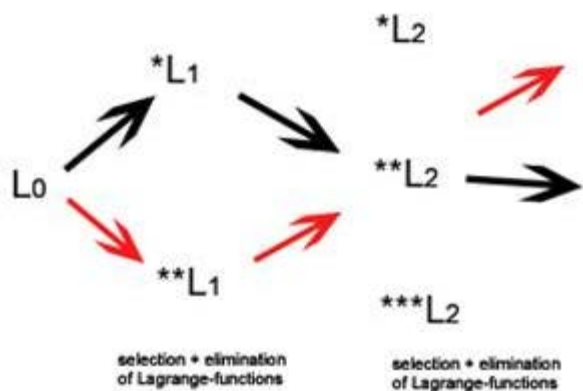
The only requirement for possible histories L0,* L1,**L2,...(black arrow) or L0,**L1,**L2,...(red arrow) is that each history forms an evolutionary sequence.

Discussion and comparison with Darwinian evolution

Biological evolution proceeds by mutation and selection. In the above scheme a theory L(n) will be extended by considering a set of valid alternatives for L(n+1) (mutation) and then subjecting these alternatives to selection criteria, i.e. by imposing symmetry conditions(e.g. general covariance, gauge symmetry etc. For survival the Lagrange-function must exhibit the correct symmetry adapted to its environment, i.e. the local physical reality it represents.

Possible' histories of theories' emerge, depending on the selection-criteria (symmetry requirements) applied at each step. There is no preferred lineage for the history of theories.

It is interesting to know that in the history of physics theories were actually discovered in precisely this way. Schroedinger used the variational principle to derive the time-independent Schroedinger equation, Einstein used the above process to find the Lagrangian for the gravitational field equation etc.

Conclusion

Why is mathematics so unreasonable effective in physical theories?

Adherents to the orthodox view postulate an isomorphism between the physical reality 'out there' and mathematical objects of the ultimate theory of physics (or even believe in the identity of the two). The task of the physicist, according to this view, is to 'decode' physical reality and to represent it in mathematical language. In their view the postulated isomorphism makes understandable the success of physics as witnessed since Newton's Principia.

In my view this picture is much too static, restricted to a mere mapping of physical reality here and mathematical objects there. The actual historic process of physics tells us a different story.

All physical theories (including future ones) contain only partial truth, but physics has a powerful methodology which permits us to generate new, more comprehensive theories from older, restricted theories. Above I have given a precise meaning to this process by pointing out selection principles of Lagrange-functions as a mechanism to construct new physical principles. The fundamental role of mathematics in physics is twofold:

1. To provide selection criteria for possible theories (consistency, invariance properties etc.)
2. To encode and pass on information about functional dependences between theories.

A nouvelle, more dynamic view of physics emerges: physics is not the decoding of one fundamental, underlying reality but the

successive exploration of a network of local ontologies.

Epilogue

Consider this situation: Alice is co-author of a paper on a new exact solution of Einstein's field equation. As a Platonist she believes that she has discovered pre-existing solution to Einstein's equations. Bob, one of the other authors and a naturalist, believes that the new solution was 'evoked' by Einstein in 1915. Finally Chloe believes that the solution exists only as a construction on a sheet of paper.

Who is right and should they proceed with publication? Answer by a Darwinian mathematician: Yes, of course, publish or perish! Mathematics is about passing on information!