About the Nature of Time

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October 2, 2008

Abstract

The identification of the experienced time t_e and geometric time t_q involves well-known problems. Physicist is troubled by the reversibility of t_q contra irreversibility of t_e , by the conflict between determinism of Schrödinger equation and the non-determinism of state function reduction, and by the poorly understood the origin of the arrow of t_q . In biology the second law of thermodynamics might be violated in its standard form for short time intervals. Neuroscientist knows that the moment of sensory experience has a finite duration, does not understand what memories really are, and is bothered by the Libet's puzzling finding that neural activity seems to precede conscious decision. These problems are discussed in the framework of Topological Geometrodynamics (TGD) and TGD inspired theory of consciousness constructed as a generalization of quantum measurement theory. In TGD space-times are regarded as 4-dimensional surfaces of 8-dimensional space-time $H = M^4 \times CP_2$ and obey classical field equations. The basic notions of consciousness theory are quantum jump and self. Subjective time is identified as a sequence of quantum jumps. Self has as a geometric correlate a fixed volume of H- "causal diamond"-defining the perceptive field of self. Quantum states are regarded as quantum superpositions of space-time surfaces of H and by quantum classical correspondence assumed to shift towards the geometric past of H quantum jump by quantum jump. This creates the illusion that perceiver moves to the direction of the geometric future. Macroscopic quantum coherence and the identification of space-times as surfaces in H play a crucial role in this picture allowing to understand also other problematic aspects in the relationship between experienced and geometric time.

1 Introduction

The notion of time remains one of the most problematic concepts of physics. In classical physics the different properties of the time of Newton's equations and thermodynamical time are puzzling. In special relativity and general relativity the notion of simultaneity becomes a problematic concept and challenges the naive Newtonian view about time flow as a motion of 3-D time=constant snapshot of 4-D space-time. This notion is also in conflict with general coordinate invariance. In quantum measurement theory the localization of the state function reduction process into a finite space-time volume is in conflict with the determinism of Schrödinger equation. In biology the presence of self-organization processes like self assembly challenge second law of thermodynamics in short time scales. In neuroscience the finding of Libet suggesting that neural activity seems to precede conscious decision forces to give up the notion of free will or the naive identification of experienced and geometrical time.

In this essay I will consider a new view about time based on Topological Geometrodynamics [13], which can be regarded as an attempt to unify fundamental interactions assuming that space-times are representable as 4-dimensional surfaces of certain higher-dimensional space-time $H = M^4 \times CP_2$ (M^4 denotes 4-D Minkowski space and CP_2 complex projective space of 2 complex dimensions) fixed by the requirement that the theory explains standard model symmetries and provides a geometrization of classical gauge fields and gravitational fields.

The construction of quantum TGD leads to a radical revision of space-time concept (many-sheeted space-time and topological field quantization), and forces also to generalize the original view about

imbedding space. p-Adic physics as physics of cognition and intentionality is part of TGD inspired theory of consciousness and the need to fuse real and p-adic physics to single coherent whole forces to revise the notions of number and space-time. It has been also necessary to replace the standard positive energy ontology with what I call zero energy ontology. These generalizations are of special importance in TGD inspired theory of consciousness and of quantum biology.

There are several first principle approaches to quantum TGD and following gives only a very concise summary of them.

- 1. Generalization of Einstein's program of geometrizing classical physics so that quantum theory can be seen as a theory of *classical* spinor fields in the world of classical worlds (WCW) consisting of light-like 3-surfaces and possessing Kähler geometry [4, 5, 6]. By general coordinate classical physics becomes an exact part of quantum theory in a well-defined sense. A geometrization of Fermi statistics is obtained, and the Clifford algebra associated with the spinors of WCW can be regarded as a direct sum of von Neumann algebras known as hyper-finite factors of type II₁ (HFFs) closely related to quantum groups and non-commutative geometry;
- 2. Quantum TGD as almost topological field theory (TQFT) with fundamental objects identified as light-like 3-surfaces and having generalized super-conformal symmetries as symmetries [7, 8]: the notion of braid is the basic building block of this approach;
- 3. There are two kinds of conformal symmetries corresponding to the boundary of light-cone of Minkowski space and light-like 3-surfaces, and these symmetries alone dictate to high degree the physics. Quite recently it turned that also a symplectic analog of conformal field theory emerges naturally in TGD framework (super-canonical symmetries) and this led to a concrete proposal for how to construct n-point functions needed to calculate M-matrix [13];
- 4. Physics as a generalized number theory involving three different threads corresponding to need fuse real and various p-adic physics to single coherent whole by using a generalization of number concept obtained by gluing reals and various p-adic number fields and their extensions together along rationals and common algebraics [10]; the observation that standard model symmetries and dynamics of quantum and classical TGD are to high degree dictated by classical number fields [11]; and the ideas inspired by the notion of infinite prime [12];
- 5. The identification of WCW Clifford algebra elements as hyper-octonion (subspace of complexified octonions spanned by real unit and octonionic imaginary units multiplied by the commuting additional imaginary unit) valued conformal fields having values in HFF provides a justification for the concept of number theoretic braid needed both in the fusion of real and p-adic physics and in TGD as almost TQFT approach;
- 6. The hierarchy of Planck constants realizing quantum criticality [9] forces a generalization of the notion of imbedding space by replacing it with a book like structure having as its pages singular coverings and factor spaces of H and allowing to realize geometric correlates for the choice of quantization axis in quantum measurement: the particles at different pages of this book are "relatively dark" since they do not possess local interaction vertices which means a radically new manner to interpret dark matter;
- 7. Zero energy ontology and the notion of finite measurement resolution formulated in terms of inclusions of HFFs fix quantum dynamics highly in terms of Connes tensor product allowing to interpret quantum theory as a square root of thermodynamics [13, 8]: finite measurement resolution has number theoretic braid as its space-time correlate so that various approaches to TGD are closely related;
- 8. Quantum theory of consciousness as a generalization of quantum measurement theory to include observer to the theory [14].

The notion of number theoretic braid is a common denominator of various approaches to quantum TGD and leads to effective discretization of the imbedding space which is however due to the finite measurement resolution and number theoretic constraints rather than being something fundamental so that there are no problems with standard conservation laws.

The article series about TGD and its applications to biology and consciousness [13, 14, 15, 16, 17, 18, 19, 20] gives an overall view about quantum TGD. In the following I will concentrate only on the aspects of quantum TGD relevant for the notion of time. I will first describe zero energy ontology and p-adicization program and after that consider the problem of time.

2 The most recent vision about zero energy ontology and padicization

The generalization of the number concept obtained by fusing real and p-adics along rationals and common algebraics is the basic philosophy behind p-adicization. One must be able to speak about rational points common to real and various p-adic variants of H. The basic objection is the necessity to fix some special coordinates in turn implying the loss of a manifest general coordinate invariance. The isometries of the imbedding space could save the situation provided one can identify some special coordinate system in which isometry group reduces to its discrete subgroup. The loss of the full isometry group could be compensated by assuming that WCW is union over sub-WCW:s obtained by applying isometries on basic sub-WCW with discrete subgroup of isometries.

The combination of zero energy ontology realized in terms of a hierarchy of causal diamonds (CDs) and hierarchy of Planck constants providing a description of dark matter and leading to a generalization of the notion of imbedding space suggests that it is possible to realize this dream. The article [13] provides a brief summary about recent state of quantum TGD helping to understand the big picture behind the following considerations.

2.1 Zero energy ontology briefly

- 1. The basic construct in the zero energy ontology is the space $CD \times CP_2$, where the causal diamond CD is defined as an intersection of future and past directed light-cones with time-like separation between their tips regarded as points of the underlying universal Minkowski space M^4 . In zero energy ontology physical states correspond to pairs of positive and negative energy states located at the boundaries of the future and past directed light-cones of a particular CD.
- 2. CD:s form a fractal hierarchy and one can glue smaller CD:s within larger CD along the upper light-cone boundary along a radial light-like ray. This construction recipe allows to understand the asymmetry between positive and negative energies and why the arrow of experienced time corresponds to the arrow of geometric time and why the contents of sensory experience is located to so narrow interval of geometric time. One can imagine evolution to occur as quantum leaps in which the size of the largest CD in the hierarchy of personal CD:s increases in such a manner that it becomes sub-CD of a larger CD. p-Adic length scale hypothesis [13] follows if the values of temporal distance T between tips of CD come in powers of 2^n : $T = 2^n T_0$. All conserved quantum numbers for zero energy states have vanishing net values. The interpretation of zero energy states in the framework of positive energy ontology is as physical events, say scattering events with positive and negative energy parts of the state interpreted as initial and final states of the event.
- 3. In the realization of the hierarchy of Planck constants $CD \times CP_2$ is replaced with a Cartesian product of book like structures formed by almost copies of CD:s and CP_2 :s defined by singular coverings and factors spaces of CD and CP_2 with singularities corresponding to intersection $M^2 \cap CD$ and homologically trivial geodesic sphere S^2 of CP_2 for which the induced Kähler form vanishes. The coverings and factor spaces of CD:s are glued together along common $M^2 \cap CD$. The coverings and factors spaces of CP_2 are glued together along common homologically non-trivial geodesic sphere S^2 . The choice of preferred M^2 as subspace of tangent space of X^4 at all its points and interpreted

as space of non-physical polarizations, brings M^2 into the theory also in different manner. S^2 in turn defines a subspace of the much larger space of vacuum extremals as surfaces inside $M^4 \times S^2$.

- 4. Configuration space (the world of classical worlds, WCW) decomposes into a union of sub-WCW:s corresponding to different choices of M^2 and S^2 and also to different choices of the quantization axes of spin and energy, color isospin and hyper-charge for each choice of this kind. This means breaking down of the isometries to a subgroup. This can be compensated by the fact that the union can be taken over the different choices of this subgroup.
- 5. p-Adicization requires a further breakdown to discrete subgroups of the resulting sub-groups of the isometry groups but again a union over sub-WCW:s corresponding to different choices of the discrete subgroup can be assumed. Discretization relates also naturally to the notion of number theoretic braid.

Is it really possible to speak about zero energy states for a given sector defined by generalized imbedding space with fixed M^2 and S^2 ? Classically this is possible and conserved quantities are well defined. In quantal situation the presence of the light-cone boundaries breaks full Poincare invariance although the infinitesimal version of this invariance is preserved. Note that the basic dynamical objects are 3-D light-like "legs" of the generalized Feynman diagrams glued together along their ends at generalized vertices.

2.2 Definition of energy in zero energy ontology

The approach relying on the two super conformal structures of quantum TGD gives hopes of defining the notion of energy for positive and negative energy parts of the state.

- 1. Since light-like 3-surfaces assignable to incoming and outgoing legs of the generalized Feynman diagrams are the basic objects, one can hope of having enough translational invariance to define the notion of energy. If translations are restricted to time-like translations acting in the direction of the future (past) then one has local translation invariance of dynamics for classical field equations inside δM_{\pm}^4 as a kind of semigroup. Also the M^4 translations leading to interior of X^4 from the light-like 2-surfaces surfaces act as translations. Classically these restrictions correspond to non-tachyonic momenta defining the allowed directions of translations realizable as particle motions. These two kinds of translations have been assigned to super-canonical conformal symmetries at $\delta M_{\pm}^4 \times CP_2$ and and super Kac-Moody type conformal symmetries at light-like 3-surfaces. Equivalence Principle in TGD framework states that these two conformal symmetries define a structure completely analogous to a coset representation of conformal algebras so that the four-momenta associated with the two representations are identical [7].
- 2. The condition selecting preferred extremals of Kähler action is induced by a global selection of $M^2 \subset M^4$ as a plane belonging to the tangent space of X^4 at all its points [7] and interpreted as a plane of nonphysical polarizations so that direct connection with number theory and gauge symmetries emerges. The M^4 translations of X^4 as a whole in general respect the form of this condition in the interior. Furthermore, if M^4 translations are restricted to M^2 , also the condition itself rather than only its general form is respected. This observation, the earlier experience with p-adic mass calculations, and also the treatment of quarks and gluons in QCD encourage to consider the possibility that translational invariance should be restricted to M^2 translations so that mass squared, longitudinal momentum and transversal mass squared would be well defined quantum numbers. This would be enough to realize zero energy ontology. Encouragingly, M^2 appears also in the generalization of the causal diamond to a book-like structure forced by the realization of the hierarchy of Planck constant at the level of the imbedding space.
- 3. That the cm degrees of freedom for CD would be gauge like degrees of freedom sounds strange. The paradoxical feeling disappears as one realizes that this is not the case for sub-CD:s, which indeed can have non-trivial correlation functions with either upper or lower tip of the CD playing a role

analogous to that of an argument of n-point function in QFT description. One can also say that largest CD in the hierarchy defines infrared cutoff.

2.3 p-Adic variants of the imbedding space

The need to fuse p-adic physics with TGD emerged originally from the discovery that p-adic mass calculations based on p-adic thermodynamics give excellent predictions for elementary particle masses if one assumes p-adic length scale hypothesis stating that primes near integer powers of 2 are physically favored [13]. Later came the interpretation of p-adic physics as cognition cognition and intentionality. The following somewhat technical construction of p-adic variants of the imbedding space provides new insights concerning the understanding of the arrow of geometric time.

- 1. Rational values of p-adic coordinates are non-negative so that light-cone proper time $a_{4,+} = \sqrt{t^2 z^2 x^2 y^2}$ is the unique Lorentz invariant choice for the p-adic time coordinate near the lower tip of CD. For the upper tip the identification of a_4 would be $a_{4,-} = \sqrt{(t-T)^2 z^2 x^2 y^2}$. In the p-adic context the simultaneous existence of both square roots poses additional conditions on T. For 2-adic numbers $T = 2^n T_0$, $n \ge 0$ (or more generally $T = \sum_{k \ge n_0} b_k 2^k$), would allow to satisfy these conditions, which would be one additional reason for $T = 2^n T_0$ implying p-adic length scale hypothesis. The remaining coordinates of CD are naturally (hyperbolic) cosines and sines of the spherical coordinates θ and ϕ (hyperbolic angle $\eta_{\pm,4}$).
- 2. The existence of the preferred plane M^2 of un-physical polarizations would suggest that 2-D light-cone proper times $a_{2,+} = \sqrt{t^2 z^2}$ $a_{2,-} = \sqrt{(t-T)^2 z^2}$ can be also considered. The remaining coordinates would be naturally $\eta_{\pm,2}$ and cylindrical coordinates (ρ, ϕ) .
- 3. The p-adically transcendental values of a_4 and a_2 are literally infinite as real numbers and could be visualized as points in infinitely distant geometric future so that the arrow of time might be said to emerge number theoretically.
- 4. The selection of the preferred quantization axes of energy and angular momentum unique apart from a Lorentz transformation of M^2 would have purely number theoretic meaning in both cases. One must allow a union over sub-WCWs labeled by points of SO(1,1). This suggests a deep connection between number theory, quantum theory, quantum measurement theory, and even quantum theory of mathematical consciousness.
- 5. In the case of CP_2 there are three real coordinate patches involved [21]. The compactness of CP_2 allows to use cosines and sines of the preferred angle variable for a given coordinate patch.

$$\xi^{1} = tan(u)exp(i\frac{(\Psi + \Phi)}{2})cos(\frac{\Theta}{2}) ,$$

$$\xi^{2} = tan(u)exp(i\frac{(\Psi - \Phi)}{2})sin(\frac{\Theta}{2}) .$$
 (1)

The ranges of the variables u, Θ, Φ, Ψ are $[0, \pi/2], [0, \pi], [0, 4\pi], [0, 2\pi]$ respectively. Note that u has naturally only positive values in the allowed range. S^2 corresponds to the values $\Phi = \Psi = 0$ of the angle coordinates.

6. The rational values of the (hyperbolic) cosine and sine correspond to Pythagorean triangles having sides of integer length and thus satisfying $m^2 = n^2 + r^2$ ($m^2 = n^2 - r^2$). These conditions are equivalent and allow the well-known explicit solution [22]. One can construct a p-adic completion for the set of Pythagorean triangles by allowing p-adic integers which are infinite as real integers as solutions of the conditions $m^2 = r^2 \pm s^2$. These angles correspond to genuinely p-adic directions having no real counterpart. Hence one obtains p-adic continuum also in the angle degrees of freedom. Algebraic extensions of the p-adic numbers bringing in cosines and sines of the angles π/n

lead to a hierarchy increasingly refined algebraic extensions of generalized imbedding space. Since the different sectors of WCW directly serve as correlates of selves, this means a direct correlation with the evolution of the mathematical consciousness. Trigonometric identities allow to construct points which in the real context correspond to sums and differences of angles.

7. Negative rational values of the cosines and sines correspond as p-adic integers to infinite real numbers and it seems that one use several coordinate patches obtained as copies of the octant $(x \ge 0, y \ge 0, z \ge 0,)$. An analogous picture applies in CP_2 degrees of freedom.

3 Zero energy ontology, self hierarchy, and the notion of time

Consider now the formulation of TGD inspired quantum theory of consciousness [14] and quantum biology [15] in terms of zero energy ontology.

One should understand the asymmetry between positive and negative energies and between two directions of geometric time at the level of conscious experience, the correspondence between experienced and geometric time, and the emergence of the arrow of time. One should explain why human sensory experience is about a rather narrow time interval of about .1 seconds and why memories are about the interior of much larger CD with time scale of order life time. One should have a vision about the evolution of consciousness: how quantum leaps leading to an expansion of consciousness occur.

Negative energy signals to geometric past - about which phase conjugate laser light represents an example - provide an attractive tool to realize intentional action as a signal inducing neural activities in the geometric past (this would explain Libet's classical findings), a mechanism of remote metabolism, and the mechanism of declarative memory as communications with geometric past. One should understand how these signals are realized in zero energy ontology and why their occurrence is so rare.

In the following I try to demonstrate that TGD inspired theory of consciousness and quantum TGD proper indeed are in tune.

3.1 Causal diamonds as correlates for selves

Quantum jump as a moment of consciousness, self as a sequence of quantum jumps integrating to self, and self hierarchy with sub-selves experienced as mental images, are the basic notions of TGD inspired theory of consciousness. In the most ambitious vision self hierarchy reduces to a fractal hierarchy of quantum jumps within quantum jumps.

It is natural to interpret CD:s as correlates of selves. CDs can be interpreted either as subsets of the generalized imbedding space or as sectors of WCW. Accordingly, selves correspond to CD:s of the generalized imbedding space or sectors of WCW, literally separate interacting quantum Universes. The spiritually oriented reader might speak of Gods. Sub-selves correspond to sub-CD:s geometrically. The contents of consciousness of self is about the interior of the corresponding CD at the level of imbedding space. For sub-selves the wave function for the position of tip of CD brings in the delocalization of sub-WCW.

The fractal hierarchy of CDs within CDs is the geometric counterpart for the hierarchy of selves: the quantization of the time scale of planned action and memory as $T(k) = 2^k T_0$ suggest an interpretation for the fact that we experience octaves as equivalent in music experience.

3.2 Why sensory experience is about so short time interval?

CD picture implies automatically the 4-D character of conscious experience and memories form part of conscious experience even at elementary particle level. Amazingly, the secondary p-adic time scale of electron is T=0.1 seconds defining a fundamental time scale in living matter. The problem is to understand why the sensory experience is about a short time interval of geometric time rather than about the entire personal CD with temporal size of order life-time. The explanation would be that sensory input corresponds to subselves (mental images) with $T \simeq .1$ s at the upper light-like boundary

of CD in question. This requires a strong asymmetry between upper and lower light-like boundaries of CD:s.

- 1. This suggests that the addition of CD:s within CD:s in the state construction must always glue them to the upper light-like boundary of CD along light-like radial ray from the tip of the past directed light-cone. This conforms with the classical picture according to which classical sensory data arrives from the geometric past with velocity which is at most light velocity.
- 2. One must explain the rare but real occurrence of phase conjugate signals identified as negative energy signals propagating towards geometric past. These are possible when the sub-CD is glued to both the past and future directed light-cones at the space-like edge of CD along light-like rays emerging from the edge. Remote metabolism would represent a particular instance of this situation. Negative energy signals as a basic mechanism of intentional action would explain why living matter seems to be so special.
- 3. Geometric memories would correspond to the lower boundaries of CD:s and would not be in general sharp because only the sub-CD:s glued to both upper and lower light-cone boundary would be present. A temporal sequence of mental images, say the sequence of digits of a phone number, could correspond to a sequence of sub-CD:s glued to the upper light-cone boundary.
- 4. Sharing of mental images corresponds to a fusion of sub-selves/mental images to single sub-self by quantum entanglement: the space-time correlate could be flux tubes connecting space-time sheets associated with sub-selves represented also by space-time sheets inside their CD:s. These "episodal" memories could correspond to CD:s at upper light-cone boundary of CD.

Interesting questions relate to how dynamical selves are.

- 1. Is self doomed to live inside the same sub-WCW eternally as a lonely god? This question has been already answered: there are interactions between sub-CD:s of given CD, and one can think of selves as quantum superposition of states in CD:s with wave function having as its argument the tips of CD, or rather only the second one since T is assumed to be quantized.
- 2. Is there largest CD in the personal CD hierarchy of self in an absolute sense? Or is the largest CD present only in the sense that the contribution to the contents of consciousness coming from very large CD:s is negligible? Long time scales T correspond to low frequencies and thermal noise might mask these contributions. Here however the hierarchy of Planck constants and generalization of the imbedding space could come in rescue by allowing dark EEG photons to have energies above thermal energy.
- 3. Can selves evolve in the sense that the size of CD increases in quantum leaps so that the corresponding time scale $T=2^kT_0$ of memory and planned action increases? Geometrically this kind of leap would mean that CD becomes a sub-CD of a larger CD either at the level of conscious experience or in absolute sense. The leap can occur in two senses: as an increase of the largest p-adic time scale in the personal hierarchy of space-time sheets or as increase of the largest value of Planck constants in the personal dark matter hierarchy. At the level of individual orgainism this would mean emergence of new lower frequencies of generalized EEG and levels of personal dark matter hierarchy with larger value of Planck constant.

3.3 Arrow of time

In the standard materialistic ontology experienced time and geometric time are identified. In the naivest picture the flow of time is interpreted in terms of the motion of 3-D time=constant surface of space-time towards geometric future. This identification is plagued by several difficulties. In special relativity the difficulties relate to the impossibility to define the notion of simultaneity in a unique manner: the replacement of time=constant 3-surface with past directed light-cone assignable to the world-line of

observer resolves this problem. In general relativity the problem is that past light-cones need make sense only locally. In quantum measurement theory the observed localization of the state function reduction to a finite space-time volume is in conflict with the determinism of Schrödinger equation.

TGD forces a new view about the relationship between experienced and geometric time. Although the basic paradox of quantum measurement theory disappears the question about the arrow of geometric time remains.

- 1. Selves correspond to CD:s the own sub-WCW:s. These sub-WCW:s and their projections to the imbedding space do not move anywhere. Therefore standard explanation for the arrow of geometric time cannot work.
- 2. If quantum jump corresponds to a shift of quantum superposition of space-time sheets towards geometric past in the first approximation (by quantum classical correspondence), one can understand the arrow of time. Space-time surfaces simply shift backwards with respect to the geometric time of the imbedding space and therefore to the 8-D perceptive field defined by the CD. This creates in the materialistic mind a kind of temporal variant of train illusion. Space-time as 4-surface and macroscopic and macro-temporal quantum coherence are absolutely essential for this interpretation to make sense.
- 3. Why this shifting should always occur to the direction of geometric past of the imbedding space? Certainly the asymmetric construction of zero energy states should correlate with the preferred direction. Cognition and time relate to each other very closely and the required fusion of real physics with various p-adic physics of cognition and intentionality could also relate to the asymmetry. Indeed, in the p-adic sectors the transcendental values of p-adic light-cone proper time correspond to literally infinite values of the real valued light-cone proper time so that most points of p-adic space-time sheets defining correlates of thoughts and intentions reside always in the infinite geometric future in the real sense. Cognition and intentionality would break the symmetry between positive and negative energies and geometric past and future, and the breaking of arrow of geometric time would be induced by intentional action and reflect the basic aspects of cognitive experience.

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