

Lessons from failures to achieve what was possible in the twentieth century physics

Vesselin Petkov
Science College, Concordia University
1455 De Maisonneuve Boulevard West
Montreal, Quebec, Canada H3G 1M8
E-mail: vpetkov@alcor.concordia.ca

In recent years physicists have started to express publicly their concerns that not everything has been quite fine with the advancement of physics in the last several decades [1]. Although not all agree with such concerns I think it is an undeniable fact that in this period there has been no revolutionary breakthrough in fundamental physics of the type of relativity and quantum physics. A convenient answer might be that more experimental evidence is required to make a breakthrough possible. But what if it turns out that all conditions necessary for a new physical theory have already been present but physicists have been failing to process successfully the existing theoretical and experimental evidence? A concise analysis of several missed opportunities in the twentieth century physics due to what I think are inadequate views on doing physics seems to suggest that this possibility should be taken seriously.

Physicists generally agree that physics describes and tries to explain the world. However, this appears to be the only consensus. Physicists' views on the nature of physical theories vary widely but I think the views which might prevent physicists who hold them from arriving at new results are those that regard our theories only as good descriptions of the world and that insist on not considering the “most successful abstractions to be real properties of our world” [2] (one can even come across extreme views that physics tells nothing about the *existence* of what it describes). I think these are unproductive views negatively affecting not only the advancement of physics, but also the physicists who share them and who often pay a high personal price. Here are two examples from the history of the twentieth century physics.

It was precisely the view, that successful abstractions should not be regarded as representing something real, that prevented Lorentz from discovering special relativity. He believed that the time t of an observer at rest with respect to the aether (which is a genuine example of reifying an unsuccessful abstraction) was the true time, whereas the quantity t' of another observer, moving with respect to the first, was merely an abstraction that did not represent anything real in the world. Lorentz himself admitted the failure of his approach [3]:

The chief cause of my failure was my clinging to the idea that the variable t only can be considered as the true time and that my local time t'

must be regarded as no more than an auxiliary mathematical quantity. In Einstein's theory, on the contrary, t' plays the same part as t ; if we want to describe phenomena in terms of x', y', z', t' we must work with these variables exactly as we could do with x, y, z, t .

The second example is especially relevant now since this year we celebrate the one hundredth anniversary of the publication of Minkowski's paper "Space and time" in 1909 [4]. A century after Minkowski, for some physicists spacetime is nothing more than a four-dimensional mathematical space which does not represent a real four-dimensional world. In a recent *Reference Frame* N. David Mermin wrote that "spacetime is an abstract four-dimensional mathematical continuum" [2] and pointed out that regarding spacetime only as an abstract concept was a good example of not "inappropriately reifying our successful abstractions" [2]. However, if this were the case and spacetime were merely a mathematical space with no counterpart in the world, we would celebrate Poincaré, not Minkowski, because it was Poincaré who first noticed (before July 1905) that the Lorentz transformations had a geometric interpretation as rotations in what he seemed to have regarded as an *abstract* four-dimensional space [5, p.168].

The reason of why Poincaré failed to develop that idea seems to be even more complicated than that in the case of Lorentz. Poincaré appeared to have seen nothing revolutionary in the idea of a mathematical four-dimensional space since he believed that our physical theories are only convenient descriptions of the world and therefore it is really a matter of *convenience* which theory we would use in a given situation. Poincaré's failure to comprehend the profound physical meaning of the principle of relativity and of the geometric interpretation of the Lorentz transformations is one of the examples in the history of physics when an inadequate philosophical position prevents a scientist (even as great as Poincaré) from making a discovery.

This should be specifically stressed because physicists often think that they do not need any philosophical position for their research. In order to make sure that inadequate philosophical views do not prevent us (especially young physicists) from arriving at important new results I think it would be helpful if from time to time we recall similar cases from the history of science or at least what the philosopher Daniel Dennett said on this issue: "Scientists sometimes deceive themselves into thinking that philosophical ideas are only, at best, decorations or parasitic commentaries on the hard, objective triumphs of science, and that they themselves are immune to the confusions that philosophers devote their lives to dissolving. But there is no such thing as philosophy-free science; there is only science whose philosophical baggage is taken on board without examination" [6].

Had Minkowski believed, like Poincaré, that uniting space and time into a four-dimensional space was only a convenient mathematical abstraction, he would not have written a paper whose title and content were devoted to something the main idea of which had already been published by Poincaré two years (and probably written three years) before Minkowski's talk on space and time given in September 1908, and would not have begun his paper with the now famous introduction, which unequivocally announced the revolution in our views on space and time: "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of

union of the two will preserve an independent reality” [7, p. 75].

Unlike Poincaré, Minkowski appears to have realized that special relativity, particularly relativity of simultaneity (which implies the existence of *many* spaces), would be *impossible* in a three-dimensional world. *As a three-dimensional space constitutes a single class of absolutely simultaneous events, if the world were three-dimensional (i.e., if spacetime were not representing a real four-dimensional world and were just an abstract mathematical space), there would exist one absolute space and one absolute class of simultaneous events in contradiction with relativity.* If one tends to think that Minkowski had not done more than Poincaré on the unification of space and time and that relativity does not force us to regard the world as four-dimensional, it would be refreshingly helpful to examine the argument in the italicized sentence (which appears to have made Minkowski realize the profound physical meaning of relativity of simultaneity) and to try to demonstrate that the theory of relativity is possible in a three-dimensional world.

When we take into account that relativity of simultaneity does imply that, having different classes of simultaneous events, observers in relative motion have *different spaces*, we can better understand why Minkowski noted “neither Einstein nor Lorentz made any attack on the concept of space” [7, p. 83] and emphasized that “We would then have in the world no longer *the* space, but an infinite number of spaces, analogously as there are in three-dimensional space an infinite number of planes. Three-dimensional geometry becomes a chapter in four-dimensional physics. You see why I said at the outset that space and time are to fade away into shadows, and that only a world in itself will subsist” [7, pp. 79-80].

A hundred years ago Minkowski gave us the correct relativistic picture of the world where the geodesic worldtubes of what we perceive as freely moving three-dimensional macroscopic bodies are in reality four-dimensional objects, not “abstract geometric constructions” [2]. That macroscopic physical bodies are real four-dimensional worldtubes is the essence of Minkowski’s explanation of the deep physical meaning of length contraction – as shown in Fig. 1 of his paper (the right-hand part of which is reproduced in Fig. 1 here) length contraction would be *impossible* if the worldtubes of the two Lorentzian electrons, represented by the vertical and the inclined bands in Fig. 1, did not exist and were nothing more than abstract graphical constructions. To see this more clearly consider only the electron represented by the vertical worldtube. The three-dimensional cross-section PP , resulting from the intersection of the electron’s worldtube and the space of an observer at rest with respect to the electron, is the electron’s proper length. The three-dimensional cross-section $P'P'$, resulting from the intersection of the electron’s worldtube and the space of an observer moving with respect to the electron, is the relativistically contracted length of the electron measured by that observer. Minkowski stressed that “this is the meaning of Lorentz’s hypothesis of the contraction of electrons in motion” [7, p. 82] and “that the Lorentzian hypothesis is completely equivalent to the new conception of space and time, which, indeed, makes the hypothesis much more intelligible” [7, p. 81]. The worldtube of the electron must be real in order that length contraction be possible because, while measuring the same electron, the two observers in relative motion measure *two* three-dimensional electrons represented by the cross-sections PP and $P'P'$ in Fig. 1.

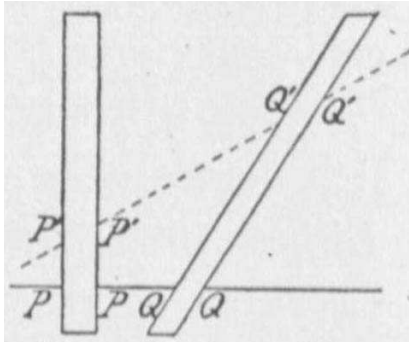


Figure 1: In Minkowski's paper "Space and Time" Fig. 1, which is reproduced here, represents two Lorentzian electrons by their worldtubes or as Minkowski called them "world bands".

This is not so surprising when one takes into account relativity of simultaneity and the fact that an extended three-dimensional object is defined in terms of *simultaneity* – all parts of the electron taken simultaneously at a given moment. If the worldtube of the electron were indeed an abstract geometric construction and what existed was a single three-dimensional electron (a single class of simultaneous events) represented by the proper cross-section PP , both observers would measure the same three-dimensional electron, i.e. the same class of simultaneous events in contradiction with relativity.

1. Missed opportunities due to failing to resolve the issue of the dimensionality of the world according to relativity. One of the main failures of the twentieth century physics is not taking seriously Minkowski's arguments for the reality of spacetime or as he called it perhaps more appropriately "the absolute world." For some physicists this is not a failure at all since for them the question of whether or not spacetime is an abstract concept or represent a real four-dimensional world is not a physical question since it rather belongs to philosophy. Such a view is worrisome for several reasons. First, I wonder how many physicists would subscribe to this view if directly asked whether the dimensionality of the world is a philosophical question (especially now when physicists freely talk about multidimensional spaces and the multiverse). Second, I think such a view deprives physicists from arriving at a deeper understanding of what relativity is telling us about the world. Third, this particular view has been responsible for not arriving at some important results as I intend to show here.

1.1. Dimensionality of the world and inertia. Inertia has been an open question for centuries and a golden opportunity to finally start resolving it in the twentieth century has been omitted because of an inadequate (and therefore unproductive) view on the nature of physical theories, namely the view that spacetime and worldtubes are theoretical constructions that do not represent anything real.

If Minkowski's explanation of length contraction is carefully examined, understood, and adopted, it becomes clear that this relativistic effect would be impossible if the worldtube of the contracted body did not exist. Then, when the reality of the worldtube of an accelerating body is taken into account, one can easily link two facts; (i)

the body resists its acceleration, and (ii) the worldtube of the accelerating body is deformed (not geodesic). As a real four-dimensional worldtube should resist its (static) deformation (like any deformed three-dimensional rod) it is natural to ask whether inertia originates from the four-dimensional stress arising in the statically deformed worldtube.

Calculations (i) in terms of the four-dimensional stress tensor, (ii) in the case of the classical electron, and (iii) a semi-classical calculation in quantum field theory, all appear to show that the static restoring force “arising” in the deformed worldtube of an accelerating body has the form of the inertial force [8, Chap. 9]. In the case of a body at rest on the Earth’s surface its deformed worldtube gives rise to a static restoring force that is precisely $\mathbf{F} = m\mathbf{g}$. So taking into consideration the reality of worldtubes of macroscopic physical bodies implies that the equivalence of inertial and gravitational forces and masses may be explained by four-dimensional stresses arising in the deformed worldtubes of non-inertial bodies (i.e. in the worldtubes that are deviated from their geodesic shape). The worldtube of a body falling in a gravitational field is geodesic (*not* deformed) and the body does not resist its fall.

Minkowski would be thrilled to examine the possibility that inertia may be another manifestation of the four-dimensionality of the world, in addition to the kinematic relativistic effects, which he believed were such manifestations.

In fact, I think there were two other failures to initiate the assault on the origin of inertia in the twentieth century. Those missed opportunities had been caused, in my view, mostly due to underestimation of the role of conceptual analyses in physics. Since regarding inertia as originating from a four-dimensional stress that arises in the deformed worldtubes of non-inertial macroscopic bodies provides only a phenomenological picture of what causes a body to resist its acceleration, the next question will be to reveal what gives rise to the four-dimensional stress. As the three-dimensional stress in a deformed three-dimensional rod is caused by electromagnetic forces acting on the atoms of the deformed rod that are deviated from their equilibrium positions, it is natural to search for a similar mechanism in the case of the four-dimensional stress.

The first attempt to explain the origin of inertia, that could be regarded as a candidate for such a mechanism, started on its own long before Minkowski. In 1881 Thomson [9] first realized that a charged particle was more resistant to being accelerated than an otherwise identical neutral particle and conjectured that inertia can be reduced to electromagnetism. This conjecture had been developed in the framework of the classical electron theory into what is now known as the classical electromagnetic mass theory of the electron (for details see [10, 11, 12]). In this theory inertia is regarded as originating from the interaction of the electron with its own electromagnetic field.

The electromagnetic mass theory of inertia correctly predicts the experimental fact that at least part of the inertia and inertial mass of every charged particle is electromagnetic in origin. As Feynman put it: “There is definite experimental evidence of the existence of electromagnetic inertia – there is evidence that some of the mass of charged particles is electromagnetic in origin” [13]. And despite the fact that, at the beginning of the twentieth century, many physicists recognized “the tremendous importance which the concept of electromagnetic mass possesses for all of physics”

[14], it has been inexplicably abandoned after the advent of relativity and quantum mechanics. And that happened even though the classical electron theory predicted that the electromagnetic mass increases with increasing velocity *before* the theory of relativity, yielding the correct velocity dependence, and that the relation between energy and mass is $E = mc^2$ [13, pp. 28-3, 28-4] (in fact, the relationship between mass and energy contained a factor of 4/3, which prompted Feynman to write "It is therefore impossible to get all the mass to be electromagnetic in the way we hoped. It is not a legal theory if we have nothing but electrodynamics" [13, p. 28-4]; but the factor of 4/3 was accounted for [10]).

In my view, there had been no justification for abandoning the classical electron theory without a thorough analysis of how it made the predictions mentioned in the above paragraph and especially its major prediction of the electromagnetic mass of charged particles. A completely wrong theory cannot make so many correct predictions. Today "the state of the classical electron theory reminds one of a house under construction that was abandoned by its workmen upon receiving news of an approaching plague. The plague in this case, of course, was quantum theory. As a result, classical electron theory stands with many interesting unsolved or partially solved problems" [15, p. 213].

The other failure to start dealing with the open question of inertia is caused, I think, again by underestimation of the role of conceptual analyses in physics. According to the Standard Model the fundamental interactions are mediated by the exchange of virtual quanta. However, it appears a rigorous conceptual analysis of how exactly the exchange of virtual quanta gives rise to the electromagnetic forces, for example, has never been carried out. Having a detailed description of the mechanism of interaction between particles in the Standard Model would have made it possible to examine whether it can provide an explanation of the origin of inertia.

To see why one should expect that inertia might be explained by the Standard Model assume that the force between two charges can indeed be explained in terms of recoils of virtual quanta. Then it turns out that the same mechanism that accounts for inertia and mass of the classical electron – the interaction of the electron charge with its distorted field – appears to lead to contributions from all interactions to the inertia and mass in the framework of the Standard Model (which might explain why the electron theory correctly predicted the existence of electromagnetic inertia).

Consider a free particle, whose constituents are involved in electromagnetic, weak, and strong interactions. The recoils the particle suffers due to the emission and absorption of the virtual quanta of the electromagnetic, weak, and strong "fields" of its own constituents cancel out precisely. However, taking into account the general-relativistic frequency shift of the virtual quanta that are *absorbed* by a non-inertial elementary particle (accelerated or supported in a gravitational field) implies that the recoils imparted on the particle by the absorbed ("blue" and "red" shifted) virtual quanta do not cancel out. That imbalance in the recoils gives rise to a self-force which is electromagnetic, weak, and strong in origin and which acts on the non-inertial particle [8, Chap. 9]. As a result the particle resists its acceleration or its being prevented from falling in a gravitational field. In other words, the non-inertial particle resists its deviation from its geodesic path, which implies that the self-force manifest itself

as the inertial force. This effect may account for the inertia and mass of elementary particles involved in the interactions that the Standard Model deals with and might either complement the Higgs mechanism or provide an alternative mechanism.

1.2. *Worldtubes and anisotropic velocity of light in non-inertial reference frames.* The missed opportunity to have a complete understanding of propagation of light in non-inertial reference frames is another example of underestimation of the power of conceptual analysis and not taking seriously Minkowski's view that physical objects are worldtubes in spacetime. Try to recall how many books on relativity define *average anisotropic velocity of light* in accelerating frames or in a gravitational field. Of course, the Sagnac effect and the time delay (Shapiro) effect, for example, are nicely discussed but they have never been explained properly in terms of the average anisotropic velocity of light.

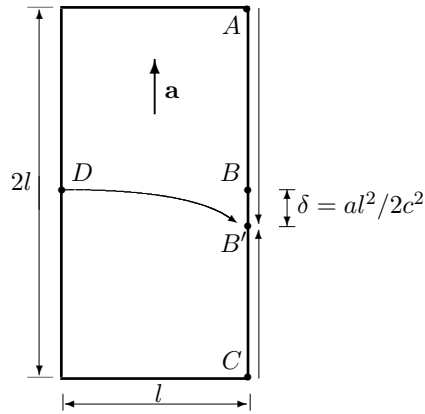


Figure 2: Propagation of light in an accelerating Einstein elevator.

Although deriving the expression for the average anisotropic velocity of light is straightforward [8, Chap. 7] realizing the need for such a velocity requires just a little trust in conceptual analyses – the type of analyses that made Galileo and Einstein, for instance, such great scientists.

Consider the famous example of Einstein's elevator – accelerating or at rest in a gravitational field. Fig. 2 depicts an accelerating elevator in which two light rays emitted from A and C toward the middle point of the elevator B are introduced (in addition to the original horizontal light ray) in order to demonstrate that the average velocity of light in an accelerating frame is not c (but the local velocity is, of course, c). The three light rays are emitted *simultaneously* in the elevator from points D , A , and C toward point B . Let I be an inertial reference frame instantaneously at rest with respect to the elevator (i.e., the instantaneously comoving frame) at the moment the light rays are emitted. As I and the non-inertial frame N of the elevator share a common instantaneous three-dimensional space and therefore common simultaneity at the moment the three light signals are emitted, the emission of the rays is simultaneous in N as well as in I . At the next moment an observer in I sees that the three light rays arrive simultaneously not at point B , but at B' , since for the time $t = l/c$ the light rays travel toward B , the elevator moves a distance $\delta = at^2/2 = al^2/2c^2$. As

the simultaneous arrival of the three rays at point B' as viewed in I is an absolute fact due to its being a *single* event, it follows that the rays arrive simultaneously at B' as seen in N as well. Since for the *same* coordinate time $t = l/c$ in N , the three light rays travel *different* distances $DB' \approx l$, $AB' = l + \delta$, and $CB' = l - \delta$, before arriving simultaneously at point B' , an observer in the elevator concludes that the propagation of light is affected by the elevator's acceleration. As seen in Fig. 2 an observer in N concludes (to within terms $\sim c^{-2}$) that the *average* velocities of the light rays propagating from A to B' and from C to B' are slightly greater and smaller than c , respectively.

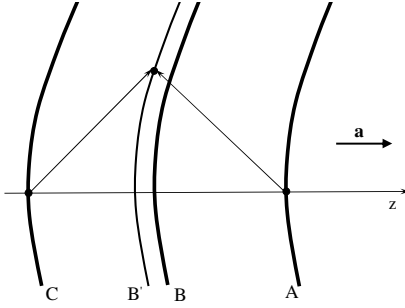


Figure 3: The simultaneous arrival of the vertical light rays at point B' in Fig. 2 is self-evident when it is taken into account that the points A , B , C , and B' are worldlines, which are *curved*.

One can arrive at the idea that the average velocity of light in non-inertial frames is not c also by taking into account that the light rays and the points A , B , C , and B' are in reality worldlines in spacetime. It is clearly seen in Fig. 3 that the light rays emitted from A and C arrive simultaneously at B' , not at B , due the fact that the worldlines of the points A , B , C , and B' are *curved*. When the curvature of the worldtubes of the emitter and the receiver in the gravitational redshift (Pound–Rebka) experiment is taken into account one can easily overcome a widely shared misconception – that the redshift effect proves the curvature of spacetime [16]. What it proves is that the worldtubes of the emitter and the receiver are curved (not geodesic).

Overlooking the introduction of an average velocity of light in non-inertial frames has led to another missed opportunity – a derivation of an anisotropic volume element, which is necessary for calculating electromagnetic phenomena in such frames. Using the anisotropic volume element in those calculations simultaneously solved two old problems – the appearance of (i) a factor of $1/2$ in Fermi's potential of a charge in a gravitational field, and (ii) a factor of $4/3$ in the expression of the self-force acting on the classical electron [8, Chap. 8].

1.3. Worldtubes and the discovery of general relativity. This section is specifically devoted to the young generation of physicists. It is sometimes claimed that future researchers cannot be taught how to make discoveries since discoveries are logical jumps coming as sudden insights to individual researchers. Often general relativity is given as an excellent example. Moreover, Einstein himself described the realization that a person falling from the roof of a house does not feel the force of gravity as the happiest

though in his life. Such insights do exist in the history of science but not all advances in science follow that pattern – it is sufficient to mention how conceptual analyses helped Galileo to arrive at the idea of inertial motion (by disproving Aristotle’s view of motion) and at his principle of relativity. Even Einstein himself convincingly proved that conceptual analyses are physics at its best by his thought experiments.

General relativity is indeed an excellent example but not of the view that discoveries are results of happy insights; it is an example of making discoveries by having a productive view on the nature of physical theories and by employing a rigorous conceptual analysis of the available theoretical and experimental evidence.

Minkowski’s representation of special relativity makes it possible to arrive at the idea of gravity as a manifestation of the non-Euclidean geometry of spacetime not just naturally, but inescapably. A conceptual analysis of Newton’s gravitational theory could and should have revealed, long before Einstein realized it, that a falling body offers no resistance to its acceleration. This means that the body is not subjected to any gravitational force, which would be necessary if the body resisted its fall. Therefore, the falling body moves non-resistantly, by inertia. But how could that be since it accelerates?

Taking seriously the existence of worldtubes and its implication that inertia originates from a four-dimensional stress that arises in the deformed worldtube of a non-inertial body provide a radical resolution – the worldtube of the falling body should be curved (reflecting the fact that the body accelerates), but not deformed (accounting for the fact that the body does not resist its acceleration). Such a worldtube can exist only in a curved spacetime where the geodesic worldtubes of bodies moving by inertia (non-resistantly) are *curved but not deformed*.

2. The major missed opportunity in quantum physics. No one knows what the quantum object is. What is worse is that the standard interpretation of quantum mechanics tells us that we cannot say or even ask anything about the quantum object between measurements. In this sense, I think, Einstein was right that quantum mechanics is essentially incomplete. It is unrealistic to assume that an electron, for example, does not exist between measurements. But if it exists, it is something and we should know what that something is.

Although relativity does not fully apply at the quantum level since its equations of motion manifestly fail to describe the behaviour of quantum objects, spacetime does seem to be the arena of the quantum world as well. Then the question is: “Are quantum objects also worldlines in spacetime?” It is well known that the answer is negative.

This can be demonstrated in the case of interference experiments performed with single electrons [17]. In such double–slit experiments accumulation of successive single electron hits on the screen builds up the interference pattern that demonstrates the wave behaviour of *single* electrons. When looking at the screen, every single electron is detected as a localized entity and the natural question is whether the electron was such an entity before it hit the screen. Our intuition leads us to assume that if the electron hits the screen as a localized entity, it is such an entity at *every* moment of time, which means that the electron exists *continuously* in time as a localized entity. But if this were the case, every single electron would behave as an ordinary particle

and should go only through one slit and no interference pattern would be observed on the screen. Therefore, the electron cannot be a localized entity at all moments of time, i.e. it cannot be a worldline in spacetime.

If this had been realized (and it could have been realized if more trust had been put in conceptual analyses), it could not have been so difficult to ask whether an electron exists *continuously* in time if it is not a worldline. Such a question could have provided a new perspective at looking at quantum phenomena and apparent paradoxes.

3. Possible missed opportunities to test string theory. I think string theory is perhaps the best example of overestimation of the predictive power of mathematics in physics. String theory constitutes an unprecedented case in the history of physics – while string theorists all admit that their theory has not yet been experimentally confirmed they behave as if this has already been done (or will certainly be done) and string theory has been treated on equal footing with the accepted physical theories.

A necessary condition that should be met by any newly proposed physical theory is not contradicting the existing reliable theoretical and experimental evidence. It is precisely here where I think opportunities to test string theory might have been missed. Let me give just one example. While string theory has extensively studied how the interactions in the hydrogen atom can be represented in terms of the string formalism, I wonder how string theory would answer a much simpler question – what should be the electron in the ground state of the hydrogen atom in order that the hydrogen atom does not possess a dipole moment in that state?

References

- [1] L. Smolin, *The Trouble With Physics: The Rise of String Theory, The Fall of a Science, and What Comes Next* (Houghton Mifflin Harcourt, Boston 2006)
- [2] N. David Mermin, *Physics Today*, May 2009, p 8
- [3] H. A. Lorentz, *The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat*, 2nd ed. (Dover, Mineola, New York 2003) p 321
- [4] H. Minkowski, “Raum und Zeit”, *Physikalische Zeitschrift* **10** (1909) pp 104–111; *Jahresbericht der Deutschen Mathematiker-Vereinigung* **18** (1909) pp 75–88
- [5] H. Poincaré, “Sur la dynamique de l’électron”, *Rendiconti del Circolo matematico Rendiconti del Circolo di Palermo* **21** (1906) pp 129–176
- [6] D.C. Dennett, *Darwin’s Dangerous Idea: Evolution and the Meanings of Life* (Simon and Schuster, New York 1996) p 21
- [7] H. Minkowski, “Space and Time.” In: H.A. Lorentz, A. Einstein, H. Minkowski, and H. Weyl, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity* (Dover, New York 1952) pp 75–91
- [8] V. Petkov, *Relativity and the Nature of Spacetime*, 2nd ed. (Springer, Heidelberg 2009)
- [9] J.J. Thomson, *Phil. Mag.* **11**, 229 (1881)
- [10] F. Rohrlich, *Classical Charged Particles* (Addison-Wesley, New York 1990) p 218
- [11] M. Jammer, *Concepts of Mass in Contemporary Physics and Philosophy* (Princeton University Press, Princeton, 2000) p 34
- [12] J.W. Butler, *Am. J. Phys.* **37**, 1258 (1969)
- [13] R.P. Feynman, R.B. Leighton and M. Sands, *The Feynman Lectures on Physics*, Vol. 2 (Addison-Wesley, New York 1964)
- [14] E. Fermi, *Rend. Acc. Lincei* (5) **31**, 184; 306 (1922)
- [15] P. Pearle, Classical Electron Models. In: *Electromagnetism: Paths to Research*, ed. by D. Teplitz (Plenum Press, New York 1982) pp 211–295
- [16] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation* (Freeman, San Francisco 1973) p 189
- [17] A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki, and H. Exawa, Demonstration of single-electron buildup of an interference pattern, *American Journal of Physics* **57** (1989) pp 117-120