PHYSICS LIVES IN FORM HEAVEN

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Abstract. Many mathematicians are Platonists in the sense of believing that their major concepts and theorems are discovered rather than invented. It is claimed here that the initial foundational source of those apparently spaceless and timeless mathematical ideas is the invariant Vacuum of physical space-time. This constitutes a non-classical yet "real" Form Heaven for fundamental physics and is a storehouse for all the knowledge of the physical constants, laws, and particles of physics. The intricate structure of the Vacuum is common to all intelligences in our universe and helps to constrain the reality of their emergent knowledge. A reductionist view begins with the basic set of quantum fields living in the Vacuum leading to more complex forms emerging from these fundamentals (protons, nuclei, atoms, molecules). These entities are quantum, and their nature along with the fundamental fields might be said to live in an unusual "square-root of reality." Mathematics applies logic, intelligence and abstraction to world patterns and then generalizes at will forming abstractions of abstractions. But the field of mathematical-physics continually cross-fertilizes math and physics modestly limiting their divergence.

1. Introduction

Focus for a moment on a simple question, "Does pi (π = C/D) exist before we discover it? And if it does, where does it exist?" Historically, our knowledge of pi comes from performing measurements in our various environments and doing practical calculations. We use pi when we deal with circles, circumferences, and areas or volumes of spheres or cylinders. And then, later on, after much development, pi can also appear from a multitude of other activities such as the summing of series. The classical physical world hints at pi in many ways: Nature has spherical planets and stars, planetary orbits, periodic vibrations, spherical droplets of water, and progressing phases of waves as examples. No matter where they are, intelligent creatures trying to understand the universe will find pi useful and intriguing. Nature doesn't explicitly or overtly know pi, but Nature can be codified by rational communities trying to understand the patterns of Nature.

The Greek philosopher Plato (~ 424 - 348 BCE) stressed the importance of relatively spaceless and timeless abstract ideas relating to numbers, geometry, nature and ethics and why these concepts seemed to be universal over the world then known to the Greeks. The ability of different people to independently re-discover or "instantiate" some of these apparently pre-existing ideas was believed to be due to "remembering" them from a time before birth when souls had contact with ideas in Plato's heaven. We now tend to view this idea as silly

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mysticism except for the nagging awareness of its general popularity across more than two millennia and its widespread popularity among respected mathematicians up to the present day. Contributing to their belief may be the desire to think that mathematical progress is indeed discovery rather than invention. We wish here to update and apply Plato's idea to modern physics and mathematics and make the "problem of universals" into "the universe."

The set of all invariant abstract ideas could be called Plato's "Form Heaven." In our modern era, the important concepts or forms of mathematics and physics have complexity and invariance that goes far beyond anything Plato could have imagined. The original intent of his Forms was that they be abstract properties, that they transcend particular instantiations, that they be pure and "perfect models" or causal templates, that there is some sense in which they are objectively "real," and that they have some sort of a hierarchy of connectedness down to instantiated objects [1]. They should not have particular places or times of existence, they are beyond localization in space or in time. The basic mathematical forms of Plato's day are still important to us today: numbers, perfect circles, spheres, geometric "platonic" solids, lines, triangles and the Pythagorean theorem. But we have now gone well beyond that. The ranking or value of a Form should reflect its degree of invariance and how generalizable it is from a fundamental reductionist sense. We may no longer consider examples like people, dogs, the color green, hair, wood, or air as being invariant enough to call Forms because some of these may be restricted too narrowly to Earth and its biology and culture. We want much wider invariance than that; and, in the following, we wish to broaden the concept of the invariance of Forms from the ancient Greek world up to the presently known universe within our particle horizon. Although concepts like reincarnation and the soul were popular in Greek culture, we tend to avoid them now − and, from our latest knowledge of cosmology, we would place some new limits on words like eternal or immutable. Plato's insistence that Form Heaven is not in space-time might also be loosened because modern physics now might actually identify it with the structures of the space-time Vacuum.

A standard objection to Platonism is asking the obvious question, where is this world of Forms? Ignoring Plato's answer, there are several possibilities. One is that it emerges and lives in the minds and culture and literature of a world community of very smart, inquisitive, international, rational, abstract thinking people (such as the mathematicians of our planet). So, mathematical Form Heaven results from "shining the light of intelligence" onto a given habitat. We may also ask the following question: Suppose there is a set of intelligent, independent, technological, alien civilizations scattered throughout our universe. Would we expect them to eventually evolve a mathematics structure and set of theorems approximately isomorphic to our own? (focusing on the most important theorems out of millions). And would their physicists eventually come up with something isomorphic to our standard models of cosmology and particle-physics? We feel slightly braver in posing such a question now that we have actually discovered nearly 2000 exoplanets [2], think that the total number of planets in our universe is extremely large (e.g., 10^{24}), and have slightly greater comfort in the Drake equation These may be standard questions for SETI

(Search for Extra-terrestrial Intelligence, for "ETI"). Our knowledge is still weak, so this may be just a thought-question ("Gedanken"). But, we would probably all find it easy to believe that the equivalent of numbers like 2, 3, $\frac{1}{2}$, π, $\sqrt{2}$ and e would exist in all ETIs – they are just too useful and important to bypass. (And, for physics, we can imagine that knowledge of universal constant values for c, h, G, q_e, k_B, m_e, m_p, N_o, a_o and α_{EM} should eventually appear.) If we can agree this far, how difficult is it to further imagine that ideas like the Euler equation $e^{i\pi} = -1$ or the Pythagorean theorem $a^2 + b^2 = c^2$ will also be universal? And from the Pythagorean theorem, we can generalize metrics to $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$.

2. Mathematics

A common definition of mathematics is "an abstract representational system used in the study of numbers, shapes, structure, change and the relationships between these concepts [3]." It is an interdisciplinary language that has reason built into it; and this purity of reasoning is why Plato valued mathematics so highly. A favorite definition is from Paul Halmos, "Mathematics is the logical dovetailing of a carefully selected sparse set of assumptions, with their surprising conclusions, via a conceptually elegant proof. Simplicity, intricacy, and above all, logical analysis are the hallmark of mathematics [4]."

It is said that, "Mathematical platonism enjoys widespread support and is frequently considered the default metaphysical position with respect to mathematics" [5]. Consider the emphasis on the phrase "there exists" (e.g., an infinite number of prime numbers, $\exists\ \mathbb{P} \subset \mathbb{N}$, $\#\{\mathbb{P}\} = \aleph_o$, $\exists\, p \in \mathbb{P}$) . We take the "existence" of these objects seriously. Can we also imagine that basic proofs for an infinite set of prime numbers might pre-exist in the book of the universe [6]? (e.g., Euclid's (∼ 300 BCE) finite set of the first r primes followed by a new number $n = p_1 p_2 ... p_r + 1$ which may have an additional new prime divisor).

In his book, The Road to Reality, about mathematics and the laws of the universe, Roger Penrose says, "Platonic existence, as I see it, refers to the existence of an objective external standard that is not dependent upon our individual opinions nor upon our particular culture [7]." He devotes a whole section on whether the "Platonic world of mathematical forms" is real and decides that it is in the sense of the "objectivity of mathematical truth." Kurt Gödel also believed in the objectivity of mathematics, that an abstract realm existed, and that the only valid philosophy of mathematics was Platonism. He was a theist who believed that intellectual mathematical intuition is a kind of sense that enables us to perceive Platonic concepts which are really "out there" [8]. And Paul Erdős believed in the pre-existence of a transfinite Book that contains the most elegant and perfect proofs of all mathematical theorems [6].

Pure mathematicians would say that mathematics is pure math, although this emphasis only dates back to about 1800. Mathematical knowledge is only concerned with the realm of thought. It attempts to not consider direct application; but, mysteriously, the purity of one era sometimes becomes the application of a following era (e.g., number theory was once pure but now also applies to computer encryption). From the time of Karl Weierstrass, we focus on mathematical analysis and rigorous proof from axioms. Jean Dieudonné (and the highly abstract and rigorous French Bourbaki school) stated in 1962 that mathematical progress has almost nothing to do with physical applications [9]. But, twenty years later, modern physics again entered the picture. Michael Atiyah said that since about 1980, "some of the most exciting developments in mathematics have arisen from the interface with physics and particularly quantum field theory" ("QFT") [19]. The Fields Medal is sometimes called the Nobel Prize of mathematics. So far, Edward Witten is the only practicing physicist who has won this award (in 1990 for his 1981 proof of the positive energy theorem of general relativity). But eight other winners did work partly related to physics (delta functions, quantum groups, PDE's, Ising model, Boltzmann equation, renormalization, Brownian motion, and general relativity). Peter Woit says "Mathematics is a science, but it is not an empirical science. It insists on precise thought, rigor, clarity, high standards of proof and debate among an international community. New mathematics is motivated by numbers and geometry and also by theoretical physics" [10] (e.g., quantum field theory and string theory).

3. Physics

"The goal of physics is to study entities of the natural world, existing independently from any particular observer's perception, and obeying universal and intelligible rules [11]." We are aware that our physical laws, particles, and the constants of Nature are universal and invariant over space and time. So, it has always been clear that physical Forms are discovered rather than created by people. We know this largely due to the spectra of electromagnetic radiation detected from very distant sources and from the success of the standard model culminating in the discovery of the Higgs particle. And our ΛCDM concordance model of cosmology is now pretty solid largely due to the study of cosmic microwave black body radiation (CMB, e.g., via the Planck mission) [12].

Formerly, physical entities were said to 'exist' because they had mass. After Einstein, we might say they exist because they have energy equivalence (e.g., $E = mc^2$ and $E = h\nu$). We believe that photons and electric fields exist because they can deliver energy even though their mass is zero. A present concern is whether information also has any real existence. A problem in physics is that we presently seem to have two worlds: the classical world (largely composed of particles created long ago) and the "quantum world" (currently either coming into being or the not yet energized forms of the Vacuum). Plato's Forms were originally conceived as beginning in abstractions from the classical world where we now speak of Newtonian mechanics and gravitation applied largely to macro-bodies. The quantum world should perhaps really be called a "pre-quantum" world because its equations stop short of the actual transfer of quanta. It is a strange non-classical existence possibly described as a sort of "square-root of reality" discussed more below. We refer to this world as "real," but that is a horribly overused word that should better be called quantum-real or "qureal" instead to separate it from a usually understood bias of being classical.

Apart from solid-state physics (condensed matter), physicists generally have a reductionist perspective: work towards the bottom and then build up from there. Elementary particles and fields represent the present bottom rung of this ladder. Our past history biases us towards visualizing classical particles when thinking about 'particle physics.' But fundamental particle physics is now discussed in books on 'quantum field theory" (QFT). The belief in "a pure fields view" has developed during the past three decades. "At the high energy end, most quantum field theorists agree for good reasons that relativistic quantum physics is about fields and that electrons, photons, and so forth are epiphenomena, namely excitations (waves) in the fundamental universal fields" [15]. Quantum fields exist in space-time; but we need to talk about the nature of that existence. The reason that all electrons are the same ($N_e \sim 10^{80}$) is that they are all excitations of the same pervasive electron field. A general view is that a quantum field is an entity existing at each point of space which regulates the creation and annihilation of particles − one field for each type of particle. QFT treats fields as the knowledge embedded in the Vacuum of how to make any particle providing that adequate energy and quantum numbers are available to do so [16]. Some say that even in usual QM there is really 'no evidence for particles' [14] [15].

Frank Wilczek noted that a new term was needed which is broader and more relevant to physics and QFT than the old ideas of aether, plenum, substance, vacuum, space-time, or world-stuff [17]. He uses the word **'Grid'** as a "multilayered, multicolored cosmic superconductor" including quantum fluctuations, a superconducting condensate, a weak superconducting Higgs condensate, Einstein's metric field ($g_{\mu\nu}$), the dark energy cosmological constant grid density (Λ), and "chiral symmetry-breaking condensate consisting of quark-antiquark pairs." It is recognized that general relativity is really an "ethereal theory of gravitation." Grid superconductivity gives masses to particles created by weak bosons, and particles are relatively localized disturbances in the Grid. Some might add that the smoothly distributed cosmic black-body background (CMB) is also a modern version of an aether with a locally preferred frame corresponding to the expanding cosmic fluid. Wilczek's picture is further encouraged by the experimental finding of the 125 GeV resonance appearing to be the standard model Higgs boson (CERN-LHC, 4-July, 2012).

As a recent example of the hidden "causal templates" of the space-time Vacuum, consider particle-antiparticle colliders producing what might be called "pure energy" which in turn can then lead to myriad possible output particles of precisely defined types apparently emerging out of the Vacuum itself. Since the earth rotates and orbits, the real historical set of collision points of these colliders have been sweeping out corkscrew paths covering large samples of space and over a long time implying that this production is spaceless and timeless. There is a beautiful plot released by CERN LHC showing quark-mesons produced by the Vacuum as seen by an increasing total mass of di-muons, $\mu^+\mu^-$, "A Lovely Dimuon Mass Spectrum" [18] . The spectra of events per GeV begins with lower energy at left showing spikes in cross-section for production of mesons called η, ρ, ω for $u\bar{u}, d\bar{d}$.

Then there are the unflavored quarkonia $q\bar{q}$ mesons: the ϕ meson for strangeness $s\bar{s}$, and then charmonium J/ψ for $c\bar{c}$ followed by Υ or $b\bar{b}$ and its excited states. Finally there is a huge spike for the neutral weak Z^o boson near 92 GeV. These particles are spewed forth when the Vacuum is stimulated. From a separate reference, a very similar plot of particle production cross-section also results from electron-positron e^+e^- collisions [13] additionally showing a high energy hump for W^+W^- production. We would believe that pumping energy into any point in the universe would enable the production of these same particles and deduce that the Vacuum of space-time holds the pre-existing knowledge of all these particles and more.

Plato would also not have anticipated the world of identical particles. All of the universe's muons are the same, each of its protons is the same, each of its ground state gold atoms is the same (isotope with neutron number say at 118). When we instantiate a physical form, it is not an impure poor-copy; it is as pure as the abstract forms themselves. And, experiments also show that these objects are quantum too − at least up to macro-molecule size like C_{60} carbon buckyballs. They are 'de-localized' entities. An experiment in 2013 [25] demonstrated nano-particle de Broglie matter-wave interference of macromolecules above 10,000 amu! It is not presently clear where a dividing line may be between these quantum objects and so-called classical objects. Could it be that the physical world is all-quantum? Or might an upper mass limit be near the "Planck mass" ($\sim \mu$ gram)?

4. Quantum Mechanics and "The Square Root of Reality":

Quantum mechanics provided us a strange new world where "reality" became hard to define and complex numbers became a necessity. After eighty years, there is still an intense on-going debate about the nature of the quantum state, ψ (is it "ontological" or "epistemological" or perhaps some blend of both?). As an interesting example, suppose that a minority view called the "Transactional Interpretation" has some validity [21]. In this TI world, ψ is an "offer wave" from an emitter to possible absorbers. A confirming wave ψ^* goes back in time from an absorber to the emitter resulting in a handshaking "transaction" with weight $\psi^*\psi$ which provides an explanation of the Born Rule. In this picture, the reality of a quantum state or wave function ψ is something like the "sound of one hand clapping." That is a very unusual kind of "reality," and the Form Heaven of Physics has a reality similar to this.

Take the Born rule seriously as having sub-quantum-real ('qureal') wavefunctions needing to be 'squared' to become classical candidate entities. Classically recognizable probability may be given by $P = \psi^*\psi$, where psi lives in a new sub-world resembling the pulling apart of classical reality into two "square-root" (or 'star-root') complex number parts. So, electron spin as classically real or vector-like fails to agree with observation, but quaternions or gamma matrices fit needs better. Discussions of the Born rule go from wavefunction to detection probability with a selection criterion that is unspecified and likely random. Here we wish conceptually and heuristically to go backwards, from classical to sub-quantum.

As a first example, one occasionally used representation of a single photon (the 'Riemann-Silberstein' form) is found by taking the "square-root" (or "star-root") of its supposed energy density: $\psi^*\psi \propto (\epsilon_o/2)(E^2 + c^2B^2)$ becomes $\psi = \sqrt{\epsilon_o/2}\,(E \pm icB)$ [27]. The 'star root' operation is of course not unique and not well-defined, it is intended to only be heuristic: star-root ${}^*\sqrt{Prob} = P^{*/2} = \psi$. In a similar vein, the 'Dirac program' essentially derives from taking the square root of the d'Alembertian [20]; or we can consider the Dirac equation as the 'star root' of the Klein-Gordon equation: $Dirac = (KG)^{*/2}$, i.e.,

$$(1) \qquad \left[\partial^\mu \partial_\mu + \left(\frac{mc}{\hbar}\right)^2\right]\psi = 0 \qquad \xrightarrow{*/2} \qquad i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0$$

where the γ^μ 's are the 4×4 unitary Dirac matrices. Instead of just complex conjugation (or *'starring'*), higher dimension quantum spaces can use matrices with conjugate transpose or 'Hermitian Adjoint,' $A^\dagger$. The 'star-root' idea can go further, for example into the realm of quantum cosmology with supersymmetry where it is said that supergravity($N = 1$ SUGRA) naturally provides a Dirac-like 'square-root' of gravity [23]. And some programs for unifying general relativity and quantum mechanics use "tetrads" which can be thought of as the square-root of the metric, $g_{\mu\nu}$.

Hypercomplex numbers can have convenient application in the classical world with examples including quaternions ($\mathbb{H}$, $basis = \{1,\ i, j, k\}$) for 3D-rotation, electromagnetism, and relativity. And their use eventually led to the development of more conventional vector analysis which was easier to use. But the use of hypercomplex numbers becomes a necessity in the quantum world. The algebra of hypercomplex quaternions and Dirac matrices are examples of Clifford algebras (e.g., $C_o = \mathbb{R},\ C_1 = \mathbb{C},\ C_2 = \mathbb{H},\ C_3 = Pauli,\ C_4 = \gamma's$) .

Similarly, the physics of fields usually begins with a Lagrangian written in terms of energies and interactions. These in turn contain "squares" of fields such as the free gauge field of electromagnetism and the gauge part of the weak action [24]:

$$(2) \qquad \mathcal{L}_{EM} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} = \frac{(E^2 - B^2)}{2}, \qquad \mathcal{L}_{weak-W} = -\frac{W^a_{\mu\nu}W^{a\mu\nu}}{4}$$

Again, what is E_i or W^a_ν all by itself? Well, the fermion-boson interactions mean something: $\mathcal{L}_{int} = -J^\mu A_\mu$ [e.g., like the eA part of the Aharonov-Bohm (momentum phase change) effect; but unless it is electromagnetic frame dragging, it is hard to interpret in words].

5. Conclusion

We can now identify Plato's Form Heaven for fundamental physics with something more tangible than previous 'spirit or mental worlds.' However, these Forms of physics are objectively real in space-time in a strange way. They are the information and potential to create elementary particles; but, without energy, they are not the particles themselves. The knowledge of this world can be plucked out by hitting the Vacuum with pulses of energy. Unlike Plato, the resulting instantiations of each type of Form are themselves identical and pure as are the contents of the Vacuum itself. But, until detected, these instantiations are quantum objects lacking classical reality. The objects and the Forms have a new type of existence similar to the "square root of reality." Quantumness is preserved up to the size of macro-molecules. Beyond that, it is undecided if larger objects are classical or not ("macrorealism").

Many mathematicians generally believe that their basic theorems and concepts are pre-existing Platonic Forms which are discovered rather than invented. To some degree, mathematics is abstracted from physical reality because the regularity, repeatedness and symmetry of Nature is fruitfully expressible in the language of mathematics. There are regions of overlap between math and physics, and this overlap region of mathematical-physics has to be compatible with both. Lack of compatibility can lead to a modification of one side or the other. Historically, physics and pure mathematics are relatively free to diverge and grow apart. But then, unexpectedly, the evolving physics finds that some previously pure math can be usefully applied. And the math finds that new physics has some aspects that deserve to be better explored mathematically (and such development can be better funded). They cross-fertilize each-other. They find that they are not entirely separated but can play together.

What motivates mathematicians and physicists to devote their lives essentially to the study of these Forms? Transcendence and connectedness. We sense that we are participating in a huge world beyond our own limited experiences. We sense that the intelligences in the universe might discover the same truths we value; so we have a cosmic sharing. Without overtly expressing it, the physicist senses Einstein's "Cosmic Religious Feeling" [22] which can be essentially summarized as rational "Deep Nature Appreciation."

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