LOST IN MATH... AND MEASUREMENTS

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Based on physical sensations the human brain manufactures mental structures leading to a theoretical framework that is used to make sense of life and external reality. Such framework turns out to be our physical understanding. To quantify and model physical systems humans have also invented both mathematics and experimental instruments. The physics developed in the last decades has trusted its validity on these two aspects above our physical understanding. I argue that undecidability and unpredictability might not be relevant for physics. Math and measurement are not the sole players in apprehending natural reality. Physical understanding should play a prominent role in physics if we wish to make headway.

I. INTRODUCTION

A. Physical understanding vs mathematical understanding

Our physical understanding of nature is strongly rooted on sensory perception. Our senses provide us with information of different nature such as on shape, color, light intensity, motion, space, weight, sound, temperature, force, taste, smell, and so on. From these stimuli, or raw data, our nervous system generates physical representations of reality that produce the sensation of an outside and 'physical' world. As our brain develops, from the early stages of growth of a human being, it gives rise to a 'theoretical framework' aiming at making sense of life and reality. This framework is confronted every single day with experiences and is continuously refined as new information enters our brain. It is useful to tell false from real, truth from lie, and reality from fiction.

Thus humans inherently associate physical reality with what our senses detect. Our senses work as transducers transforming signals of different kinds to electrochemical signals. And just as a television transforms electromagnetic waves into images and sounds our brain transforms those incoming signals into physical representations: perhaps a smell, an image or a sound pattern. Once these constructions are stored and classified in our memory according to our theoretical framework, the next step is to process, compute or manipulate this information and release an output. To exteriorize our thoughts and sensations our brain decodes the output into complex electrochemical signals that are transported to the rest of the body by the peripheral nervous system through efferent neurons. Eventually, these signals are converted into mechanical motion by our muscles or vocal strings; for instance when we speak. Language can be understood as a system of patterns with specific rules (grammar) aiming at expressing thoughts, ideas, and sensations in different ways.

For many centuries we humans have helped ourselves with the use of abstract constructions that have been quite useful to model and quantify our physical perceptions. For instance, a circle or sphere, may symbolize the shape of the sun, a differential equation represents change, and numbers are used to describe quantities. As such, many of the mathematical properties of these structures are inherited to the 'physical' structures. Mathematical structures are founded on several aspects such as quantity, logic, shape, motion, topology, symmetry, etc.; aspects that many times have a physical counterpart. For this reason mathematics can be used by our brain as an alternative way of representing and codifying our understanding of reality. For instance, m can symbolize the matter content of an body and is mathematically represented as a real number. So, from this perspective, one can say that the work of a mathematician is then to invent mathematical structures based on rules of logic, some definitions, and a series of axioms. Physicists in turn, use these constructions as raw material to build models of reality.

B. Measurement

The task of a physicist is then to create a link between the physical world and the mathematical one. Using math physicists seek to represent the physical world in abstract constructions. Since mathematics can be used to compute physical quantities, it is necessary to measure them. Centuries ago, however, humanity realized that our senses are quite limited to test the world. To extend the scope of our senses we have developed theories, instruments, and experimental methods that not only survey areas of the physical world that our senses cannot reach but also quantify natural phenomena. In this respect, Lord Kelvin once expressed:

If you can not measure it, you can not improve it. I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind.

Measurement consists in defining a physical magnitude as unit that is used as standard for comparison with another magnitude of the same nature. Quantification is necessary to gain scientific knowledge following Lord Kelvin's quote. When we measure a magnitude we obtain a number that is multiplied by the unit of measurement. The information contained in a measurement is interpreted in a theoretical framework, for even the instrument of measurement and the physical magnitude in question are also defined within the theoretical framework. For instance, the unit of length is the metre which is defined as the distance travelled by a ray of light in 1/299 972 458 s. This definition in turn implies the equation of speed, i.e., $v = ds/dt$ which is part of the theoretical framework where space (s) and time (t) are understood as fundamental physical quantities. So, if data are explained by the framework, we have not gained new knowledge, otherwise we would be forced to either modify our framework or create a new one that explains the results of our observations. So a measuring instrument can be seen as an extension of our senses and as a method of quantifying a particular physical magnitude. According to the International System of units, there are seven fundamental physical magnitudes, namely: space (metre), time (second), mass (kilogram), electrical current (ampere), absolute temperature (kelvin), quantity of substance (mole), and intensity of luminosity (candela). From these we can express any other physical magnitude. Each of these magnitudes have an operational definition that also depends on a theoretical framework.

C. The problem

The way physical theories have been constructed is an admixture of both physical and mathematical understanding. In the XVI and XVII centuries, physical theories had little mathematics and a large quantity of physical content but gradually this started changing in the XVIII and XIX centuries. As time went by, the amount of mathematics started to rival the physical understanding. In the last two centuries, physics became mainly expressed in mathematical language to such degree that traditional physicists (Einstein et al.) had trouble adhering to the highly abstract level of doing physics. Max Tegmark is today one of the champions in favor of eliminating all the physical understanding (what he calls baggage) from theories (4). He has advocated the fundamental view that physical reality is in fact a mathematical structure. We should not be surprised of this for, as we have discussed above, mathematics is just another approach our brain uses to understand reality. Nevertheless, in my view, physical understanding can still make considerable contributions to physics. The history of physics has shown that sometimes mathematical structures are limited for they cannot see what physical understanding can. It has been argued that it is impossible to find a physical parallel to mathematical objects. This may be right, but the opposite can also be true; not all physical representations have a mathematical counterpart. For several centuries we have witnessed how many branches of mathematics have not spawned from pure mathematical arguments but from physical observations. Just to mention an example: The observation of solitary waves gave rise to the theory of solitons that we used every day in several fields of physics. So both fields have benefited mutually.

A couple of years ago, Sabine Hossenfelder published a book (1) where she expresses her thoughts on how physicists have given too much importance to mathematical beauty and symmetry for the construction of physical theories to such degree that these principles seem to have digressed physicists from the 'right' path. Earlier in 2006, Peter Woit and Lee Smolin published their opinions on this matter, taking string 'theory' as case for argument of two aspects: First, as case for exemplifying how mathematical beauty has been used as a pillar for the erection of modern physical theories; and, second, as a case for expressing publicly a deep concern on the deep crisis in theoretical physics arguing that string theory, despite its mathematical beauty, makes no testable predictions, snatching legions of smart young talents to a theory of apparently nothing (2; 3). Smolin and others wonder why no other Einstein has appeared (11). Perhaps this crisis was caused in view of the fact that physics has given to much prominence to mathematical representation. To my knowledge, the crisis is not merely new but in reality dates back, at least, to the 1950s.

Having said that, I would like to add that physical understanding can still be of great value for attaining new breakthroughs not only in physics but also in science. From the history of physics we can retrieve several stories that exemplify how physical understanding has been successfully used to obtain outstanding progress in the development not only of new physical theories but also of new fields of mathematics. So, despite that Gödel and Turing have shown that mathematics have limitations for computability, predictability, and decidability, such limitations not necessarily apply to physical understanding.

II. PHYSICAL UNDERSTANDING ON TOP OF MATH

I shall bring up some instances just to set the case about two aspects of physical understanding: First to substantiate that physical understanding is in many cases put on top of mathematical understanding; and, second, to show that many breakthroughs in physics have not been guided by mathematical elegance or symmetry, and so far, are not subject to Gödel and Turing's theorems. With this I do not wish to underestimate the prediction power of mathematical modelling and I do not want to replace mathematical theories with physical interpretations rather I argue that both approaches are not exclusive but complementary since both are employed to understand, in one way or another, reality. I agree that a theory must be written in mathematical terms but I also support the view that we have to work out, in parallel to the mathematical understanding, a physical picture of nature.

A. Radius of a quantum dot

There is an academic problem in nanotechnology that deals with the radius of a quantum dot whose band gap is found to be:

$$E_{g,QD} = E_{g,Bulk} + \frac{C}{r^2} + \frac{B}{r} + D,$$

where r is the radius of the quantum dot, $E_{g,Bulk}$ the band gap in bulk, and B, C, D are constants. The solution can be found by applying the expression:

$$r = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}.$$

After replacing the values, it turns out that there are two solutions given for the $\pm$ signs. Usually, if we take the minus sign, the radius turns out to be negative. Since quantum dots are material objects, they are extended objects of finite size, therefore one rules out negative radius as physically meaningless despite that mathematically speaking that solution is also legitimate. From here we see that physics is not algebra though physics is written in mathematical terms following mathematical rules. The true physical knowledge is that material objects are extended objects and therefore a negative radius is considered as a fiction. If we just followed the math we would have thought that negative radius were also possible in nature, but thanks to our physical understanding we avoid falling into mathematical fallacies. Indeed, the opposite can also occur, that is why mathematical and physical understanding are complementary.

B. Wavenumber of an electron

A similar lesson can be drawn from the problem of an electron confined in a one-dimensional box of side a. According to quantum mechanics, electrons can be represented as plane waves and the proposed solution for this problem is

$$\psi(x) = Ae^{ikx} + Be^{-ikx}.$$

Here k is the wavenumber and A and B are constants representing the electron wave amplitude which can be determined by the boundary conditions: $\psi(x \leq 0) = \psi(x \geq a) = 0$. Applying these conditions we get:

$$\psi(0) = A + B = 0; \qquad \psi(a) = A2i\sin(ka) = 0.$$

The second expression is satisfied for $ka = n\pi$ with $n = 0, 1, 2, 3 \ldots$. However, the case $n = 0$ is neglected based on physical grounds; because $k = 0$ would imply that there is no wave associated with the electron in contradiction to our initial assumption that there is an electron in the box. Although mathematically speaking this possibility is consistent, it is nevertheless physically meaningless. Thus, we are again resorting to physical understanding to make sense of mathematical results. Once again, the option $n = 0$ can be seen as a mathematical fallacy.

C. Louis De Broglie on matter waves

Quantum mechanics differs from classical mechanics in several aspects. First of all, classical physics assumes that the measurement process does not affect the pristine state of a system under study. Another aspect is that energy in classical physics is considered continuos which in quantum mechanics is commonly found to be quantized. One of the most significant realizations of the founders of quantum mechanics is that light exhibits two physical facets. Light in some cases, such as in the interference effect, behaves as wave; and in some others, such as in the photoelectric effect, as particle. Louis De Broglie noticed that both aspects were apparently inherent to light. By a deep physical reflection he came up with a marvellous idea. In his book he expressed his thoughts on this matter as follows (7):

But if for a century we have neglected too much the corpuscular aspect in the theory of light in our exclusive attachment to waves, have we not erred in the opposite direction in the theory of matter? Have we not wrongly neglected the point of view of waves and thought only of corpuscles? These are the questions the author of this book set himself some years ago in reflecting upon the analogy between the principle of least action and the principle of Fermat and upon the meaning of the mysterious quantum conditions introduced into intra-atomic dynamics by Planck, Bohr, Wilson and Sommerfeld.

By reasoning which will be studied in this volume, we may arrive at the conviction that it is necessary to introduce waves into the theory of matter and to do it in the following way....

By this simple reasoning and without resorting to mathematical symmetry (as opposed to Dirac discovery of antimatter) but to physical symmetry, De Broglie was able to develop his famous relation: $p = h/\lambda$. This example teaches us that physical understanding can some times see where math can not.

III. LOST IN MATH AND MEASUREMENTS

A. Physics is about truth, isn't it?

Physics as a science has been considered as a factual science. Despite that physics is publicized in textbooks and mass media as a single entity there are some internal divisions that sometimes struggle with each other. Physics is divided in three large areas that try to work together for the progress of such entity. These areas are theoretical physics, experimental physics, and computational physics. Theoreticians usually do not visit labs; they spend most of their time studying advanced mathematics and physical theories with the aim of developing models to fit experimental data. Experimental physicists, on the other side, rarely develop a theory. They are quite busy designing experiments, conducting measurements, and commissioning new instruments to test nature. Experimentalist take the models developed by theoreticians and apply them to interpret their observations. Computational physicists are in the middle of both, they apply theories and develop computational models to test theories without the need of carrying out experiments. At the end, the last word on whether a theory is scientifically valid, is given by experimental outcomes; but these alone are meaningless. Measurements only give raw data, just in the same way that our senses detect information from the outside world. The meaning of this information is granted by the theoretical framework. The continuous feedback between theory and observation is what has been given physics its current status as one of the most prominent sciences in human history.

Experimentalists are usually guided by theories in their endeavour to test reality. Theoreticians on the contrary, seem to be happy developing heavily abstract theories without worrying much on the physical interpretation of their theories. It is generally thought that the physical interpretation is irrelevant for what matters is that theory simply fits measurements. A quote from Richard Feynman expresses this feeling quite well:

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.

Most theoreticians are influenced by a way of doing physics embedded in a mote: 'shut up and calculate'. The 'shut up' means 'stop talking/philosophizing' and the 'calculate' means 'put your words in a mathematical model, or compute'. This precept usually derides physical understanding as mere philosophy. Feynman also expressed that nobody understand quantum mechanics. What he really meant is that nobody has a physical understanding of QM, however there is a mathematical-experimental understanding of QM. In my view, many have not realized that mathematical and physical understandings are two ways of approaching the truth. Science is about true knowledge, that is why we can use it to tell real from fiction. For if science is not about truth, then scientific activity becomes meaningless and in that case I should not be writing this essay. Thus, we should use any tool our brain has to understand and seek for true knowledge, whether it is mathematical or physical.

B. Absolutism vs Relativism

I would like to add a discussion on an important topic where physical understanding sees something that math doesn't. With this I hope to set the case that, mathematical undecidability and unpredictability are not key factors to greatly impact the progress of physics provided that we grant our physical understanding a higher rank in the way of doing physics.

1. Rotational motion

It is well known that general relativity is not a fully relativistic theory. The adjective 'general' was Einstein's adjective to generalize special relativity to all frames of reference, including accelerated frames (8). Julian Barbour and other specialists have recognized this 'weakness'; as such they have spent most of their lives, without success, trying to develop a fully relativistic theory that embodies the essence of Mach's principle (9; 10). In an attempt to explain the famous bucket experiment, devised by Newton; Mach argued (paraphrasing) that the water in the bucket would raise if the stars rotate creating the effect of a centrifugal force. For Mach, all motion was relative and he denied the existence of absolute space on the grounds that this entity was inaccessible to experience. Largely influenced by Mach, Einstein developed his theory in an effort to eliminate any trace of the absolute frame. Did he succeeded? In practice yes, but in theory no.

Thus, according to relativists, the judgement of an event depends on the frame of reference from where the event is judged. In such conception, the statement that (1) the Earth rotates and the stars are static is as **true** as the opposite argument; that (2) the Earth is static and the stars rotate –this is pure relativity. From the standpoint of relativity both asseverations are true provided that one takes into consideration the reference system. On the contrary, for a Newtonian, asserting both statements as true is **naive**. Absolutism accepts relative motion but also claims that statement (2) is **false**. It is so, because the apparent motion of the stars is due to the absolute rotation of the Earth.

Nowadays no respectable physicist holds as true statement (2) despite that relativity theory **dictates** that both assertions are legitimate. So if the Earth's rotation represents absolute motion, are we impelled to think that Einstein was wrong? The answer to this important question does not merely reside on the mathematical formulation of the theory but also on our physical understanding; it is a physical matter that demands deeper investigation.

2. Absolute vs relative motion

Our previous discussion inevitably lead us to a long debate about relative and absolute motion, an issue that has not been settled despite that some have claimed that relativity killed absolutism. This is not so, absolutism was never knocked out but rather disdained by most theoreticians due to its opposition to Einstein's relativity. The great Steven Weinberg acknowledged that physicists do not know how to solve this problem, though it seems that the solution might be in between Newton and Einstein (5).

3. Galiean transformations

During my investigations on this subject I found a notable inconsistency in the literature. In the Principia Mathematica is clearly understood that Newton composed the first book to let the world know that relative motion and absolute motion can be distinguished by forces (that is why he put forward the bucket experiment). The famous scholium on relative and absolute motion clearly establishes that his theory is based on the existence of a universal reference system, space itself. He recognized that perhaps it would be impossible to detect it, nevertheless he assumed its existence for the sake of physical consistency in his theory. One of the consequences of this principle is the derivation, that Newton himself performed, of Galilean relativity which today we mathematically express (for motion in one direction without rotations) in terms of Galilean transformations:

$$x' = x - vt; \tag{1}$$
$$t' = t. \tag{2}$$

Here the prime variables belong to the system moving at speed v relative to the unprimed system. Indeed, the celebrated law of inertia assumes that the state of motion of a body, either at rest or in motion, is relative to the universal system and when a force is applied to that body both its absolute and relative motions can be affected. So, in such frame mechanical laws hold true as well as in any other inertial frame moving relative to the former. This formulation does not exclude the universal system but instead embraces all inertial systems. The inconsistency appeared in the famous book of Mechanics authored by L. D. Landau and E. M. Lifshitz (6). In section 3 Galileo's relativity principle in chapter 1, the authors lecture the following:

> The complete mechanical equivalence of the infinity of such [inertial] frames shows also that there is no "absolute" frame of reference which should be preferred to other frames.

This statement evidently contradicts the heart of Newton's theory. Galilean transformations per se do not discriminate among inertial systems, strictly speaking they say nothing on whether there is such frame or not. The authors are thus wrong; if the frame is inertial it should not be excluded. This same argument applies to reference systems in relativity. Lorentz symmetry does not exclude any inertial system.

In connection with this, in an address delivered in Leyden in 1920 Einstein expressed his thoughts on the luminiferous æther, an absolute frame for electromagnetic fields (15):

...The ether does not exist at all. The electromagnetic fields are not states of a medium, and are not bound down to any bearer, but they are independent realities which are not reducible to anything else, exactly like the atoms of ponderable matter. This conception suggests itself the more readily as, according to Lorentz's theory, electromagnetic radiation, like ponderable matter, brings impulse and energy with it, and as, according to the special theory of relativity, both matter and radiation are but special forms of distributed energy, ponderable mass losing its isolation and appearing as a special form of energy. More careful reflection teaches us, however, that the special theory of relativity does not compel us to deny æther...

Lorentz, Poincaré and later Paul Dirac in 1951, John Bell and others in the 1960s expressed their disagreement with Einstein's view (12–14; 16–18). In 2005 Robert Laughlin, a 1998 Nobel Laureate in physics, expressed his thoughts on this matter (19):

It is ironic that Einstein's most creative work, the general theory of relativity, should boil down to conceptualizing space as a medium when his original premise [in special relativity] was that no such medium existed [..] The word 'æther' has extremely negative connotations in theoretical physics because of its past association with opposition to relativity. This is unfortunate because, stripped of these connotations, it rather nicely captures the way most physicists actually think about the vacuum. . . . Relativity actually says nothing about the existence or nonexistence of matter pervading the universe, only that any such matter must have relativistic symmetry. [..] It turns out that such matter exists. About the time relativity was becoming accepted, studies of radioactivity began showing that the empty vacuum of space had spectroscopic structure similar to that of ordinary quantum solids and fluids. Subsequent studies with large particle accelerators have now led us to understand that space is more like a piece of window glass than ideal Newtonian emptiness. It is filled with 'stuff' that is normally transparent but can be made visible by hitting it sufficiently hard to knock out a part. The modern concept of the vacuum of space, confirmed every day by experiment, is a relativistic æther. But we do not call it this because it is taboo.

From the previous discussion we realize that mathematical beauty is not enough to tell the whole story, and to achieve a solid knowledge we should work out a physical understanding. The history of physics has shown that physical understanding is crucial to make headway in this field; otherwise we might continue lost in math and measurements.

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