A convergence of arguments from multiple disciplines—pure mathematics, cognitive development psychology, modern physics—leads to the conclusion that reality is always simultaneously analog and digital. Real numbers are reality's analog numbers. And a branch of mathematics called p-adic numbers are reality's digital numbers. There is even a branch of mathematics, called adelic mathematics, that insists that we always simultaneously use both real and p-adic numbers in order to get a complete understanding of reality.

The Pure Mathematics Argument

The pure mathematics argument for reality being simultaneously analog and digital is called Ostrowski's theorem. This is a theorem presented early in a course on p-adic mathematics, which is a form of discrete mathematics typically taught at the graduate level.

Ostrowski's theorem states that there are two and only two completions of the rational numbers for which the rational numbers form a dense subfield. One of these completions is the field of real numbers, and the other is the field of p-adic numbers for every prime number p. [1]

Clearly, we need to discuss what p-adic numbers are. But to get us motivated, let's be explicit about how exciting Ostrowski's theorem is.

For mathematicians, there is only a trivial distinction among a wide variety of digital numbers. In math-speak, there is a one-to-one correspondence, for example, between the counting numbers (1, 2, 3, ..., also known as the natural numbers) and the integers (..., -3, -2, -1, 0, 1, 2, 3, ..., negative and positive, including zero). Now this seems a bit weird, since there seem to be more integers than counting numbers. But for mathematicians, there are the same number of counting numbers as integers. It's an infinite number, which mathematicians call aleph-zero. There are aleph-zero counting numbers, and there are also aleph-zero integers.

It turns out that there are also aleph-zero rational numbers. Rational numbers include all the integers, plus any numbers (such as fractions) that can be written as ratios of integers. There is a one-to-one correspondence between the set of rational numbers and the set of integers.

But this exhausts aleph-zero. If we want to also include numbers that cannot be formed as the ratio of integers, this will bring us to a larger set of numbers, an infinite set of numbers that has more numbers in it than aleph-zero. We have all been taught that this larger set of numbers, which includes numbers such as the square root of 2 which cannot be written as the ratio of integers, is called the set of real numbers. The cardinality of the real numbers is greater than the cardinality of rational numbers. There are more real numbers than there are rational numbers.

The set of rational numbers is incomplete. The set of real numbers completes the set of rational numbers. We are very comfortable with real numbers, even if we don't have a deep theoretical understanding of their theoretical basis.

Real numbers seem so real.

But Ostrowski's theorem tells us that we shouldn't avoid the second alternative for completing the rational numbers. There are, after all, only two alternatives. If we hadn't been so p-adic-averse, we probably wouldn't have had such a hard time seeing how obvious it is that reality is always simultaneously analog and digital. This is why Ostrowski's theorem is so exciting.

Creating Parity Between Real and P-Adic Numbers

Before we migrate to a disucssion of what p-adic numbers are, a plea for parity needs to be made. It is only fair that we allow p-adic numbers into our lives under the same conditions that we allow real numbers into our lives.

Specifically, few of us have mastered the deeply theoretical underlying concepts of what a real number is. We are quite comfortable using real numbers, accepting real numbers, understanding real numbers, even if we don't understand the theory of Dedekind cuts that justifies and deeply explains real numbers.

So be generous in accepting p-adic numbers into your life. Allow yourself to accept a general understanding of p-adic numbers even if you learn nothing about their deep theoretical basis. When we get to our psychology discussion later on, we will see that it is p-adic numbers that are our earliest numbers. From infancy, we learn about the world p-adically.

Picasso noted that it took him a lifetime to learn how to paint like a child. [2] So revive your inner child to absorb the next section.

P-Adic Mathematics: The Mathematics of Enclosure

P-adic mathematics is an astoundingly efficient way to record hierarchical structure. Said another way, p-adic mathematics is the mathematics of enclosure.

A p-adic number tells us how many circles are enclosing how many circles are enclosing how many circles. The number of digits in a p-adic number tells us how many levels of enclosure there are. The p-adic number 7 means a single enclosure of 7 items. The p-adic number 37 means each of the original 7 items encloses 3 items. (In this sense, p-adic numbers are written backwards: the digit on the right is the largest enclosure.) The p-adic number 537 means each of the 3 items encloses 5 items.

So p-adic arithmetic maps hierarchical structure. Thinking p-adically means thinking about hierarchy, about what's enclosing what. Unlike real mathematics, p-adic mathematics is not measured. In math-speak, real mathematics is metric, but p-adic mathematics is ultrametric. P-adic mathematics notes only levels of hierarchy. In p-adic mathematics, two items at the same level of hierarchy are the same size as each other. P-adically, no measurement distinguishes items within the same level of hierarchy. [3]

Even if this briefest of introductions to p-adic numbers has succeeded in giving you an intuition for p-adic numbers, you might still be left scratching your head wondering why you should care. So just a reminder of what we've established so far: There are two and only two complete ways to look at the world mathematically. One (real mathematics) is analog. The other (p-adic mathematics) is digital. Adelic mathematics is the mathematics that insists on always recording both the real representation and the p-adic representation for every prime number p.

Cognitive Development

Jean Piaget was a Swiss epistemologist whose observations of cognitive development remain today at the core of psychology's understanding of how, from infancy onward, children develop their adult ways of thinking, their formal operations of cognition.

Piaget never heard of p-adic mathematics. He also has been criticized for developing his theories in a nonscientific manner, not using the scientific methods of experimentation, but rather developing his theories from watching children develop from infancy onward, including his own children, no less.

Piaget considered himself an epistemologist first, with his primary interest being to draw conclusions about the nature of knowledge by observing how children's cognition develops.

Piaget observed that our earliest concepts are what he called topological, not metric. Our earliest concepts are those involving enclosure. We first learn about the world by learning about boundaries and relationships. It is only later in our cognitive development that we begin to focus on measurement of the concrete world. [4]

Piaget's specific methodology by which cognition develops is by the dual processes of assimilation and accommodation. Assimilation means that we take an observation or experience and add it to our existing conceptual structure, enhancing the structure that was already in place, but not transforming that structure into a new conceptual structure. Accommodation, on the other hand, means that the observation or experience has been so novel or discordant that we cannot absorb it into our existing conceptual structure. Instead, we must modify our conceptual structure, accommodating our world view to incorporate the latest novel, discordant event.

P-adic mathematics straightforwardly models Piaget's processes of assimilation and accommodation. Assimilation means one of the digits of the prior p-adic number has gotten larger, but we still have the same number of digits. Accommodation in its simplest form means that we have one more digit, one more level of hierarchy. Accomodation can also take other forms: A segment of the prior p-adic number can be preserved but encapsulated within a different enclosure. Or levels of enclosure can collapse into a larger single enclosure.

The point is that Piaget's model of cognition is a p-adic model. The mind interfaces with the real world, recording the world's real, metric, analog structure, and also making sense of this same world by digitally recording its structure, using p-adic mathematics, the mathematics of enclosure.

Remember that Piaget considered himself an epistemologist first, drawing conclusions about the nature of knowledge from his observations of human cognitive development. There is one more astounding correspondence that Piaget's epistemology gives us. This has to do with how Piaget viewed what guides the continual back-and-forth interplay between the dual processes of assimilation and accommodation.

For Piaget, this interplay is inherently unstable. An observation can trigger assimilation or it can trigger accommodation. But imediately, the cognitive structure is disrupted by another observation.

Piaget observed that the driving force in cognition is a drive for what he called equilibration, a momentary balance, a momentary resolution at a higher level, which was soon to again be disrupted, to again become unbalanced, only this time at a higher level of understanding. [5]

For Piaget, this drive to a higher level of equilibration is the driving force not only of cognition but also of life. This concept corresponds remarkably to the driving principle we will find in our adelic theory of physics, topological geometrodynamics. We'll largely postpone this discussion until we establish more physics, but for now we'll say that the principle from physics is the negentropy maximization principle, the tendency to select the route that maximizes information content (minimizes entropy).

Pure mathematics, cognitive development psychology, and (next) adelic physics all lead us to always understand reality as simultaneously analog and digital.

Adelic Physics

To gain an appreciation for what adelic physics can do for us, consider the Planck length, physics' smallest distance. Using real mathematics, all physicists agree that this is a lower bound on measurement. There can be no length smaller than the Planck length.

But p-adic mathematics, adelic mathematics' other half, is ultrametric. The Planck length is not relevant. Length is a metric concept, deriving from real mathematics' norm being based on the absolute value calculation. P-adic mathematics' norm is not based on the absolute value; it is based on the position of the last digit to the right in the p-adic number. The more digits to the right that a p-adic number has, the larger is the p-adic number.

So arguably, the Planck length is not a p-adic restriction. This is arguable rather than certain, because it's possible that a theory of adelic physics might somehow bring the Planck length's real restriction into the ultrametric calculations. But clearly this is not a logical necessity. The p-adic analysis is independently created, and certainly a well-developed system of adelic physics could operate with lengths that are unmeasurable but nevertheless exist. Equally clearly, this is a very obvious possibility for explaining dark matter—not measured and not measurable, but existing nevertheless.

In a previously published work [6], the author discusses an extensive search of the physics literature to find theories of physics that reject mainstream physics' approach toward string theory and toward reductionist solutions, such as the Higgs mechanism, to the question of quantum gravity. It is a booklength discussion to present the details of this search process and to present the criteria by which the several hundred largly hidden radical theories may be judged. But generally speaking, the best theory is the one that presents a coordinated framework for reconciling relativity with quantum physics, for explaining phenomena of mainstream physics that mainstream models do not satisfactorily explain (for example, dark matter, dark energy, and entanglement), and for proposing a physics-based model of the mind.

Clearly, the hidden radical theories of *New Physics and the Mind* are not for everyone. In particular, most mainstream physicists reject out of hand a role for physics in explaining the mind. However, one of these theories—the theory topological geometrodynamics (TGD, developed by Finnish physicist Matti Pitkänen)—stands out as the number one hidden radical theory of new physics and the mind.

TGD is extensively developed over thousands of pages [7] and therefore has a scope much larger than can be presented in this paper. TGD is a theory of particle physics, cosmology, quantum biology, and consciousness. It is based on adelic mathematics, incorporating both continuous analog real mathematics and discrete digital p-adic mathematics. TGD is the detailed implementation of the concept that reality is always simultaneously analog and digital.

The key to TGD's successful adelic model is a reconfiguration of the physical model of spacetime. This reconfiguration then becomes the foundation for TGD's applications to physics, biophysics, and the mind. The reconfigured spacetime is one that permits easy application of p-adic mathematics, because TGD's spacetime is hierarchically orgainized as enclosings of standard physics' 4-dimensional Minkowski spacetime M^4_+ . Minkowski spacetime is 4-dimensional, incorporating time, and its axes are not quite perpendicular, because they are adjusted for special relativity. The plus sign means both space and time are measured from the big bang.

In TGD, spacetime is not simply 4-dimensional M^4_+ spacetime, with objects situated at particular coordinates within an M^4_+ spacetime that surrounds the object. Instead, each TGD object, starting at the elementary particles, is its own macroscopic and macrotemporal event in bounded M^4_+ spacetime. An atom also exists as its own bounded M^4_+ spacetime, linked to the elementary particles that it incorporates by a microscopic and microtemporal wormhole in 4-space. This wormhole has the simplest possible 4-dimensional structure, which can be visualized as two perpendicular circles, but don't forget that they're perpendicular in 4-space not 3-space. This microscopic, microtemporal 4-space is labeled CP_2 space, the complex projective space of two complex dimensions.

This hierarchical structure builds up to the molecule and beyond, all the way to the universe. Thus TGD spacetime is the 8-dimensional space $M^4_+ \times CP_2$, using real mathematics (extended, of course, to complex mathematics, since both Minkowski spacetime and complex projective space include imaginary numbers). It is also quite easy to see how this TGD spacetime is straightforwardly mapped p-adically, since p-adic mathematics is the mathematics of enclosure, and since the 8-space's configuration is all about levels of enclosings and enclosures.

Thus TGD provides a fully developed model of reality that is always both analog and digital. This dual analog/digital nature is not an incidental feature of TGD. It is the heart of TGD and what gives TGD its power to solve modern physics' mysteries where standard physics has failed.

Digital Mind Math

Although TGD is a much larger topic than can possibly be incorporated within this brief paper, we close by introducing TGD's application to consciousness and the mind. It is through the adelic approach to reality—the approach that always sees reality as simultaneously both analog and digital—that TGD is able to create a model of how we think, using both real and p-adic mathematics.

A number of theorists have observed that p-adic mathematics is the natural mathematics of cognition. For example, mathematician and physicist Andrei Khrennikov has written extensively about the use of p-adic mathematics in physics and in cognitive modeling. Khrennikov recently published some thoughts on modeling cognition in ultrametric mental space using category-encoding balls [8], in Springer's recently introduced journal *p-Adic Numbers, Ultrametric Analysis and Applications*. This has many similarities to Pitkänen's model, which Khrennikov references as independently developed. What Pitkänen's TGD adds is its fully developed, entirely specific mathematical model, based on applying adelic mathematics to $M^4_+ \times CP_2$ space.

In TGD, cognition proceeds according to the core quantum process that mainstream physics recognizes as the quantum jump. Note that this is not the construct that some theorists have proposed, in which the mind, through the brain, taps into physics at the traditionally understood quantum level. Rather, the TGD model is that the core quantum process takes place at all levels, from elementary particles through the universe, within physics, biophysics, and cognition.

In the TGD model of cognition, thinking starts at the last thought, then from that starting point develops a configuration space of every possible next thought, selecting as the next thought to experience one that tends toward negentropy maximization (that is, maximization of the information content). Thoughts are hierarchies of bounded 4-dimensional sequences: a person interacting with another person enclosed in a room in a building at a certain location, for example. The full experience of this thought is real, but the ability to remember and analyze thoughts and hypothesize about next possible thoughts is such a mass of real data that thinking can proceed only because p-adic mathematics allows such efficient capturing of the hierarchies of information.

Reality is always simultaneously both analog and digital.

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