Incompleteness Implications for a Theory of Everything

By Donald G. Palmer

Abstract:

Built upon formal mathematical systems, physical models are subject to the limitations of those systems. Gödel's Incompleteness theorems state these formal systems are incomplete, and therefore so are the physical models built upon them. Expanding the axioms of the formal mathematical system can resolve the incompleteness of the original system, even though the expanded system remains incomplete. Given these two levels, of physical models built upon mathematical tools, we should consider whether we have adequate mathematical tools to describe physical reality and whether we have an accurate model of reality. Can we find aspects of reality that expand our models and are there expanded mathematical tools for us to build new models upon?

Part I – Incompleteness and Models

I take, as my starting point, Rebecca Goldstein's tribute to Kurt Gödel "Incompleteness: The Proof and Paradox of Kurt Gödel" [5]. Dr. Goldstein provides a metamathematical and philosophical perspective of Gödel's Incompleteness theorems indirectly asking how these concepts impact human knowledge and understanding. She gives evidence of both Gödel and Einstein's belief in a world (abstract and physical respectively) beyond our senses, towards which we strive to apprehend. They rejected the prevailing positivist perspective which both men fought against their entire lives. Each held there is more to the universe than what we can perceive and measure and we should continue to strive toward a full understanding of the universe 'out there' – abstract or physical.

Dr. Goldstein describes Gödel as having a strong Platonic perspective. Let us consider Plato's allegory of reality: What we believe to be reality are only shadows of actual events displayed on the wall of the cave we live in. An updated expansion of this Platonic perspective could be that we use human devised tools to model the shadows on the wall of the cave. Our scientific models are, therefore, attempts to model the reality we perceive, the shadows on the cave wall, rather than the true objects of reality. (Einstein might have considered quantum mechanics as a fuzzy model of shadows on the wall.)

Scientific models are couched in the language of mathematics, which consists of formal axiomatic systems. Any mathematical model will be, at the very least, constrained by the limitations of the formal axiomatic system it is couched in. All formal systems (capable of at least producing basic arithmetic) are limited by Gödel's Incompleteness theorems. They will never be able to prove all statements within the system. The equivalent statement for a scientific model is that all equations of the model are not derivable within the model. The equations can be stated and can be shown to fit our perceptions of reality, but they are not all derivable from the foundational postulates of the model.

While science might expect such models to be logically consistent and hope they are complete, Gödel's Incompleteness theorems have removed the possibility that they are complete, within themselves.

There is an upside to all this, however: Gödel's theorems imply that formal systems (and models based upon them) can be re-stated in a more expanded formal system in which the incomplete statements of the original system are provable in the expanded one. Therefore, while any model we can formally describe will be incomplete, we can always expand our formal system, re-stating our model within that expanded system. This leads to the conclusion that science will continually be incomplete and we will be required to continually expand our models to explain the reality we perceive. As a sort-of corollary: There will never be a final 'Theory of Everything' built upon any mathematical or logically formal foundation. This means there should be work for scientists far into the future.

This is how Gödel's incompleteness theorems could impact science – by indicating we must expand our postulates of the world we perceive in order to produce expanded models of the universe. And there might not be an end to this process.

Part II – The Need to Expand Our Models

It should also be easily understood that models never precisely describe what we see and experience – those shadows on the wall of the cave. Models always have limitations and our measuring tools always have limits of accuracy. We should never expect to have a model that is 100% accurate, down to whatever level of accuracy we could conceivably reach. So we are faced with theories that model our perceptions of the world, (hopefully) are consistent, are only accurate to a point, yet are incomplete. Note that this situation would remain even if we believed we were perceiving the true reality and not the shadows on the cave wall – our models remain consistent, only accurate to a point, and yet incomplete.

It is very easy to get caught believing in a model, since it can be so consistent and appear exceedingly accurate. However, since it will be incomplete (and not entirely accurate), we need to anticipate expanding the model in some way – or we need to expand the formal system underlying the model (or maybe both). Note that we have two levels to this situation – the level of the model (Einstein) and the level of the formal system (Gödel) used to specify the model. Expanding the model might mean making a fresh connection to reality, to something the model does not address or 'get right'. Expanding the underlying formal system might mean altering or adding a foundational abstract concept the formal system does not prove (an axiom). The former can be an aspect of reality such as the constancy of the speed of light giving Special Relativity, while the latter is more like changing Euclid's 5th postulate to identify consistent multi-dimensional spaces. Note that a change in the underlying formal system should directly impact any model built on that system. Conversely, making a change in the model might require some change to basic foundational axioms of the underlying formal system. This situation implies a change in the model could require changes in the underlying formal tools. This could, in turn (in a sort-of feed-back loop), impact the capabilities of the model expanding what the model can encompass – even expanding what the model can measure.

There is no requirement that the expansion discredits the model we have caught ourselves believing in, since it expands the scope of the model rather than replaces it. If the model expansion also requires a change to the underlying formal system, then a re-evaluation of the model (of the interpretation of reality via an expanded mathematical system) might be in order. Einstein started with Special Relativity, which applies to constant motion. He understood the need to expand this model to account for acceleration. In his search he realized the need to find appropriate mathematical tools for the expanded model. Euclid's 5th postulate had already been expanded and the tools were already available. These tools had not invalidated 3-D physical space (which would have upset all of physics), instead they described new spaces beyond Euclidean space (which were initially considered purely abstract having no reality to them) along with new mathematical tools. These abstract tools, in turn, allowed Einstein to complete his model of General Relativity and made the supposedly abstract spaces real.

In this example, the underlying mathematical tools had already been discovered. What if we identified such a change to our model that would require a change in the underlying formal systems, which does not yet exist? This situation would require expansion of our physical models and our underlying mathematical tools. The change in the underlying mathematical tools would, in turn, require a re-evaluation of our physical models.

As a starting point, we should first consider: What is our universe? We can respond that it is everything we perceive, however we perceive it. Then we might ask how do we understand the universe (there being the assumption that we even can). This is traditionally done through 'science' and involves modeling the universe using a set of measurements and modeling tools. Then we might step back and ask a couple open questions. Two, in particular are:

1. Do we have the proper tools to measure and model this universe?
2. Do we have the correct, or at least adequate, model of the universe?

Applying some simple thought to these two, it should be apparent that the second is dependent upon the first. For if the answer to the first is 'no', then the answer to the second should automatically also be 'no' (although a partially adequate model might still be possible). However, if the answer to the second is 'no', then we cannot really give an answer to the first.

Considering other possibilities, if the answer to the second is 'yes', then we presume the answer to the first is also 'yes', although this is not a direct relationship in the way a 'no' to the first forces a 'no' to the second. For we could have an apparently adequate model, yet unknowingly are using inadequate tools, thus not providing a correct model. This could leave us in the unforeseen situation of believing we have a proper model, without the tools to correctly identify what is not correct – a sort of "we don't know what we don't know" situation. Finally, if the answer to the first is 'yes', then at least we should be able to develop a correct model of the universe, although this response has other dependencies upon the model we develop, that would determine a 'yes' or 'no' response to the second.

Given the unforeseen situation, we need to address it by considering that we may not have the proper tools to model reality. In this case our model might appear correct and accurate, but that would be because we are lacking the proper tools to know otherwise. How might we know we could say ‘yes’ to the first? This is a difficult question to answer and, if our tools include formal systems, we might find it is unprovable from within our models. This would imply we need to step outside of our current model in order to decide if we have adequate tools for modeling the reality we perceive.

What might be an indication that we do not have adequate tools to model the universe? Today we think we have a very good model, accurate to many decimal places. As noted above, this does not guarantee we have the proper tools, since we could be deceived by the limitations of our tools and not be able to measure something our tools cannot. If our thinking is that ‘what is important is only what we measure’, then we will be caught in our own measurement tautology and completely miss aspects of the universe that are unavailable to our tools (especially mathematical). This trap is due to our being concerned only with what we can measure since what we can measure is limited by our tools. Note that this doesn’t mean we don’t perceive these aspects we cannot measure, only that our inability to measure them causes us to exclude them from our models. Such a situation is a potentially severe limitation of the positivist and Copenhagen positions.

We need to take that step back to consider wider questions, in particular: What do we perceive as our universe that we are currently unable to adequately measure?

Part III – Indications that Our Models are Inadequate

We should note how the perception of our universe, and what is in our universe, has expanded tremendously over the past 5-6 centuries. We now perceive solar systems, star clusters, galaxies and galaxy clusters – in one direction. And tissue fibers, cells, proteins, molecules, atoms and sub-atomic particles in the other. These two directions are considered a continuum of the universe and the extent of this continuum was not apparent until proper tools were developed to perceive up and down this continuum (the telescope and the microscope were the first such physical tools). It has been by investigating objects along this continuum that we have come to our current models of the universe. And, along with the physical tools, we have developed new mathematical tools, such as the calculus, vector analysis, plus real and even ‘imaginary’ numbers.

However, even using these new mathematical tools, we have not modeled the universe in such a way as to include this continuum. We have really developed many models of the universe, each tending to apply to this or that section along the continuum. We have cellular biology and molecular biology and atomic physics. Then we have meteorology, ecology and astronomy of solar systems and astronomy of galaxy clusters. So we have many models and many align along sections of this continuum that we have only recently perceived the scope of.

There have been a number of books, videos, and websites demonstrating even taking a trip along this continuum. From Kees Boeke’s *Cosmic View: The Universe in Forty Jumps* [28] to the Canadian NFB’s *Cosmic Zoom* [29], to the Huang brothers’ *The Scale of the Universe 2* [32], we can gain some perspective of this continuum, even of traveling along it.

Peter Watson, in his book '*Convergence: The Idea at the Heart of Science*' [27] provides extensive examples of science converging along this continuum. The author does not, unfortunately, appear to perceive this continuum as part of the convergence, ending up resorting to reductionism in his conclusion. The hard science part of the book does provide striking examples of disciplines spreading upward and downward along the continuum of scale.

Consider that identifying any object in the universe first requires us to define where, on this continuum, the object lies (see Fig. 1). An atom or a galaxy sit at very different places on the continuum. It is only after we have identified the object, and therefore the position of that object along this continuum of scale, that we can then consider a coordinate system to measure the position of this object relative of other objects at a similar scale. Scientific disciplines appear to limit themselves to a particular part of the continuum and even attempt to describe actions at all levels of the continuum through a single level – the smallest level. Why might this be – why have we been unable to develop a model of the universe that accounts for this continuum which we perceive as a basic aspect of reality?

Particle physics does not progress upward into molecular chemistry and microbiology in a direct way. We use mathematical tools that explicitly average out actions at one level (and thus leave out information) in order to specify actions at a larger (or 'higher') level – that of statistics. While another mathematical tool, probability, explicitly admits individual actions are not accessible to analysis (the law of large numbers). Simply by the use of these tools we explicitly are not accounting for all information in a system. This is a consequence of the mathematical tools, not of the models built using them.

Some people have suggested that the character of these tools, probability in particular, are actually part of reality. Imparting characteristics of the mathematical tools to reality is like believing the characteristics of a number are defined by the symbols used to represent that number. So '5' must be similar to 'S' or '11' to parallel line segments. Or .333… + .666… must equal .999… which is in conflict with different symbols for the same numbers which give a different result: 1/3 + 2/3 = 3/3 = 1.000… We should not confuse the characteristics of the tools with that which we model using the tools. We should also be cognizant of the limitation of those tools. This does not mean our current tools are not useful, only that they carry limitations that must be accounted for as limitations of models built with the tools.

Our lack of ability to model across the continuum, our attempts to make characteristics of our tools into reality, and our attempts to model all of reality from one level of the continuum should indicate we have hit limits to our knowledge and tools and need to address those limits. Our inability to model across this technologically perceived continuum suggests we need to expand our models (so 'no' to the second question above). The use of such tools as statistics and probability, because of the underlying mathematical limitations of the tools, essentially means we must answer 'no' to the first question above. Our attempts to model all levels from a single level indicates we have neither the appropriate measuring tools to cross the continuum nor the appropriate mathematical tools to model the reality we perceive. We have hit major limitations at both the physical model level and the mathematical tool level.

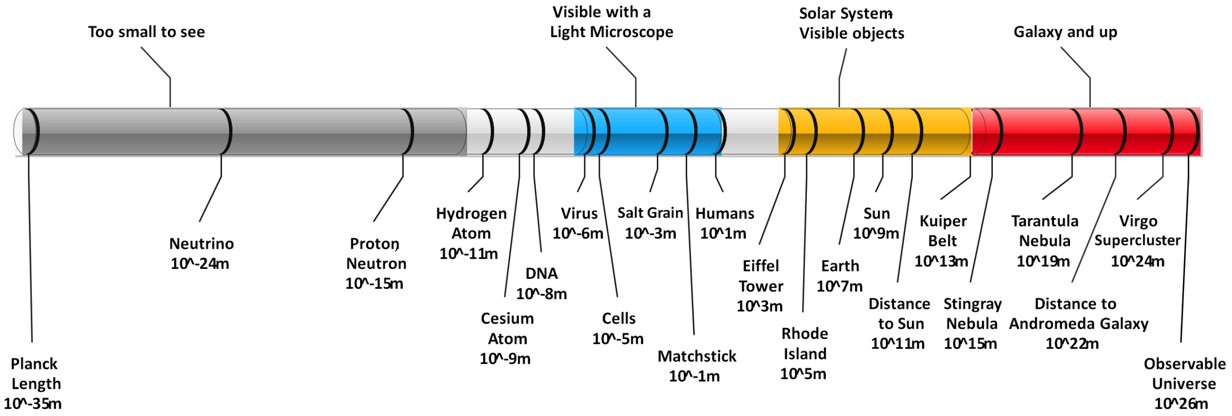

Figure 1: The continuum of Scale

Part IV – Expanding Our Models

Only recently has science begun to attempt to model across sections of this continuum – multi-scale modeling being such an example. However these attempts also are unable to cross up and down the continuum. In particular is the concern of the micro-universe not fitting well with the macro-universe (eg. Schrödinger's Cat, tools to derive an equation that directly connects the actions of genes with the development of our eye). There is also the reductionist single level conclusion that all causes can be understood entirely from the smallest level (eg. String Theory as a Theory of Everything) and therefore there cannot be causes from the larger to the smaller (have you even watched a large scale particle physicist run an experiment on tiny particles?). These concerns (strongly) suggest we do not have an appropriate understanding of this scale continuum and have not presented a model that adequately accounts for it.

What if we want to locate an atom in the pen on the table on the second floor of a building at 8th and Pine against the location of the surface of the sun. Simply identifying 3 positional measurements will not quantify the relative differences in scale of these two physical objects. A fourth measurement is required to locate an object in the expanded universe we are now able to perceive – with our new technological perception tools. Our models do not include such a 4th positional measure.

We perceive these differences in scale, so why do we not measure it? If we lack the appropriate tools, especially mathematical tools, to measure this aspect we perceive, then we will not be able to add it to our models and the models will be missing crucial aspects of reality. Here we have identified something we perceive that is missing from our models and something for which we do not have adequate tools to measure. At a time when the reigning philosophy is to only consider what we can measure, we have hit that unforeseen situation, which might be better stated as 'we don't know what we cannot measure'.

We could, therefore, expand our model of the universe by incorporating the unaddressed aspect of continuum of scale to our model. Note that this doesn't change what we perceive of the universe, just how we model it. This change in the model will, however, require a re-examination of how we interpret items and actions of the world, using this new model.

Just for fun, let us consider a few possible results of including this continuum.

- As with our other continuums, an object spreads across some 'length' of this continuum. So the sun is at some scale, say 1.39 x 10^6 kilometers, and also spreads across scale – all the way down to compressed atoms.
- When we touch a finger to a pane of glass we should believe our eyes and senses that show us touching the glass. Touching a pane of glass would occur as actions at multiple levels and combining these levels involves a 4-dimensional model with touching represented by a surface that includes all these levels of scale with the objects at each level.
- Consider that one unit along this dimension of scale might be 10 meters, while two units would be 10^2 = 100 meters and three units would be 10^3 = 1000 meters. So 'length' along this dimension of scale would appear to us as a power series length – at least relative to our traditional measurements. This does not match with our traditional concept of 'length' and is likely why we cannot properly measure it.
- A constant velocity, in this scale continuum direction, would appear to us as a constant acceleration. An object moving only in this direction, appearing motionless in our traditional three, would appear to be accelerating at a constant rate – yet not moving. We would normally say there is a constant force on the object – which we usually call 'gravity'.
- Movement of the larger objects in our universe in this continuum direction might cause us to think (in our old model) that there is extra energy in the universe. This would be unperceived energy, indirectly understood. However the new model could explain it as movement of the larger objects along this scale continuum.

We now have identified something we perceive of the universe that we either cannot, or have great difficulty, modeling. In particular, we appear to have great difficulty measuring across the continuum of scale, which has led us to attempt to explain all reality from a single (smallest) level. This situation is precisely the type of concern that should question whether we have adequate tools for modeling the universe.

While 'all we know' cannot be formalized, we can continually expand our formal systems to capture more of what we know, which allows us to expand what we know. That is the implication of Gödel's theorems to human knowledge. There is a positive feed-back loop of learning, formally modeling that learning, then using the models to expand our learning – that has no limit. This just might be the future of science.

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